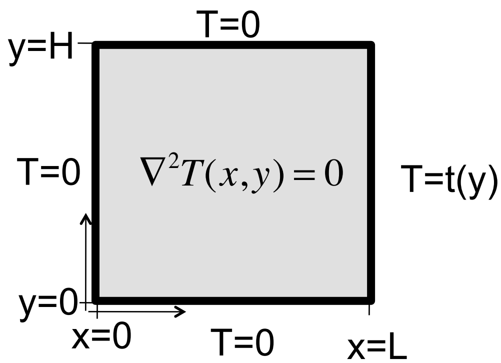
Rectangular plate, with temperature fixed at edges:



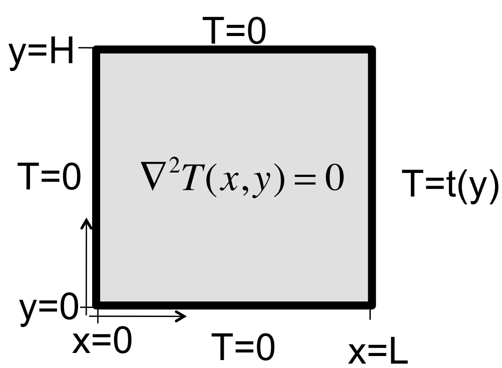
When using separation of variables, so T(x,y)=X(x)Y(y), which variable (x or y) has the sinusoidal solution?

A) X(x) B) Y(y) C) Either, it doesn't matter
D) NEITHER, the method won't work here
E) ???

2- 1

Trial solution: $T(x,y)=(Ae^{kx}+Be^{-kx})(C\cos(ky)+D\sin(ky))$

Applying the boundary condition T=0 at
i) y=0 and ii) y=H gives
(in order!)

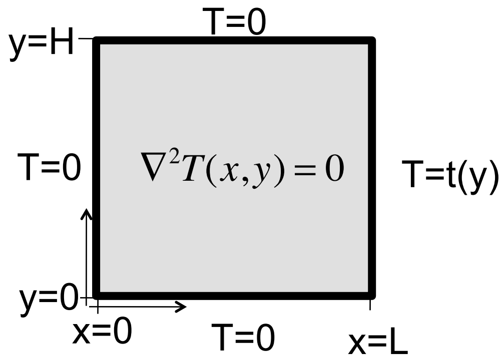


A) i) k=n π/H, ii) A=-B
B) i) k=n π/L, ii) D=0
C) i) A=-B, ii) k=n π/H
D) i) D=0, ii) k=n π/L
E) Something else!!

2- 2

Trial solution: $T(x,y)=(Ae^{kx}+Be^{-kx})(C\cos(ky)+D\sin(ky))$

Applying the boundary condition T=0 at
i) y=0 and ii) y=H gives
(in order!)

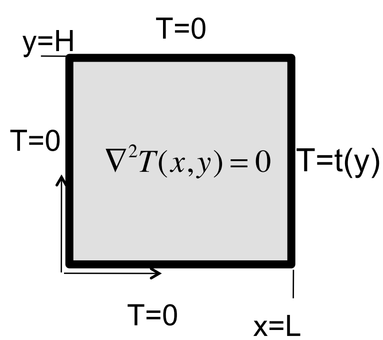


A) i) $k=n\pi/H$, ii) $A=-B$
B) i) $k=n\pi/L$, ii) $D=0$
C) i) $A=-B$, ii) $k=n\pi/H$
D) i) ~~D=0~~, ii) ~~k=n π/L~~ $\longrightarrow$ i) $C=0$ ii) $k = n\pi/H$
E) Something else!!

2- 3

Trial solution: $T_n(x,y)=(A_n e^{n\pi x/H}+B_n e^{-n\pi x/H})(\sin n\pi y/H)$

Applying the boundary condition T(0,y)=0 gives...



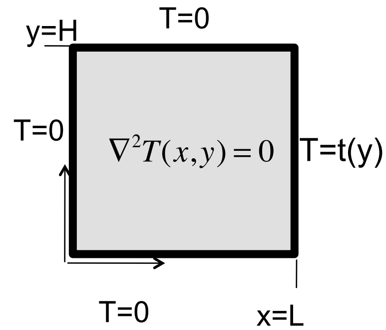
A) $A_n=0$
B) $B_n=0$
C) $A_n=B_n$
D) $A_n=-B_n$
E) Something entirely different!

