In a Phys 1110 exam, a student produced the following solution.
Is the final solution correct?
If NOT, does that mean the initial equation must have been wrong?
If NOT, and assuming the initial equation is NOT wrong,
find the error using dimensional analysis.

$$[M] = M,$$
$$[g] = L/T^2$$
$$[h] = L,$$
$$[\omega] = 1/T$$
$$[v] = L/T,$$
$$[R] = L$$
$$[I] = ML^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$
$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\omega^2$$
$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\left(\frac{v^2}{R}\right)^2$$
$$gh = \frac{1}{2}v^2 + \frac{1}{2}v^4$$

Suppose you solve an ODE for a particle's motion, and find
$x(t) = c(t-t_0)$ What can you conclude?

A) This particle is responding to a constant force
B) This particle is free (zero force)
C) This particle is responding to a time varying force
D) ???

Suspose you solve an ODE for a particle's motion, and find $x(t) = bt^2$.

What can you conclude?

A) This particle is responding to a constant force
B) This particle is free (zero force)
C) This particle is responding to a time varying force
D) ???

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Suppose you solve an ODE for a particle's motion, and find : $x(t) = c(1 - e^{-t/\tau})$

What can you conclude?

A) This particle is responding to a constant force
B) This particle is free (zero force)
C) This particle is responding to a time varying force
D) ???

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For an object of "diameter D",
$f_{linear}=bv=\beta D$

$f_{quad} = cv^2 = (1/2)c_0\, A\, \rho_{air}\, v^2$

For a sphere in air, $f_{quad}/f_{linear} \approx (1600\ s/m^2)\, Dv$

Which form of drag dominates most sports events?

A) linear
B) quadratic
C) ??

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B) quadratic
C) ??

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A falling mass feels a drag force $f \propto v^2$. (Up is the + direction)

Which eq'n of motion is correct?

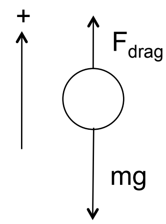
A) $dv/dt = -g + kv^2$

B) $dv/dt = +g - kv^2$

C) $dv/dt = -g + kv^2$

D) $dv/dt = +g - kv^2$

E) Other!



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An object falling in air satisfies the ODE (from Newton's 2nd law):

$$m\, dv/dt = -mg - bv$$

The equation has three dimension-ful parameters (m, g, b)

a) Use those three dimensionful parameters to create "Natural" scales of mass (M_0), length (L_0), and time (T_0)

b) Using these three natural scales, create a natural scale for velocity (V_0)

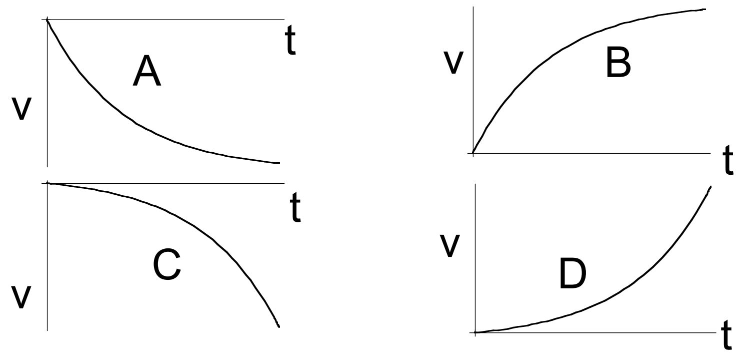
c) Define a *dimensionless velocity V* by the equation $V=v/V_0$, and rewrite the original ODE as an equation for V instead.

(This equation should contain NO parameters with dimensions, except where you have combinations of quantities that manifestly look dimensionless)

The solution to the equation describing an object falling from rest with linear air drag was

$$v_y(t) = -v_t(1 - e^{-t/\tau})$$

Which figure best shows this sol'n?



E) Other/???

