How was Midterm 2 for you?

A) *Way* too hard - no fair!

B) Hard, but fair

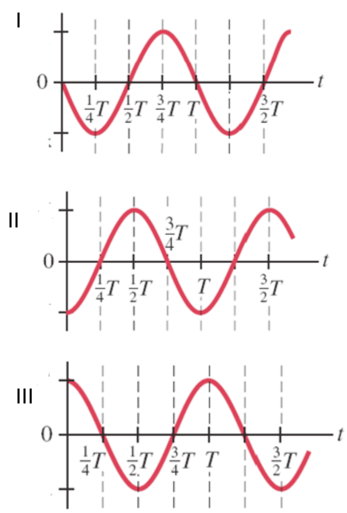
C) Seemed reasonable.

D) Easy/fair enough, thanks!

E) Almost too easy, really should make it harder next time!

Quick Review after the break

If a spring is pulled away its equilibrium point to x>0 and released from rest, the plots shown below correspond to



A) I: velocity, II:position, III:acceleration

B) I: position, II:velocity, III:acceleration

C) I: acceleration, II:velocity, III:position

D) I: velocity, II:acceleration, III:position

E) I do not have enough information

Important concepts

Angular frequency $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

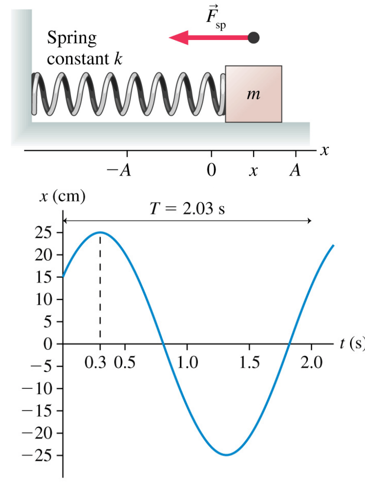
Frequency: $f = \frac{1}{T}$

Position: $x(t) = A\cos(\omega t + \delta)$

Velocity: $v(t) = -A\omega\sin(\omega t + \delta)$

Acceleration: $a(t) = -\omega^2 x(t)$

Energy: $E = -\frac{mv^2}{2} + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$



No friction implies conservation of mechanical energy

Damped Oscillations

In most physical situations, there are nonconservative forces of some sort, which will tend to decrease the amplitude of the oscillation. In many cases (e.g. viscous flow) the damping force is proportional to the speed:

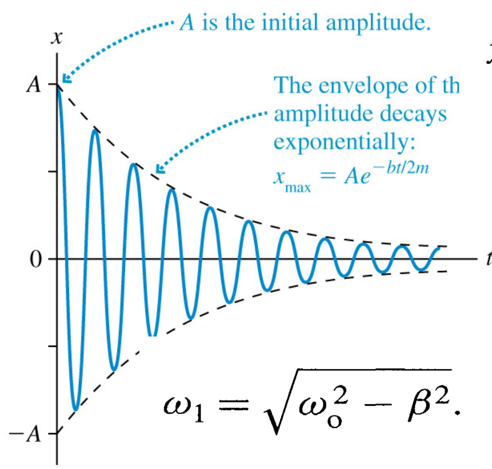
$$\boxed{\vec{\mathbf{F}} = -b\vec{\mathbf{v}}} \qquad 2\beta = b/m$$

$$F = m\ddot{x} = -kx - b\dot{x}$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0{}^2 x = 0$$

Underdamped Oscillations

An *underdamped oscillation* *with b<w$_0$*:



$$x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta).$$

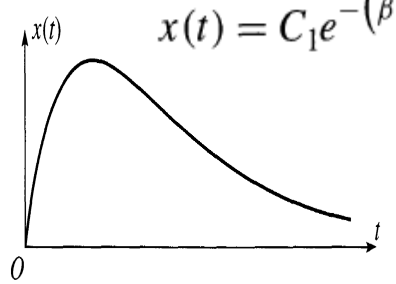
$$\beta < \omega_o$$

$$\omega_1 = \sqrt{\omega_o^2 - \beta^2}.$$

Note that damping ***reduces*** the oscillation frequency.

