

## ★ Fourier Series ★

A common way to write a function as a Fourier series is:

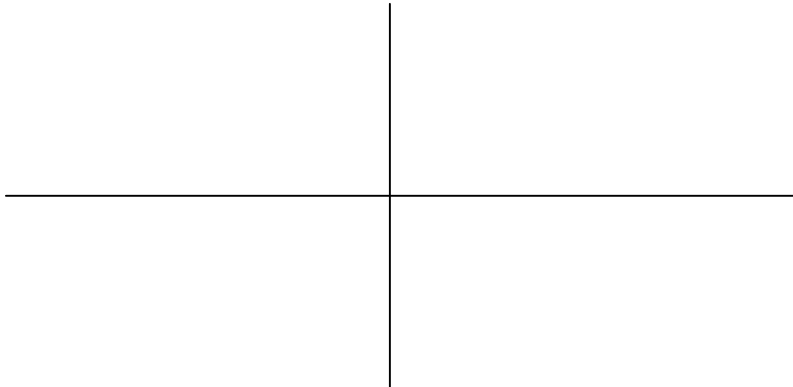
$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) \quad (1)$$

### Part 1

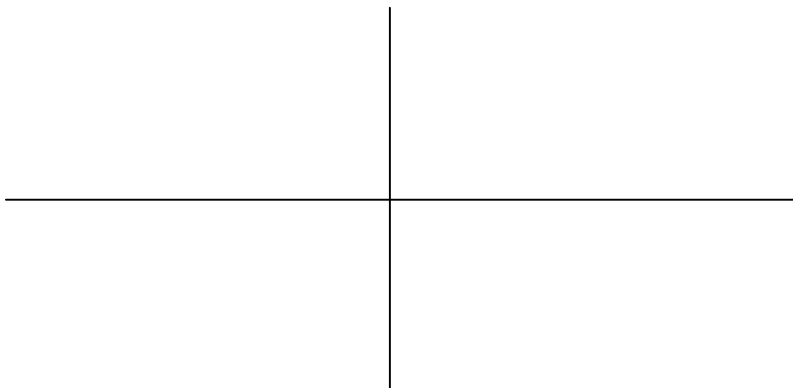
Sketch  $f(t)$  for the following cases:

(Please label the axis and include labeled tick marks)

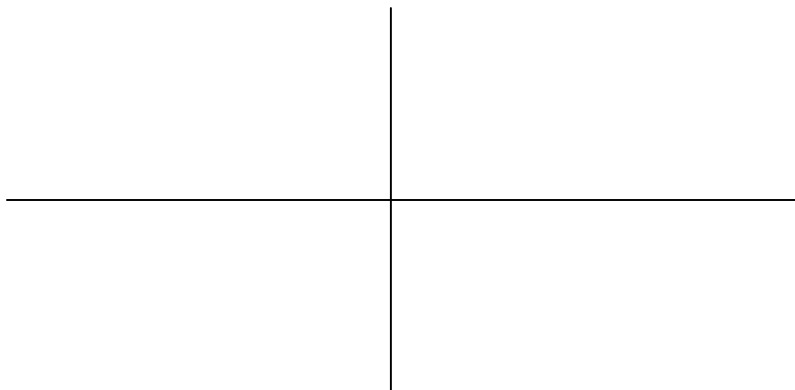
a)  $a_0 = 2$ , all the other  $a_n$ 's and  $b_n$ 's = 0.



b)  $a_0 = 1$ ,  $b_1 = 2$ , all the other  $a_n$ 's and  $b_n$ 's = 0. Assume  $\omega = 1$  in Equation (1).



c)  $a_2 = 3$ , all the other  $a_n$ 's and  $b_n$ 's = 0. Assume  $\omega = 1$  in Equation (1).



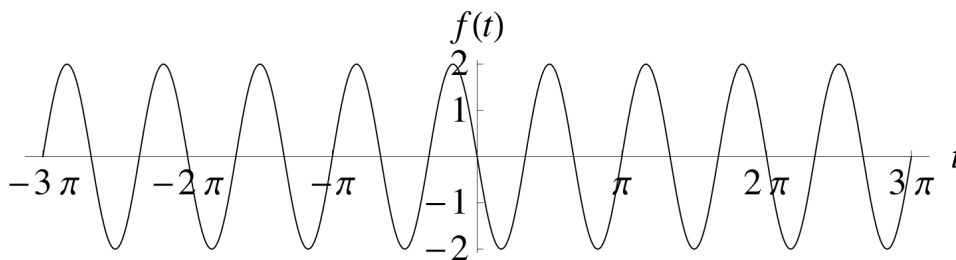
Part 2 
$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) \quad (1)$$

Given  $f(t)$ , as written or drawn below, tell us, **without calculations**, anything you know about the  $a_n$ 's and  $b_n$ 's. You can just answer qualitatively – are some definitely zero, positive, negative? Assume  $\omega = 1$  in Equation (1).

a)  $f(t) = 3\cos(17t)$

What do you know about the  $a_n$ 's and  $b_n$ 's?

