If f(t) is periodic (period T), then we can write it as a *Fourier series:*

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega t}$$

What is the formula for c_n?

A) $\int_0^T f(t) e^{-in\omega t} d\omega$

B) $\int_0^T f(t) e^{-in\omega t} dt$

C) $\frac{1}{\omega}\int_0^T f(t) e^{-in\omega t} d\omega$

D) $\frac{1}{T}\int_0^T f(t) e^{-in\omega t} dt$

E) Something else/not sure?

2- 1

Fourier Series (period T) limit, as T➔∞

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t} \longrightarrow f(t) = \int_{-\infty}^{\infty} g(\omega) e^{i\omega t} d\omega$$

$$c_n = \frac{1}{T}\int_0^T f(t) e^{-in\omega_0 t} dt \longrightarrow g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{i(\text{what})?} d(\text{what})?$$

A) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-in\omega t} dt$

B) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$

C) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-in\omega t} d\omega$

D) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-i\omega t} d\omega$

E) Something else/not sure?

2- 2

g(ω) is the *Fourier Transform* of f(t)

$$f(t) = \int_{-\infty}^{\infty} \mathrm{g}(\omega)\mathrm{e}^{i\omega t}\, d\omega$$

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega \mathrm{t}}\, d\mathrm{t}$$

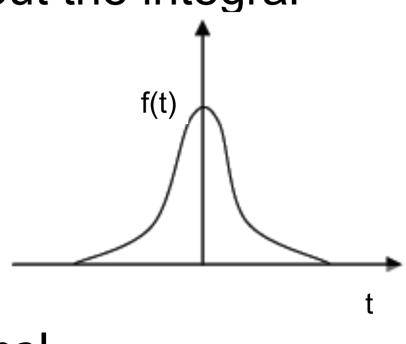
f(t) is the inverse Fourier Transform of g(ω)

2- 3

Consider the function $f(t) = e^{-t^2/b}$

What can you say about the integral

$$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\, dt$$



It is …

A)zero

B)non-zero and pure real

C)non-zero and pure imaginary

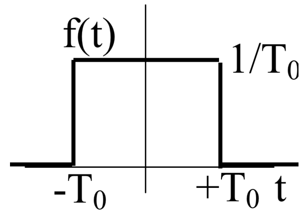
D)non-zero and complex

2- 4

50

If f(t) is given in the picture,
it's easy enough to evaluate

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$



Give it a shot!

After you find a formula, is it...

A) real and even

B) real and odd

C) complex

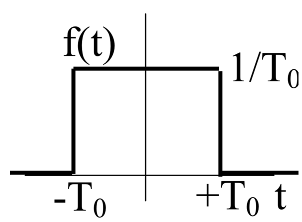
D) Not sure how to do this...

2- 5

If f(t) is given in the picture,

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

$$= \frac{1}{\pi}\frac{\sin\omega T_0}{\omega T_0}$$



What is $\lim_{(\omega\to 0)} g(\omega)$?

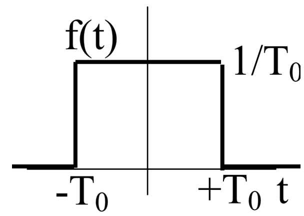
A) 0 B) infinite C) $1/\pi$ D) $1/(\pi\omega T_0)$
E) something else/not defined/not sure...

2- 6

If f(t) is given in the picture,

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

$$= \frac{1}{\pi}\frac{\sin \omega T_0}{\omega T_0}$$



What is $\lim\limits_{(\omega\to\infty)} g(\omega)$?

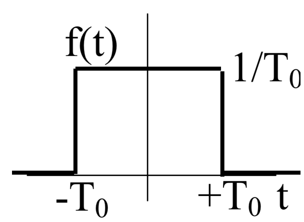
A) 0 B) infinite C) $1/\pi$ D) $1/(\pi\omega T_0)$
E) something else/not defined/not sure...