Overdamped Oscillations

An *overdamped oscillation* *with b>w$_0$*:



$$x(t) = C_1 e^{-\left(\beta - \sqrt{\beta^2 - \omega_o^2}\right)t} + C_2 e^{-\left(\beta + \sqrt{\beta^2 - \omega_o^2}\right)t}.$$

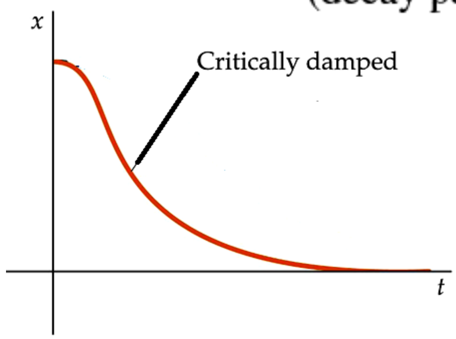
$$\text{(decay parameter)} = \beta - \sqrt{\beta^2 - \omega_o^2}$$

Critically damped Oscillations

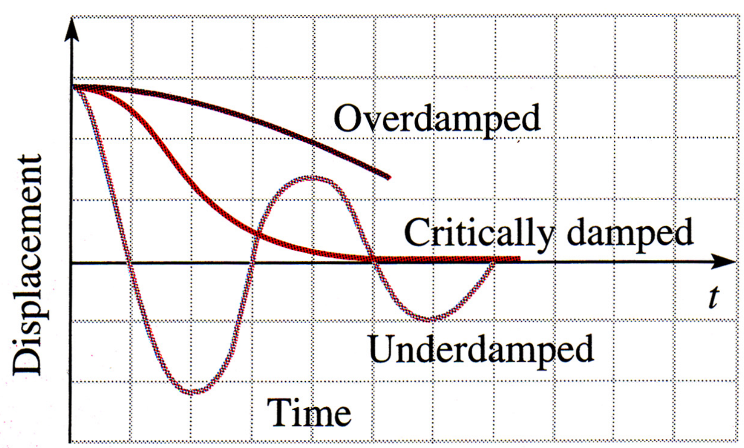
A critically damped *oscillation* *with b=w$_0$*:

$$x(t) = C_1 e^{-\beta t} + C_2 t\, e^{-\beta t}.$$

$$(\text{decay parameter}) = \beta = \omega_0$$

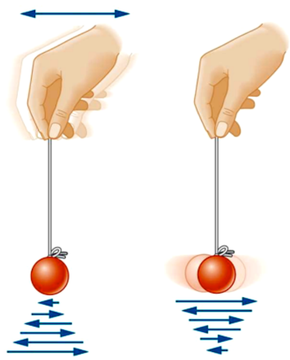


Damped oscillations



This class

Driven Oscillations & Resonance



1. Particular and homogeneous solution
2. Response to periodic driving forces: amplitude and phase.
3. Tutorial
4. Resonance: concept, Q factor

What is the particular solution to the equation
$y''+4y'-12y=5$?

A) $y=e^{5t}$
B) $y=5/12$
C) $y=a5/12$, a determined by initial conditions
D) $y=ae^{2t} +be^{-6t}+5$

Consider the following equation

$$x'' + 16x = 9\sin(5t), \quad x(0)=0,\ x'(0)=0$$

The solution is given by

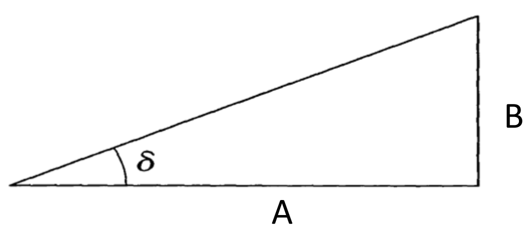
A. $x(t)=c_1 \cos[4t]+ c_2 \sin[4t]- \sin(5t)$

B. $x(t)=5/4 \sin[4t]- \sin(5t)$

C. $x(t)= \cos[4t]- \cos(5t)$

D. I do not know

The phase angle d in this triangle is



A. $\delta = \arctan\left(\frac{B}{A}\right)$ B. $\delta = arc\cot\left(\frac{A}{B}\right)$

C. $\delta = \arctan\left(\frac{A}{B}\right)$ D. More than one is correct

D. None of these are correct

Consider the general solution for an underdamped, driven oscillator:

$$x(t) = C_1 e^{-\beta t} e^{+\sqrt{\beta^2-\omega_0^2}\,t} + C_2 e^{-\beta t} e^{-\sqrt{\beta^2-\omega_0^2}\,t} + A\cos(\omega t - \delta)$$

term A   term B   term C

Which term dominates for large t?

D) Depends on the particular values of the constants

Challenge questions: Which term(s) matters most at small t? Which term "goes away" first?

