CLASSICAL MECHANICS AND MATH METHODS, SPRING, 2011

Homework 3

(Due Date: Start of class on Thurs. Jan 27)

1. Boas has a very useful formula for solving *any* linear first-order differential equation: Eq. 3.9 page 401. We casually sketched the derivation of this formula in class, but you can check now how useful it is. (8 pts) For example, consider an electric circuit containing a resistor in series with an inductor L and a source of emf $V(t) = V_0 e^{i\omega t}$. You may or may not have treated "RL" circuits in Phys 1120, but the resulting equation for the current is a pretty straightforward ODE:

$$L\frac{dI}{dt} + RI = V(t) = V_0 e^{i\omega t} \tag{1}$$

Find the current $I(t)$ as a function of time, assuming $I(0) = 0$. *(Note: Be careful - don't mix up the symbol I in I(t) (current) with the symbol I that Boas defines! By the way, in practice, because we started with a complex voltage, you would take the real part of the answer to figure out physical currents, but no need to bother with that for now)*

2. Consider a ball that moves vertically under the influences of both gravity and air resistance. For the purposes of this problem, take vertically upward as the positive direction. For each equation of motion below, determine whether that equation applies to (i) a situation in which the ball moves upward, (ii) a situation in which the ball moves downward, (iii) either of these, or (iv) neither of these. Explain your reasoning for each case. (4 pts total)

a) $m(dv_y/dt) = -mg + c_1 v_y$, b) $m(dv_y/dt) = -mg - c_1 v_y$

c) $m(dv_y/dt) = -mg + c_2 v_y^2$, d) $m(dv_y/dt) = -mg - c_2 v_y^2$

3. Consider a grain of pollen which is ejected from the anther of a plant vertically up with initial speed v_0. Assume the air resistance on the grain of pollen can be modeled to a very good approximation by a linear drag. Measuring the position of the grain, y, upward from the point of release:

(a) Find the time for the grain to reach its highest point and its position y_{max} at that time. (Note that velocity $v_y(t)$ and position $y(t)$ have been derived in the lecture notes, and the text. Of course, if you want to rederive them yourself, that's probably a very good idea, but not required for credit)

(b) Show that as the drag coefficient approaches zero your answer in part b reduces to the well know freshman physics result $y_{max} = \frac{v_o^2}{2g}$. **Hint:** If the drag coefficient is very small the terminal velocity is very big so v_0/v_{ter} is very small. Use the expansion $\ln(1+\epsilon) \sim \epsilon - \epsilon^2/2$ for $\epsilon \ll 1$.

4. A particle of mass m slides down an inclined plane under the influence of gravity. If the motion is resisted by a force $f = kmv^2$, show that the time required to move a distance d after starting from rest is

$$t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{A \sin\theta}} \tag{2}$$

where θ is the angle of inclination of the plane. What is A in this formula in terms of k and g?(12 pts).

Hint: To solve this problem you will want to know how the position of the particle depends on time. Taylor's section on "vertical motion with quadratic drag" (p. 60-61) should be extremely helpful on this problem. The math/formulas are not identical, but closely related.

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5. In your textbook (section 2.4), Taylor solves for the case of a baseball being dropped from a high tower using quadratic air resistance, $F_D = -cv^2\hat{v}$. Lets look at the case of a ball being shot *up* at an initial speed v_0.

(a) Draw a free body diagram for a ball moving vertically upwards, subject to quadratic air drag. Write down a differential equation for this situation, and solve this differential equation for v(t). Make a rough sketch of v(t) vs. t, and briefly discuss any key features.

(b) Using your result from the previous part, find an expression for the time it takes to reach the top of the trajectory. (It will look simpler if you write it in terms of terminal velocity, which satisfies $v_t^2 = mg/c$)

(c) Now download the PhET simulation http://phet.colorado.edu/en/simulation/projectile-motion.
On the top right, switch the object to baseball. This sim uses quadratic drag: $F_D = -\frac{1}{2}c_0 A\rho_{air}v^2\hat{v}$, where c_0 is called the drag-coefficient, A is the cross-sectional area of the object being shot, and ρ_{air} is the density of air = 1.3 kg /m^3. (The sim shows you the value of c_0 and diameter it has picked for a baseball, on the right side of the simulation) Use your formula in part b for "time to top" with these numbers to deduce what numerical initial velocity v_0 you need to get the ball to reach the top of its trajectory at precisely t=3 sec. Now test it, you can input v_0 into the sim and fire the cannon. Aim the cannon at 90^o (or 88-89^o if it is easier to see the trajectory) and switch on air resistance. The little + and - glasses let you zoom in or out. Does the ball reach the top at t=3 sec? (It should!)

(d) When you fired the ball on the PhET sim, did it take longer for the ball to go from the ground to the top of the trajectory or from the top of the trajectory to the ground? Explain why this is the case.

(e) Now let's look at another interesting feature of shooting an object up in the air. Start increasing the value of v_0 in the sim. Double it from what you had before, then increase it by 10, and then by 100. What is happening to the time to reach the top? Use your formal mathematical results from above to explain what is happening!

(f) Play with the PhET sim a little more and explore anything you are interested in. Write down one question that you have about what you notice while playing with the sim.

6. *Mathematica problem:* Consider a ball thrown at an angle θ above the horizontal ground with an initial speed v_0 in a medium with linear drag. For numerical solutions, Mathematica doesn't deal well with unknown symbols (!), so let's consider a particular case where $v_0 = v_{ter}$ and $v_{ter}^2/g = 1$. We know that in vacuum the maximum range is at $\theta = \pi/4$. Let's try to estimate the maximum range (and angle) when we include air resistance:

(a) Plot Eq.(2.37) in Taylor for different values of θ and try by inspection to figure out the angle at which the range is maximum. (With the assumptions above, most unit-full quantities become "one", but do be careful of the fact that e.g. $v_{0,x}$ is $v_0\cos(\theta)$, not v_0), etc.

(b) UseFindRoot to find the range when $\theta = \pi/4$. (Syntax: FindRoot[f[x]==0,{x,0.5}] finds a numerical solution for f[x]=0, starting to look near the point x=0.5)

(c) Repeat for different values of θ (homing in on a small range near the angle you estimated in part a). Continue until you know the maximum range and the corresponding angle to two significant figures. Connect to point a) and compare with the ideal vacuum values. Does your answer seem reasonable? Briefly, explain.
Extra credit Check what happens in the limit $v_0 \gg v_{ter}$ (say $v_0 = 100v_{ter}$) and $v_0 \ll v_{ter}$ (say $v_0 = 0.01v_{ter}$). Discuss if your results make sense. (+4 points)

Note: When you turn in your homework, judiciously use "print selection" on a minimalist subset of your MMA notebook so we can see your final results, and your discussion, without our having to read through multiple pages of preliminary plots and calculations.