2- 7

If f(t) is given in the picture,

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

$$= \frac{1}{\pi}\frac{\sin \omega T_0}{\omega T_0}$$



Describe (or sketch) g(ω)

Challenge: What changes if T_0 is very SMALL?
How about if T_0 is very LARGE?

2- 8

What is the Fourier transform of a Dirac delta function, f(t)=δ(t)?

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$= ?$$

A) 0
B) ∞
C) 1
D) 1/2π
E) $e^{-i\omega}$

2- 9

What is the Fourier transform of a Dirac delta function, f(t)=δ(t-t_0)?

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$= ?$$

A) $\frac{1}{2\pi}$

B) $\frac{1}{2\pi}\delta(\omega)$

C) $\frac{1}{2\pi} e^{-i\omega t}$

D) $\frac{1}{2\pi} e^{-i\omega t_0}$

E) Something else...

2- 10

The Fourier transform of $f(t) = e^{-t^2/(2\sigma^2)}$

is $g(\omega) = \sigma\sqrt{2\pi}e^{-\omega^2\sigma^2/2}$

Sketch this function

2- 11

What is the standard deviation of

$g(\omega) = \sigma\sqrt{2\pi}e^{-\omega^2\sigma^2/2}$

which is the Fourier transform of

$f(t) = e^{-t^2/(2\sigma^2)}$

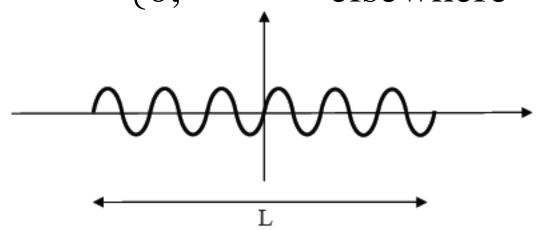
A) 1
B) σ
C) σ^2
D) $1/\sigma$
E) $1/\sigma^2$

2- 12

Consider the function f(x)

which is a sin wave of length L.

$$f(x) = \begin{cases} \sin(kx), & -\frac{L}{2} < x < +\frac{L}{2} \\ 0, & \text{elsewhere} \end{cases}$$



- Which statement is closest to the truth?

A) f(x) has a single well-defined wavelength

B) f(x) is made up of a range of wavelengths

42

Compared to the original function f(t), the Fourier transform function g(ω)

A) Contains *additional information*

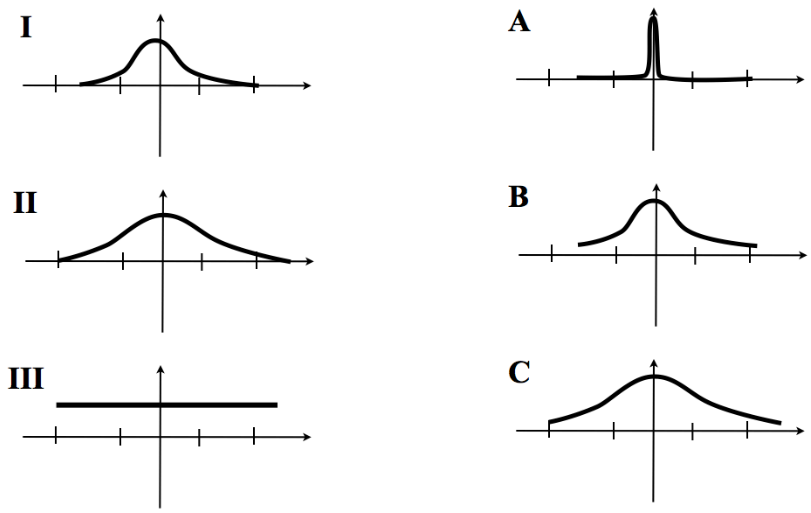
B) Contains the *same* amount of information

C) Contains *less* information

D) It depends

2- 14

Match the function (on the left)
to its Fourier transform (on the right)



2- 15