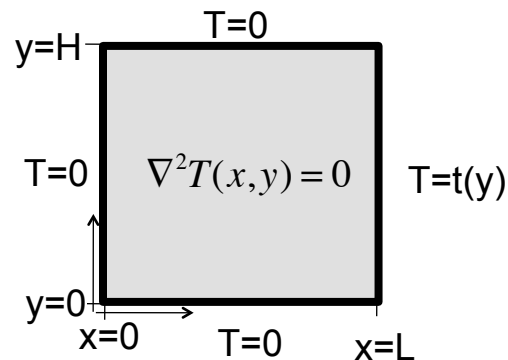


Rectangular plate, with temperature fixed at edges:



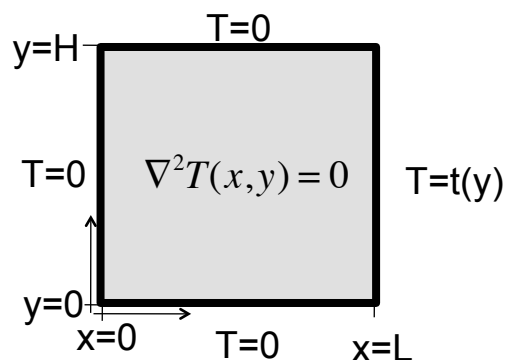
When using separation of variables, so $T(x,y)=X(x)Y(y)$,
which variable (x or y) has the sinusoidal solution?

- A) $X(x)$ B) $Y(y)$ C) Either, it doesn't matter
- D) NEITHER, the method won't work here
- E) ???

2- 1

Trial solution: $T(x,y)=(Ae^{kx}+Be^{-kx})(C\cos(ky)+D\sin(ky))$

Applying the boundary
condition $T=0$ at
i) $y=0$ and ii) $y=H$ gives
(in order!)

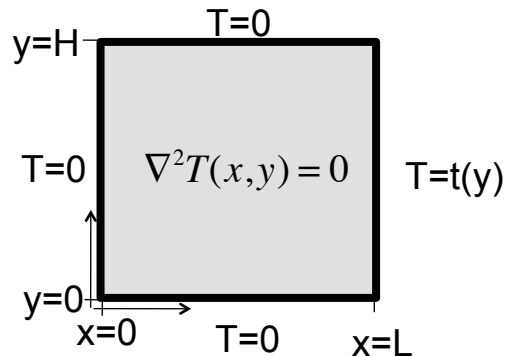


- A) i) $k=n\pi/H$, ii) $A=-B$
- B) i) $k=n\pi/L$, ii) $D=0$
- C) i) $A=-B$, ii) $k=n\pi/H$
- D) i) $D=0$, ii) $k=n\pi/L$
- E) Something else!!

2- 2

Trial solution: $T(x,y)=(Ae^{kx}+Be^{-kx})(C\cos(ky)+D\sin(ky))$

Applying the boundary condition $T=0$ at
i) $y=0$ and ii) $y=H$ gives
(in order!)



A) i) $k=n\pi/H$, ii) $A=-B$

B) i) $k=n\pi/L$, ii) $D=0$

C) i) $A=-B$, ii) $k=n\pi/H$

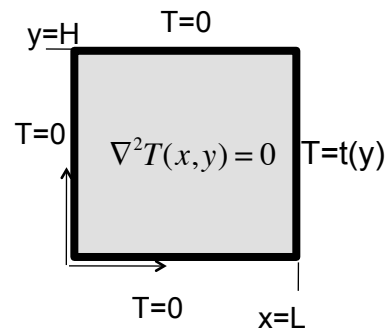
D) i) $D=0$, ii) $k=n\pi/L \longrightarrow$ i) $C=0$ ii) $k = n\pi/H$

E) Something else!!

2- 3

Trial solution: $T_n(x,y)=(A_n e^{n\pi x/H} + B_n e^{-n\pi x/H})(\sin n\pi y/H)$

Applying the boundary condition $T(0,y)=0$ gives...



A) $A_n=0$

B) $B_n=0$

C) $A_n=B_n$

D) $A_n=-B_n$

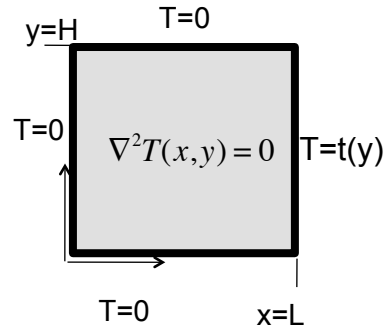
E) Something entirely different!

2- 4

Recalling $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$

Trial solution: $T_n(x, y) = A_n \sinh(n\pi x/H) \sin(n\pi y/H)$

Applying the boundary condition $T(L, y) = t(y)$ does what for us...



- A) Determines (one) A_n
- B) Shows us the method of separation of v'bles failed in this instance
- C) Requires us to sum over n before looking for A_n 's
- D) Something entirely different/not sure/...

