$$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

2- 3

Vectors, in terms of a set of basis vectors:

$$\vec{\mathbf{v}} = \sum_{i=1}^{3} v_i \, \hat{\mathbf{e}}_i$$

Inner product, or "dot product":

$$\vec{\mathbf{c}} \cdot \vec{\mathbf{d}} = \sum_{i=1}^{3} c_i \, d_i$$

To find one numerical component of v:

$$v_i = \vec{\mathbf{v}} \cdot \hat{\mathbf{e}}_i$$

2- 4

$$\vec{\mathbf{v}} = \sum_{i=1}^{3} v_i \,\hat{\mathbf{e}}_i$$

$$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

2- 5

Inner product, or "dot product" of vectors:

$$\vec{\mathbf{c}} \cdot \vec{\mathbf{d}} = \sum_{i=1}^{n} c_i \, d_i$$

If you had to make an intuitive stab at what might be the analogous inner product of *functions*, c(t) and d(t), what might you try? (Think about the large n limit?)

2- 7

Inner product, or “dot product” of vectors:

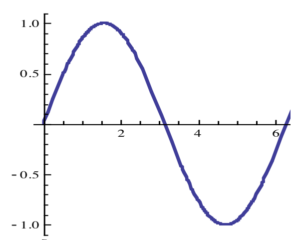
$$\vec{\mathbf{c}} \cdot \vec{\mathbf{d}} = \sum_{i=1}^{n} c_i d_i$$

If you had to make an intuitive stab at what might be the analogous inner product of *functions*, c(t) and d(t), what might you try? (Think about the large n limit?)

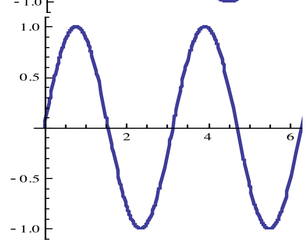
How about:

$$\int c(t)d(t)dt \qquad ??$$

2- 8



$\sin(\omega t)$

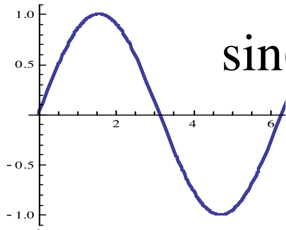


$\sin(2\omega t)$

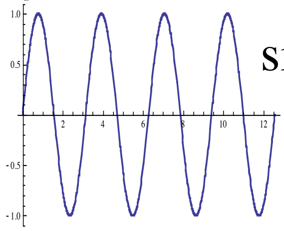
What can you say about $\int_0^T \sin(\omega t)\sin(2\omega t)dt$

A) 0 B) positive C) negative D) depends
E) I would really need to compute it...

2- 10



$\sin(\omega t)$



$\sin(mw\, t)$

If m>1, what can you guess about

$$\int_0^T \sin(\omega t)\sin(m\omega t)dt$$

A) always 0 B) sometimes 0 C)???

2- 11

Summary (not *proven* by previous questions,
but easy enough to just do the integral and show this!)

$$\int_0^T \sin(n\omega t)\sin(m\omega t)dt = 0 \quad \text{if n} \neq \text{m}$$

2- 12

Orthogonality of basis vectors:

$$\hat{\mathbf{e}}_{\mathbf{i}} \cdot \hat{\mathbf{e}}_j = 0 \qquad (\text{if } \mathrm{i} \neq \mathrm{j})$$

What does ...

$$\int_0^T \sin(n\omega t)\sin(m\omega t)\,dt = 0 \qquad (\text{if } \mathrm{n} \neq \mathrm{m})$$

suggest to you, then?

2- 13

Orthonormality of basis vectors:

$$\hat{\mathbf{e}}_{\mathbf{i}} \cdot \hat{\mathbf{e}}_j = \begin{cases} 0 \ (\text{if i} \neq \text{j}) \\ 1 \ (\text{if i} = \text{j}) \end{cases} \qquad \equiv \delta_{\mathrm{i,j}}$$

$$\frac{2}{T}\int_0^T \sin(n\omega t)\sin(m\omega t)\,dt = \begin{cases} 0 \quad (\text{if n} \neq \text{m}) \\ 1 \quad (\text{if n} = \text{m}) \end{cases} \qquad \equiv \delta_{n,m}$$

2- 14

Vectors, in terms of a set of basis vectors:

$$\vec{\mathbf{v}} = \sum_{i=1}^{n} v_i \hat{\mathbf{e}}_i$$

To find one numerical component:

$$v_i = \vec{\mathbf{v}} \cdot \hat{\mathbf{e}}_i$$

Functions, in terms of basis functions

$$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

To find one numerical component:

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt \quad (??)$$

2-16

Vectors, in terms of a set of basis vectors:

$$\vec{\mathbf{v}} = \sum_{i=1}^{n} v_i \hat{\mathbf{e}}_i$$

To find one numerical component: Fourier's trick

$$\hat{\mathbf{e}}_j \cdot \vec{\mathbf{v}} = \hat{\mathbf{e}}_j \cdot \sum_{i=1}^{n} v_i \hat{\mathbf{e}}_i$$

$$= \sum_{i=1}^{n} v_i \hat{\mathbf{e}}_j \cdot \hat{\mathbf{e}}_i$$

$$= \sum_{i=1}^{n} v_i \delta_{i,j}$$

2-17

$$\sum_{i=1}^{n} v_i \delta_{i,j} = ?$$

A) $\sum_{i=1}^{n} v_i$

B) v_i

C) v_j

D) v_n

E) Other/none of these?