2- 4

Recalling $\sinh(x)=½(e^{x}-e^{-x})$

Trial solution: $T_n(x,y)=A_n \sinh(n\pi x/H)\sin(n\pi y/H)$

Applying the boundary condition $T(L,y)=t(y)$ does what for us...



A) Determines one A_n
B) Shows us the method of separation of v'bles failed in this instance
C) Requires us to sum over n before looking for A_n's
D) Something entirely different/not sure/...

2- 5

Trial solution: $$T(x,y)=\sum_{n=1}^{\infty} A_n \sinh(n\pi x/H)\sin(n\pi y/H)$$

What is the correct formula to find the A_n's?

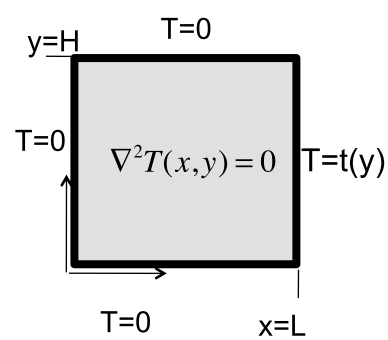
A) $A_n = \frac{2}{L}\int_0^L t(y)\sin(n\pi y/L)dy$

B) $A_n = \frac{2}{L}\int_0^L t(y)\sin(n\pi y/H)dy$

C) $A_n = \frac{2}{H}\int_0^H t(y)\sin(n\pi y/L)dy$

D) $A_n = \frac{2}{H}\int_0^H t(y)\sin(n\pi y/H)dy$

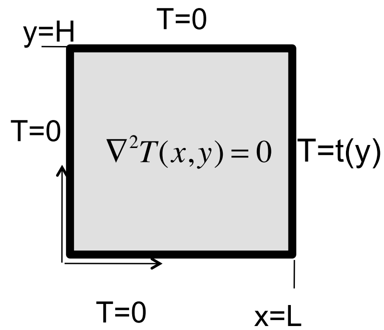
E) Something entirely different!



2- 6

Trial solution: $\mathrm{T}(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H)\sin(n\pi y/H)$

Right b'dry: $\mathrm{t(y)} = \mathrm{T}(L,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi L/H)\sin(n\pi y/H)$

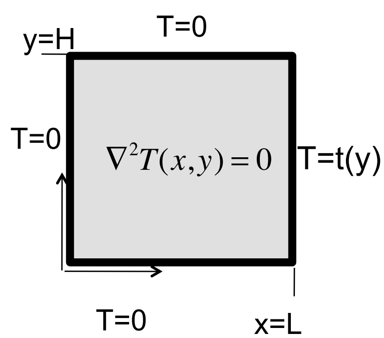


2- 7

Right b'dry: $\mathrm{T}(L,y) = \sum_{n=1}^{\infty} \underbrace{A_n \sinh(n\pi L/H)}_{b_n} \sin(n\pi y/H)$

$$t(y) = \sum_{n=1}^{\infty} b_n \sin(n\pi y/H)$$

$$\mathrm{b_n} = \frac{2}{\mathrm{H}}\int_0^{\mathrm{H}} \mathrm{t(y)\sin(n}\pi\mathrm{y/H)dy}$$



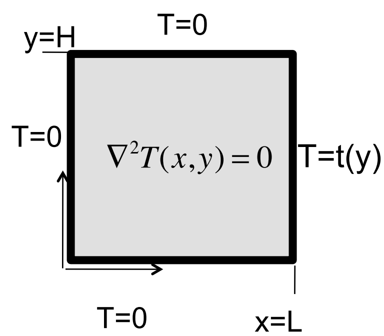
Which means

$$A_n \sinh(n\pi L/H) = \frac{2}{\mathrm{H}}\int_0^{\mathrm{H}} \mathrm{t(y)\sin(n}\pi\mathrm{y/H)dy}$$

2- 8

Solution (!!) : $$\mathrm{T}(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H)\,\sin(n\pi y/H)$$

with: $$A_n = \frac{1}{\sinh(n\pi L/H)} \frac{2}{\mathrm{H}} \int_0^{\mathrm{H}} \mathrm{t(y)\sin(n\pi y/H)dy}$$