2- 5

Trial solution: $T(x, y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H) \sin(n\pi y/H)$

What is the correct formula to find the A_n 's?

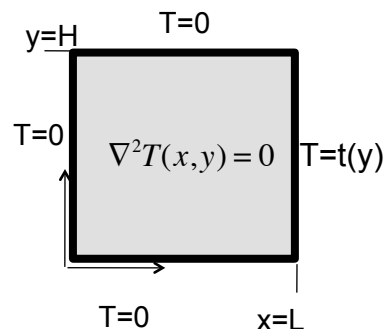
A) $A_n = \frac{2}{L} \int_0^L t(y) \sin(n\pi y/L) dy$

B) $A_n = \frac{2}{L} \int_0^L t(y) \sin(n\pi y/H) dy$

C) $A_n = \frac{2}{H} \int_0^H t(y) \sin(n\pi y/L) dy$

D) $A_n = \frac{2}{H} \int_0^H t(y) \sin(n\pi y/H) dy$

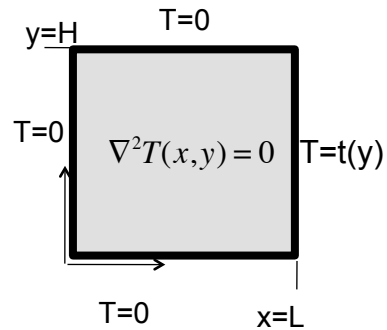
E) Something entirely different!



2- 6

Trial solution: $T(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H) \sin(n\pi y/H)$

Right b'dry: $t(y) = T(L,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi L/H) \sin(n\pi y/H)$



2- 7

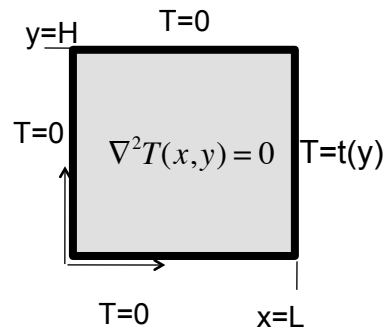
Right b'dry: $T(L,y) = \sum_{n=1}^{\infty} \underbrace{A_n \sinh(n\pi L/H)}_{b_n} \sin(n\pi y/H)$

$$t(y) = \sum_{n=1}^{\infty} b_n \sin(n\pi y/H)$$

$$b_n = \frac{2}{H} \int_0^H t(y) \sin(n\pi y/H) dy$$

Which means

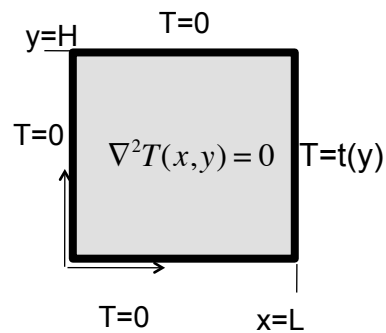
$$A_n \sinh(n\pi L/H) = \frac{2}{H} \int_0^H t(y) \sin(n\pi y/H) dy$$



2- 8

Solution (!!): $T(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H) \sin(n\pi y/H)$

with: $A_n = \frac{1}{\sinh(n\pi L/H)} \frac{2}{H} \int_0^H t(y) \sin(n\pi y/H) dy$

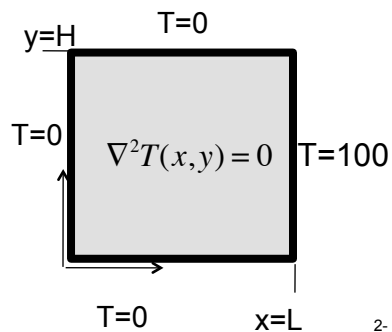


2- 9

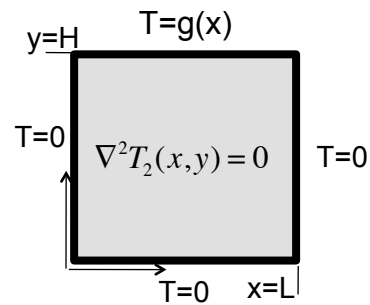
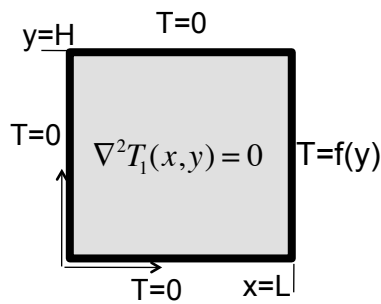
Solution (!!): $T(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H) \sin(n\pi y/H)$

with: $A_n = \frac{1}{\sinh(n\pi L/H)} \frac{2}{H} \int_0^H t(y) \sin(n\pi y/H) dy$

If e.g.
 $t(y)=100^\circ$ (a constant)...