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Consider the amplitude

$$A^2 = \frac{f_0^{\,2}}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$

In the limit as ω goes to infinity, A

A. Goes to zero
B. Approaches a constant
C. Goes to infinity
D. I don't know!

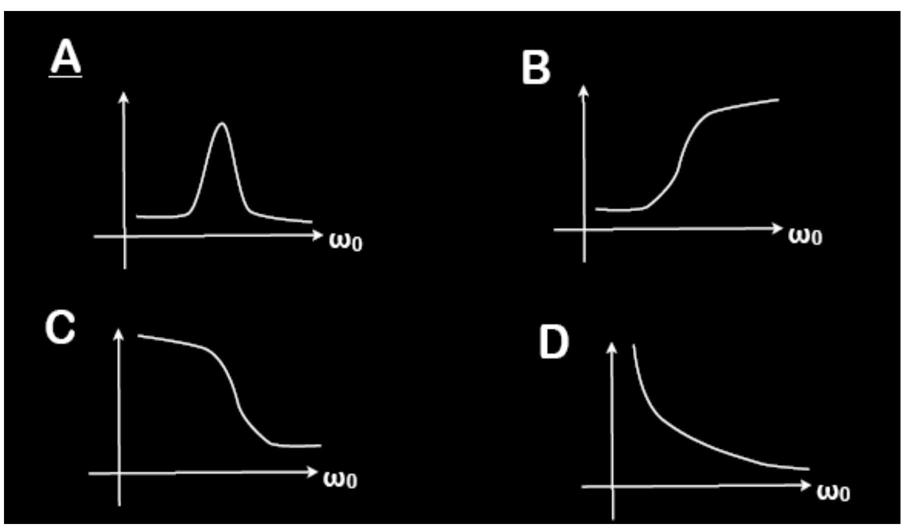
Consider the amplitude

$$A^2 = \frac{f_0^{\ 2}}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$

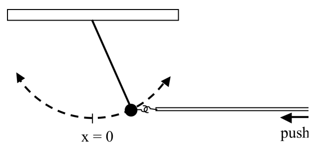
In the limit of no damping, A

A. Goes to zero
B. Approaches a constant
C. Goes to infinity
D. I don't know!

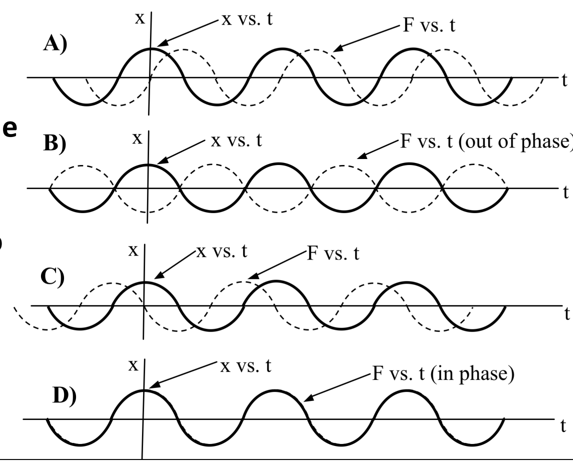
What is the shape of $$A = \frac{1}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2}}$$



A person is pushing and pulling on a pendulum with a long rod. Rightward is the positive direction for both the force on the pendulum and the position x of the pendulum mass

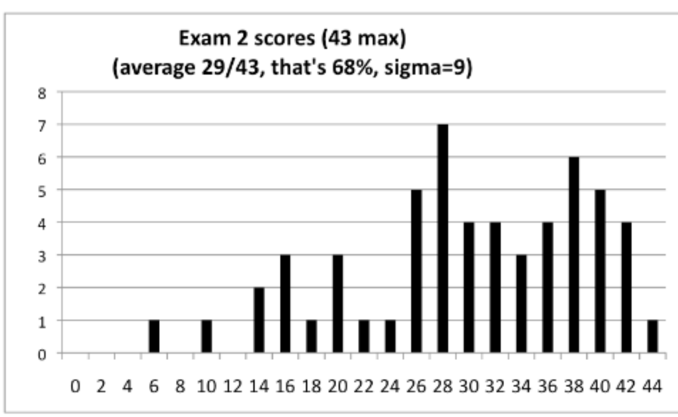


To maximize the amplitude of the pendulum, how should the phase of the driving force be related to the phase of the position of the pendulum?



Exam 2 info:

Here is a histogram of exam 2 scores. Max possible is 43 points



$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

Given the differential equation for an RLC circuit, which quantity is analagous to the damping term in a mechanical oscillator?

A) R, resistance
B) L, inductance
C) C, capacitance

Challenge questions: Which quantity is analogous to the mass

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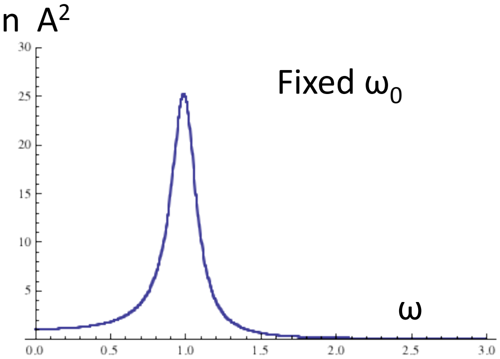
$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

What is ω_0?

