CLASSICAL MECHANICS AND MATH METHODS, SPRING, 2011

Homework 15

(Due Date: Start of class on Thurs. Apr 28)

1. Complete the online survey at

`http://www.colorado.edu/sei/surveys/Sp11/Clicker_Phys2210_sp11-post.html`

Note: You get full credit just for inputting your name and ID, but we really appreciate your thoughtful feedback!

2. (a) (2 pts) Show that if a function $f(x)$ is even, then its Fourier transform $g(\alpha)$ (as defined in Boas Eq 12.2) is also an even function.

(b) (2 pts) Show that in this case f(x) and g(α) can be written as Fourier cosine transforms and obtain Boas Eq 12.15 (Assuming that f(x) is identical to Boas' $f_c(x)$, how is $g_c(\alpha)$ from Boas' Eq 12.15 related to g(α) of Boas' Eq 12.2?

Please note the typo in the top equation of Boas 12.15, the argument of g_c should be α, not x.

3. For the function shown in Fig.1

(a) (1 pt) Qualitatively, how do you expect $g(\alpha)$, the Fourier transform of $f(x)$, to change if you make a bigger or smaller?

(b) (4 pts) Find the Fourier transform $g(\alpha)$.

(c) (2 pts) Sketch $g(\alpha)$ (or plot it in Mathematica if you prefer). Was your prediction in part (a) correct? Explain.

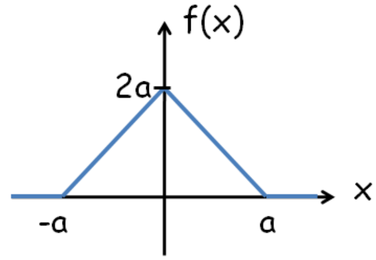

Figure 1:

4. (4 pts) The function $e^{-c|x|}$ (with c a positive, known constant) is associated with bound states in quantum mechanics, and (pretty much as always in quantum mechanics) its Fourier transform is important to know. Find the Fourier transform of $e^{-c|x|}$, sketch it, and comment on any key features.

As a mathematical aside, use your result to (easily!) compute $\int_0^\infty \frac{\cos(\alpha x)}{\alpha^2+1} d\alpha$.

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5. The potential energy of two atoms in a molecule can sometimes be approximated by the Morse function,

$$U(r) = A\left[\left(e^{(R-r)/S} - 1\right)^2 - 1\right]$$

where r is the distance between the two atoms and A, R, and S are positive constants with $S \ll R$.

(a) (4 pts) Sketch the function for $0 < r < \infty$ (or plot in Mathematica if you prefer).

(b) (2 pts) Show that the equilibrium separation r_0 at which $U(r)$ is a minimum is given by $r_0 = R$.

(c) (2 pts) Based just on your figure/sketch, what behavior do you expect for the system if it starts with no kinetic energy, but at some r close to r_0? Describe what this would mean physically for the two atoms in the molecule (i.e. tell the story of what is happening to them).

(d) (4 pts) *If our diatomic molecule has one very heavy atom, that atom can be viewed as effectively fixed at the origin, and this U(r) simply describes the potential energy of the second, lighter atom (mass m, a distance r(t) from the origin.)*

Approximate $U(r)$ for r close to r_0. Use this approximation of $U(r)$ to get a differential equation for $r(t)$. Can you tell from this differential equation if your prediction in part (c) was correct? Explain.

Extra Credit (4 pts) Find the Fourier Transform of a Gaussian function (with standard deviation σ), $f(x) = e^{-x^2/(2\sigma^2)}$, and show that it is *also* a Gaussian function. (You will need to consult an integral table for this one . See the online lecture notes for more hints)

Sketch $f(x)$ and $g(\alpha)$. What is the standard deviation of $g(\alpha)$? Does this agree with your expectations?