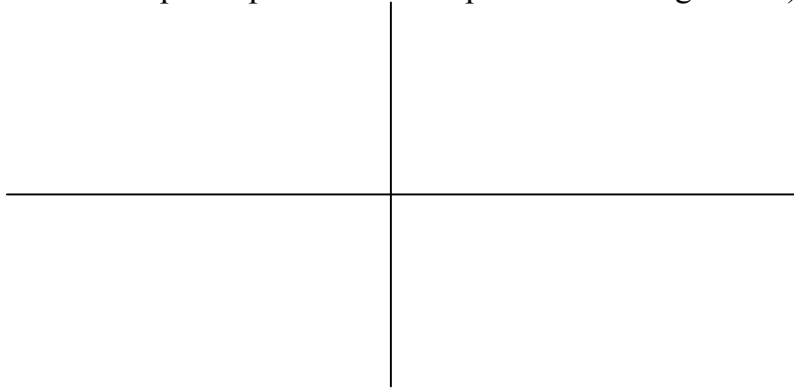
b)



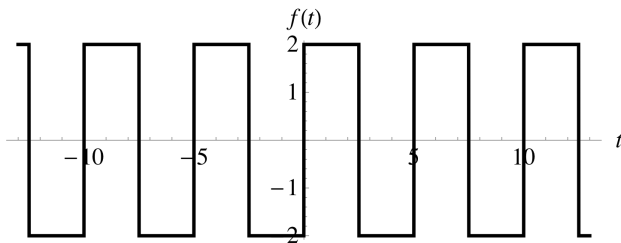
What do you know about the  $a_n$ 's and  $b_n$ 's?

c)  $f(t) = \begin{cases} 1, & (|t| < \frac{\pi}{2}) \\ -1, & (\pi > |t| > \frac{\pi}{2}) \end{cases}$ , outside this region it repeats, so  $f(t + 2\pi) = f(t)$ .

What do you know about the  $a_n$ 's and  $b_n$ 's? (Hint: it might help to draw this function over a couple of periods in both positive and negative  $t$ .)



### Part 3



Given the function above for  $f(t)$ , what would you choose for  $\omega$  in the general

form for the Fourier series:  $f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$ ?

Please explain your answer:

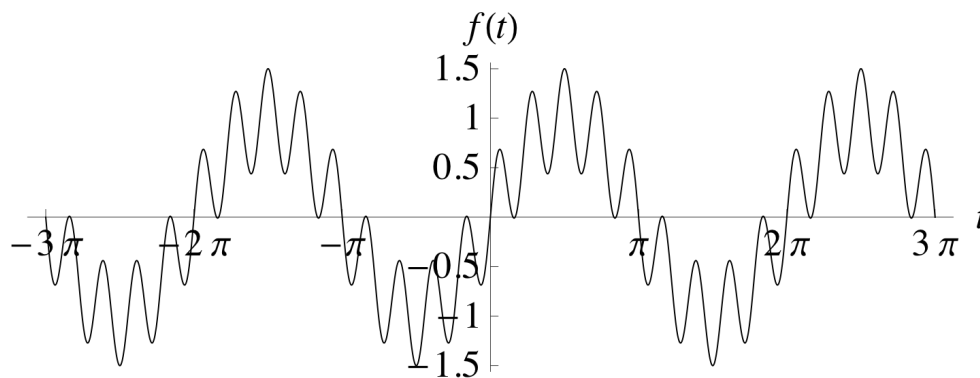
Bonus – (Do this if you finish the other parts and still have time)

$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

1) For  $f(t) = 2\sin(15t)$ , what are some possible values of  $\omega$  that would work in formula above? For some of these  $\omega$ 's, what do you know about the  $a_n$ 's and  $b_n$ 's?

2) What is the period,  $T$ , of  $f(x,t) = 3\cos\left(\frac{\pi b}{3}x + \frac{2}{\pi a}t\right)$ ?

3)



Without calculation, what do you know about the  $a_n$ 's and  $b_n$ 's for  $f(t)$ ?