A) C
B) 1/C
C) 1/Sqrt[C]
D) 1/LC
E) 1/Sqrt[LC]

20

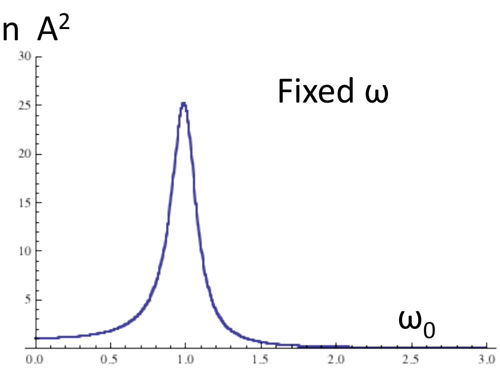
If you have a damped, driven oscillator, and you increase damping, β, (leaving everything else fixed) what happens to the curve shown?



A) It shifts to the LEFT, and the max value increases.
B) It shifts to the LEFT, and the max value decreases.
C) It shifts to the RIGHT, and the max value increases.
D) It shifts to the RIGHT, and the max value decreases.
E) Other/not sure/???

21

If you have a damped, driven oscillator, and you increase damping, β, (leaving everything else fixed) what happens to the curve shown?



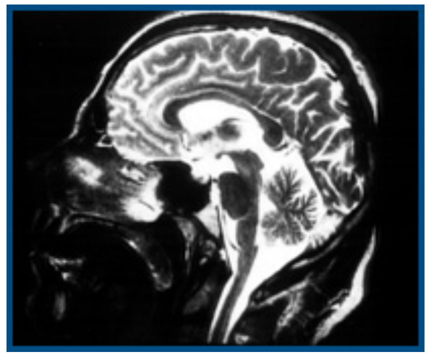
A) It shifts to the LEFT, and the max value increases.
B) It shifts to the LEFT, and the max value decreases.
C) It shifts to the RIGHT, and the max value increases.
D) It shifts to the RIGHT, and the max value decreases.
E) Other/not sure/???

22

23

Resonance phenomena occur widely in natural and in technological applications:

Emission & absorption of light
Lasers
Tuning of radio and television sets
Mobile phones
Microwave communications
Machine, building and bridge design
Musical instruments
Medicine
– nuclear magnetic resonance
magnetic resonance imaging
– x-rays
– hearing



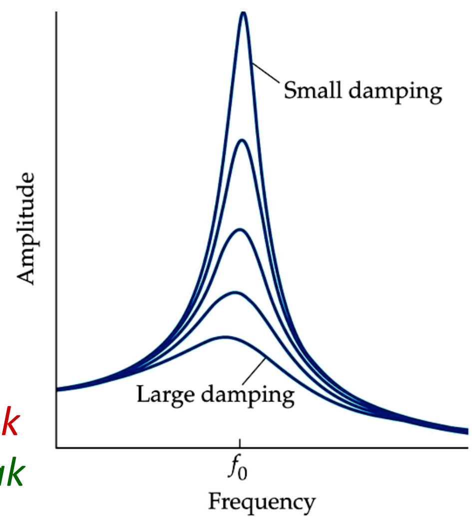
Nuclear magnetic resonance scan

CP 465

Driven Oscillations & Resonance

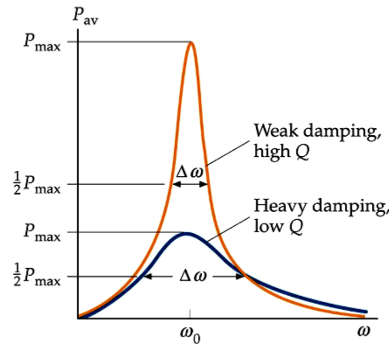
If the driving frequency is close to the natural frequency, the amplitude can become quite large, especially if the damping is small. This is called resonance.

Small damping: *sharp peak*
Large damping: *broad peak*



Driven Oscillations & Resonance

$$\boxed{\omega_0 = \sqrt{\frac{k}{m}}}$$



$$\boxed{\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}}$$

The Q-factor characterizes the sharpness of the peak

Can you break a wine glass with a human voice if the person sings at precisely the resonance frequency of the glass?

A) Sure. I could do it
B) A highly trained opera singer might be able to do it.
C) No. Humans can't possibly sing loudly enough or precisely enough at the right frequency. This is just an urban legend.