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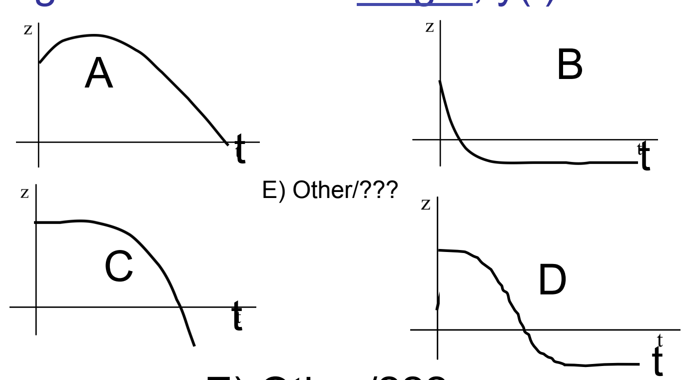
The solution to the eq'n describing an object falling from rest from h>0 with linear air drag was $v_y(t) = -v_t(1 - e^{-t/\tau})$

Which figure best shows <u>height</u>, y(t)?



E) Other/???

An object is launched directly upwards with initial speed v_0.
Compare the time t_1 to reach the top to the time t_2 to return back to the starting height.

A) $t_1>t_2$
B) $t_1=t_2$
C) $t_1<t_2$
D) Answer depends on whether v_0 exceeds v_t or not.

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A tennis ball is hit directly upwards with initial speed v_0. Compare the time T to reach the top (height H) to the time and heigh in an ideal (vacuum) world.

A) $T>T_{vacuum}$, $H\approx H_{vacuum}$
B) $T>T_{vacuum}$, $H<H_{vacuum}$
C) $T\approx T_{vacuum}$, $H<H_{vacuum}$
D) $T<T_{vacuum}$, $H<H_{vacuum}$
E) Some other combination!!

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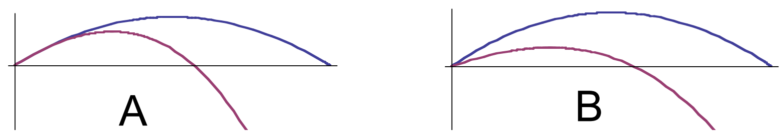
Which of these two plots seems more realistic for a tennis ball trajectory, comparing (in each graph) the path with (purple), and without (blue), drag.



C) Neither of these is remotely correct!

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With quadratic air drag, $v(t)=v_0/(1+t/\tau)$
where $\tau=m/(cv_0)$, and $c=(1/2)c_0 A \rho_{air.}$
(For a human on a bike, c is of order .2 in SI units)

Can you confirm that c is about 0.2? (c_0 is ~1 for non-aerodynamic things, and ~.1 for very- aerodynamic things.

Roughly how long does it take for a cyclist on the flats to drift down from v_0=10 m/s (22 mi/hr) to ~1 m/s?

A) a couple seconds
B) a couple minutes
C) a couple hours
D) none of these is even close.

In real life, the answer is shorter than this formula would imply. Why?

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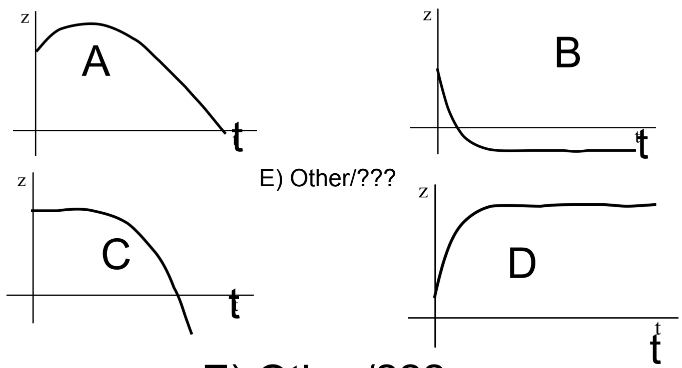
The solution to the eq'n describing an object thrown up from h>0 with linear air drag was

$$v(t) = -g\tau + \tau\left(v_0/\tau + g\right)e^{-t/\tau}$$

Which figure best shows height, z(t)?



E) Other/???

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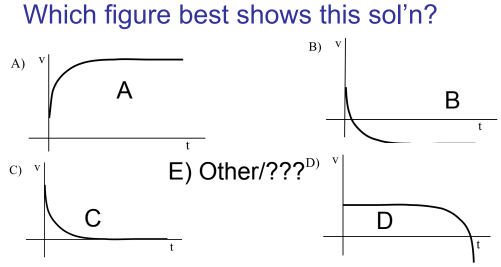
The solution for an object moving horizontally with linear air drag was

$v(t) = v_0 e^{-t/\tau}$ if $v(t=0) = v_0 > 0$

Which figure best shows this sol'n?



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