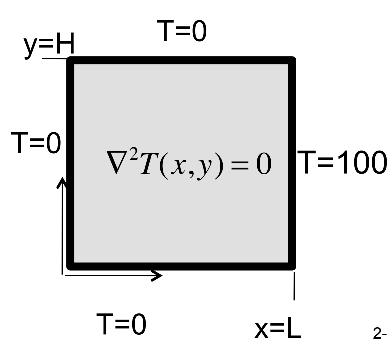


2- 9

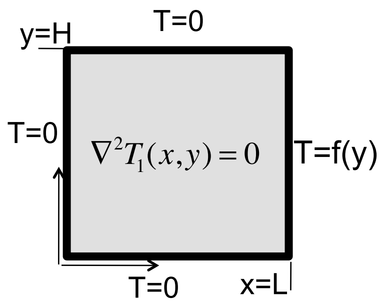
Solution (!!) : $$\mathrm{T}(x,y) = \sum_{n=1}^{\infty} A_n \sinh(n\pi x/H)\,\sin(n\pi y/H)$$

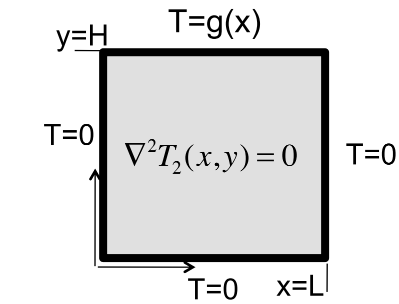
with: $$A_n = \frac{1}{\sinh(n\pi L/H)} \frac{2}{\mathrm{H}} \int_0^{\mathrm{H}} \mathrm{t(y)\sin(n\pi y/H)dy}$$

If e.g.
t(y)=100° (a constant)...



2- 10



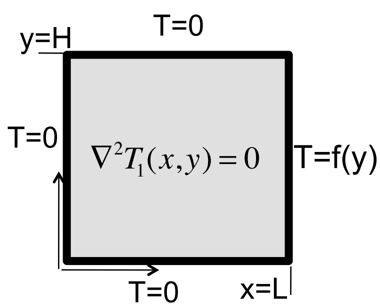


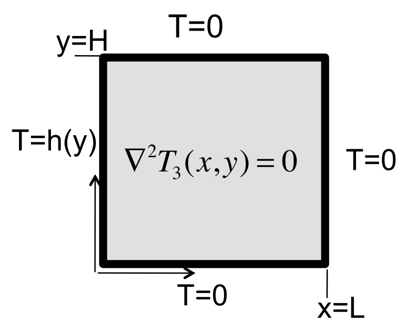
$$\mathrm{T}_1(x,y)=\sum_{n=1}^{\infty} A_n \sinh(n\pi x/H)\,\sin(n\pi y/H)$$

$$\text{with } A_n=\frac{1}{\sinh(n\pi L/H)}\,\frac{2}{\mathrm{H}}\int_0^{\mathrm{H}} \mathrm{f(y)sin(n\pi y/H)dy}$$

How would you find $T_2(x,y)$?

2- 11



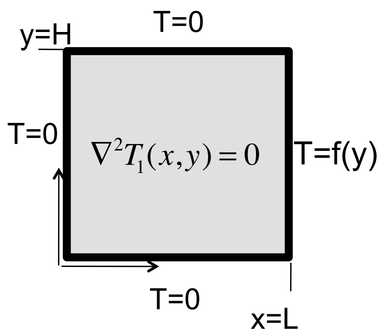


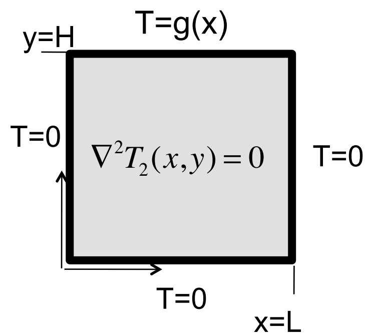
$$\mathrm{T}_1(x,y)=\sum_{n=1}^{\infty} A_n \sinh(n\pi x/H)\,\sin(n\pi y/H)$$

$$\text{with } A_n=\frac{1}{\sinh(n\pi L/H)}\,\frac{2}{\mathrm{H}}\int_0^{\mathrm{H}} \mathrm{f(y)sin(n\pi y/H)dy}$$

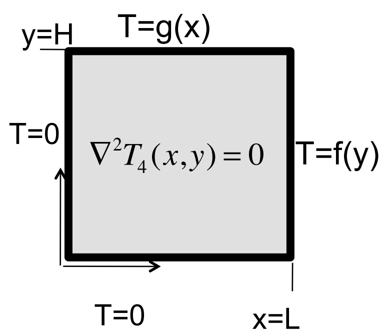
How would you find $T_3(x,y)$?