2- 10

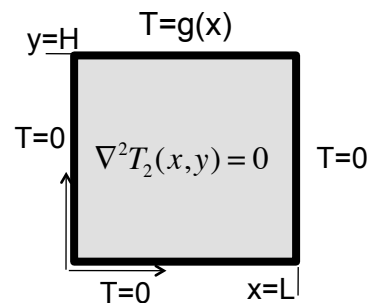
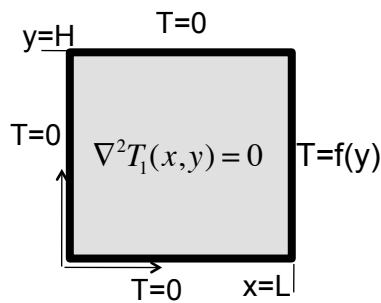


$$T_1(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H) \sin(n\pi y/H)$$

$$\text{with } A_n = \frac{1}{\sinh(n\pi L/H)} \frac{2}{H} \int_0^H f(y) \sin(n\pi y/H) dy$$

How would you find $T_2(x,y)$?

2- 11



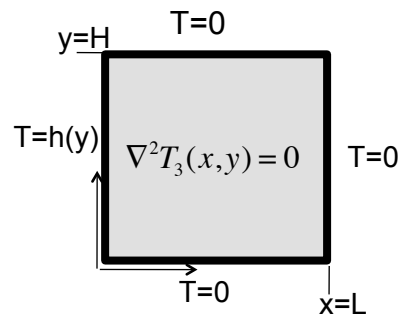
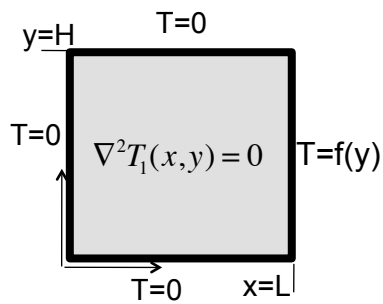
$$T_1(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H) \sin(n\pi y/H)$$

$$\text{with } A_n = \frac{1}{\sinh(n\pi L/H)} \frac{2}{H} \int_0^H f(y) \sin(n\pi y/H) dy$$

$$T_2(x,y) = \sum_{n=1}^{\infty} A'_n \sinh(n\pi y/L) \sin(n\pi x/L)$$

$$\text{with } A'_n = \frac{1}{\sinh(n\pi H/L)} \frac{2}{L} \int_0^L g(x) \sin(n\pi x/L) dx$$

2- 12

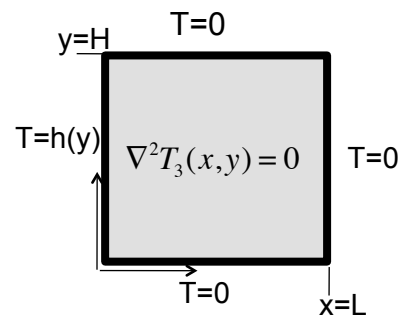
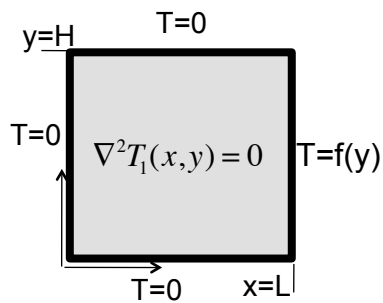


$$T_1(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H) \sin(n\pi y/H)$$

$$\text{with } A_n = \frac{1}{\sinh(n\pi L/H)} \frac{2}{H} \int_0^H f(y) \sin(n\pi y/H) dy$$

How would you find $T_3(x,y)$?

2-13



$$T_1(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H) \sin(n\pi y/H)$$

$$\text{with } A_n = \frac{1}{\sinh(n\pi L/H)} \frac{2}{H} \int_0^H f(y) \sin(n\pi y/H) dy$$

Just swap x with (L-x) (!)

2-14

$\nabla^2 T_1(x,y) = 0$

$\nabla^2 T_2(x,y) = 0$

How would you find $T_4(x,y)$?

$\nabla^2 T_4(x,y) = 0$

2-15

$\nabla^2 T_1(x,y) = 0$

$\nabla^2 T_2(x,y) = 0$

Would this work?

$T_4(x,y) = T_1(x,y) + T_2(x,y)$

A) sweet!

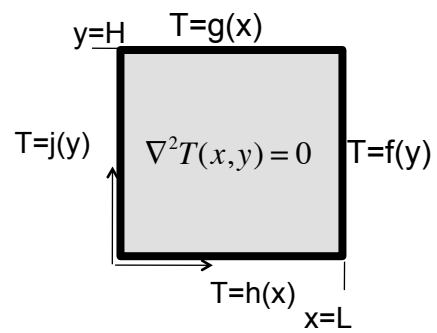
B) No, it messes up Laplace's eqn

C) No, it messes up Bound conditions

D) Other/??

$\nabla^2 T_4(x,y) = 0$

2-16



We have solved this!

2-17