What is the general solution to
$Y''(y)-k^2Y(y)=0$

A) $Y(y)=A\,e^{ky}+Be^{-ky}$
B) $Y(y)=Ae^{-ky}\cos(ky-\delta)$
C) $Y(y)=A\cos(ky)$
D) $Y(y)=A\cos(ky)+B\sin(ky)$
E) None of these or MORE than one!

2- 1

What is the general solution to
$X''(x)+k^2X(x)=0$

A) $X(x)=A\,e^{kx}+Be^{-kx}$
B) $X(x)=Ae^{-kx}\cos(kx-\delta)$
C) $X(x)=A\cos(kx)$
D) $X(x)=A\cos(kx)+B\sin(kx)$
E) None of these or MORE than one!

2- 2

When solving $\nabla^2 T(x,y)=0$, separation of variables says:
try T(x,y) = X(x) Y(y)

i) Just for practice, invent *some* function T(x,y) that is manifestly of this form. (Don't worry about whether it satisfies Laplace's equation, just make up some function!) What is your X(x) here? What is Y(y)?

ii) Just to compare, invent some function T(x,y) that is definitely NOT of this form.

Challenge questions:
1) Did your answer in i) satisfy Laplace's eqn?
2) Could our method (separation of variables) ever FIND your function in part ii above?

2- 3

When solving $\nabla^2 T(x,y)=0$, separation of variables says try T(x,y) = X(x) Y(y). We arrived at the equation

$$f(x) + g(y) = 0$$

for some complicated f(x) and g(y)

Invent *some* function f(x) and some *other* function g(y) that satisfies this equation.

Challenge question: In 3-D, the method of separation of variables would have gotten you to f(x)+g(y)+h(z)=0. Generalize your "invented solution" to this case.

2- 4

When solving $\nabla^2 T(x,y)=0$, separation of variables says try T(x,y) = X(x) Y(y). We arrived at

$$\frac{d^2X(x)}{dx^2} = cX(x)$$

and

$$\frac{d^2Y(y)}{dy^2} = -cY(y)$$

Write down the *general solution* to both of these ODEs!

Challenge: Is there any deep ambiguity about your solution?

2- 5

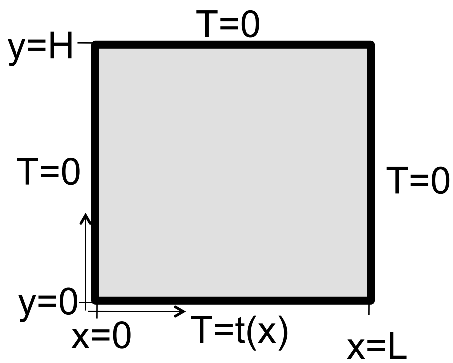
In part B of the Tutorial, you have
f(x) = Csin(kx) + D cos(kx),
with boundary conditions f(0)=f(L)=0.

Is the f(x) you found at the end unique?

A) Yes, we found it.
B) Sort of – we found the solution, but it involves one completely undetermined parameter
C) No, there are two very different solutions, and we couldn't choose!
D) No, there are infinitely many solutions, and we couldn't choose!
E) No, there are infinitely many solutions, each of which has a completely undetermined parameter!

2- 6

Rectangular plate, with temperature fixed at edges:

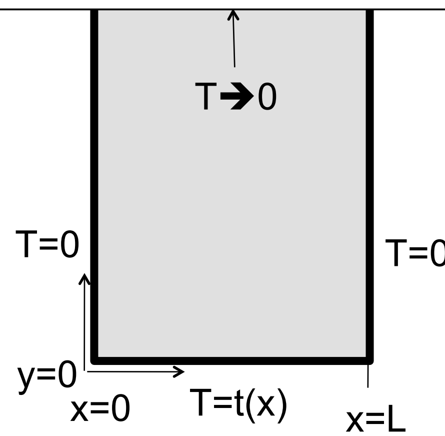


Written mathematically, the left edge tells us T(0,y)=0.

Write down analogous formulas for the other 3 edges.
These are the *boundary conditions* for our problem

2- 7

Semi-infinite plate,
with temp fixed at edges:



When using separation of variables, so T(x,y)=X(x)Y(y), which variable (x or y) has the sinusoidal solution?

A) X(x) B) Y(y) C) Either, it doesn't matter
D) NEITHER, the method won't work here
E) ???

2- 8

We are solving $\nabla^2 T(x,y)=0$, with boundary conditions:
$T(x,y)=0$ for the left and and right side, and "top" (at ∞)
$T(0,y)=0$, $T(L,y)=0$, $T(x,\infty)=0$.
The fourth boundary is $T(x,0) = f(x)$
What can we conclude about our solution $Y(y)$?

A) Cannot contain e^{-ky} term
B) Cannot contain e^{+ky} term
C) Must contain both e^{-ky} and e^{+ky} terms
D) Must contain sin(ky) and/or cos(ky) term
E) ???

2- 10

Using 3 out of 4 boundaries, we have found
$T_n(x,y) = A_n \sin(n\pi x/L)\, e^{-n\pi y/L}$

Using the bottom boundary, $T(x,0)=f(x)$, we can compute all the An's:

$$A_n = \frac{2}{L}\int_0^L f(x)\sin(n\pi x/L)\,dx$$

In terms of these (known) constants, what then is the complete final answer for $T(x,y)$?

2- 12

Using all 4 boundaries, we have found

$$T(x,y) = \sum_{n=1}^{\infty} A_n \sin(n\pi x/L)\, e^{-n\pi y/L}$$

where $A_n = \dfrac{2}{L}\int_0^L f(x)\sin(n\pi x/L)dx$

Now suppose f(x) on the bottom boundary is
T(x,0)=3sin(5 π x/L)

What is the complete final answer for T(x,y)?

2- 13