2- 18

$$\hat{\mathbf{e}}_j \cdot \vec{\mathbf{v}} = \hat{\mathbf{e}}_j \cdot \sum_{i=1}^{n} v_i \hat{\mathbf{e}}_i$$

$$= \sum_{i=1}^{n} v_i \hat{\mathbf{e}}_j \cdot \hat{\mathbf{e}}_i$$

$$= \sum_{i=1}^{n} v_i \delta_{i,j}$$

$= v_j$ D'oh!

2- 19

$$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

To find one component: **Fourier's trick**

"Dot" both sides with a "basis vector"

$$\frac{2}{T}\int_0^T f(t)\sin(m\omega t) = \frac{2}{T}\int_0^T \sum_{n=1}^{\infty} b_n \sin(n\omega t)\sin(m\omega t)$$

$$= \sum_{n=1}^{\infty} \frac{2}{T}\int_0^T b_n \sin(n\omega t)\sin(m\omega t)$$

2- 20

$$\frac{2}{T}\int_0^T f(t)\sin(m\omega t) = \sum_{n=1}^{\infty} \frac{2}{T}\int_0^T b_n \sin(n\omega t)\sin(m\omega t)$$

$$= \sum_{n=1}^{\infty} b_n \delta_{n,m}$$

2- 21

$$\sum_{n=1}^{\infty} b_n \, \delta_{n,m} = ?$$

A) $\sum_{n=1}^{\infty} b_n$

B) b_n

C) b_m

D) Other/none of these?

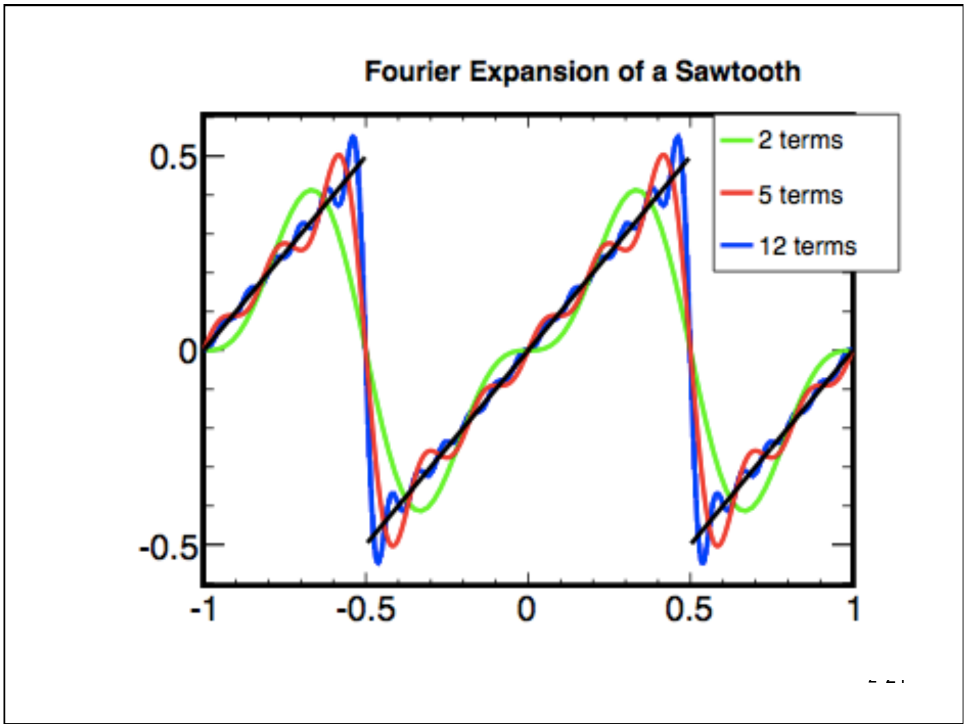
2-22

$$\frac{2}{T}\int_0^T f(t)\sin(m\omega t) = \sum_{n=1}^{\infty} \frac{2}{T}\int_0^T b_n \sin(n\omega t)\sin(m\omega t)$$

$$= \sum_{n=1}^{\infty} b_n \delta_{n,m}$$

$$= b_m$$

2-23



Fourier Expansion of a Sawtooth
2 terms
5 terms
12 terms
0.5
0
-0.5
-1
-0.5
0
0.5
1