⭑ TUTORIAL: *SEPARATION OF VARIABLES* ⭑

Solving Boundary Conditions.

When solving PDEs, you inevitably come to a point where you have a trial function with some undetermined coefficients, and a "boundary condition" that helps you pick (or constrain) those coefficients. Let's consider two common examples.

A) Your trial function is $f(x) = Ae^{kx} + B\,e^{-kx}$. You do not yet know A, B, or k.
The boundary conditions are $f(0)=0$, and $f(h)=0$, where h is a known length.

i) What does $f(0)=0$ tell you about A, B, and/or k?
(Hint: it might tell you about *some* but not *all* of them)

ii) Given the above, what additional information does $f(h)=0$ give you?

iii) Summarize: what does f(x) look like? Is it unique, or are there many possibilities?

(Check with an instructor to talk about what you have!)

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B) Your trial function is $f(x) = C\sin(kx) + D\cos(kx)$. You do not yet know C, D, or k! The boundary conditions are $f(0)=0$, and $f(L)=0$, where L is a known length.

i) Which of these two conditions do you think will be most useful to start with? Why? Start with it - what does it tell you?

ii) What additional information does the other boundary condition yield?

iii) Summarize: what does f(x) look like? Is it unique, or are there many possibilities? Discuss!

(Check with an instructor to talk about what you have!

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Several in-class clicker questions preceding the above Tutorial page:

When solving $\nabla^2 T(x,y)=0$, separation of variables says: *try* T(x,y) = X(x) Y(y)

i) Just for practice, invent *some* function T(x,y) that is manifestly of this form. (Don't worry about whether it satisfies Laplace's equation, just make up some function!) What is your X(x) here? What is Y(y)?

ii) Just to compare, invent some function T(x,y) that is definitely NOT of this form.

Challenge questions:

1) Did your answer in i) satisfy Laplace's eqn?

2) Could our method (separation of variables) ever FIND your function in part ii above?

2- 3

When solving $\nabla^2 T(x,y)=0$, separation of variables says try T(x,y) = X(x) Y(y). We arrived at the equation

f(x) + g(y) = 0

for some complicated f(x) and g(y)

Invent *some* function f(x) and some *other* function g(y) that satisfies this equation.

Challenge question: In 3-D, the method of separation of variables would have gotten you to f(x)+g(y)+h(z)=0. Generalize your "invented solution" to this case.

2- 4

When solving $\nabla^2 T(x,y)=0$, separation of variables says try T(x,y) = X(x) Y(y). We arrived at

$$\frac{d^2X(x)}{dx^2} = cX(x)$$

and

$$\frac{d^2Y(y)}{dy^2} = -cY(y)$$

Write down the *general solution* to both of these ODEs!

Challenge: Is there any deep ambiguity about your solution?

2- 5

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