2- 13

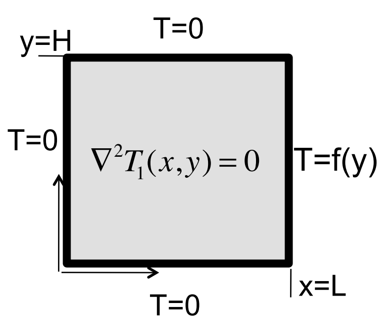


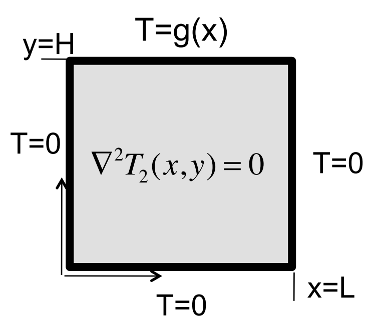


How would you find $T_4(x,y)$?



2- 15

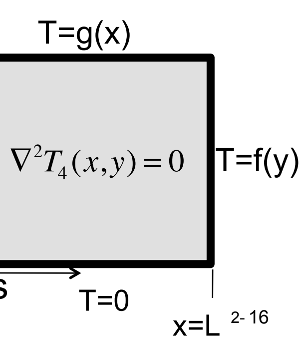




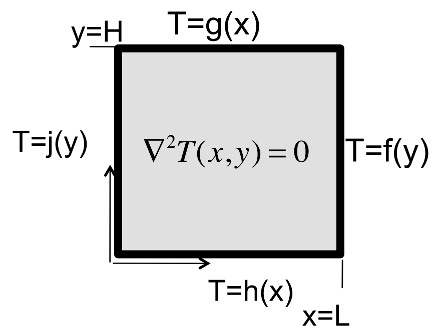
Would this work?

$T_4(x,y) = T_1(x,y) + T_2(x,y)$

A) sweet!
B) No, it messes up Laplace's eqn
C) No, it messes up Bound conditions
D) Other/??



2- 16



We have solved this!

Fourier Series

Fourier Transforms

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega t} \longrightarrow f(t) = \int_{-\infty}^{\infty} \mathrm{g}(\omega)\mathrm{e}^{\mathrm{iwt}}$$

Fourier Series

Fourier Transforms

$$f(t)=\sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t} \longrightarrow f(t)=\int_{-\infty}^{\infty} g(\omega)\mathrm{e}^{\mathrm{iwt}} d(\text{what?})$$

A) dx
B) dt
C) dω
D) Nothing is needed, just $f(t)=\int_{0}^{\infty} g(\omega)\mathrm{e}^{\mathrm{iwt}}$
E) Something else/not sure

2- 22

Fourier Series

Fourier Transforms

$$f(t)=\sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t} \longrightarrow f(t)=\int_{-\infty}^{\infty} g(\omega)\mathrm{e}^{\mathrm{iwt}} d\omega$$

$$c_n=\frac{1}{T}\int_0^T f(t)e^{in\omega_0 t}dt \longrightarrow g(\omega)\propto\int_{-\infty}^{\infty} f(t)e^{i\,\text{what?}}d\text{what?}$$

A) $\int_{-\infty}^{\infty} f(t)e^{in\omega\mathrm{t}}d\mathrm{t}$

B) $\int_{-\infty}^{\infty} f(t)e^{i\omega\mathrm{t}}d\mathrm{t}$

C) $\int_{-\infty}^{\infty} f(t)e^{in\omega\mathrm{t}}d\omega$

D) $\int_{-\infty}^{\infty} f(t)e^{i\omega\mathrm{t}}d\mathrm{t}$

E) Something else/not sure?

2- 23

Fourier Series

Fourier Transforms

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t} \longrightarrow f(t) = \int_{-\infty}^{\infty} \mathrm{g}(\omega)\mathrm{e}^{\mathrm{iwt}} d\omega$$

$$c_n = \frac{1}{T}\int_0^T f(t) e^{in\omega_0 t} dt \longrightarrow g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{i\omega \mathrm{t}} d\mathrm{t}$$

2- 24