A tennis ball is hit directly upwards with initial speed v_0.
Compare the time T to reach the top (height H) to time and height in an ideal (vacuum) world.

A) $T>T_{vacuum}$, $H \approx H_{vacuum}$
B) $T>T_{vacuum}$, $H<H_{vacuum}$
C) $T \approx T_{vacuum}$, $H<H_{vacuum}$
D) $T<T_{vacuum}$, $H<H_{vacuum}$
E) Some other combination!!

1

With quadratic air drag, $v(t)=v_0/(1+t/\tau)$
where $\tau=m/(cv_0)$, and $c=(1/2)c_0 A \rho_{air.}$
(For a human on a bike, c is of order .2 in SI units)

Can you confirm that c is about 0.2? (c_0 is ~1 for non-aerodynamic things, and ~.1 for very- aerodynamic things.

Roughly how long does it take for a cyclist on the flats to drift down from v_0=10 m/s (22 mi/hr) to ~1 m/s?

A) a couple seconds
B) a couple minutes
C) a couple hours
D) none of these is even close.

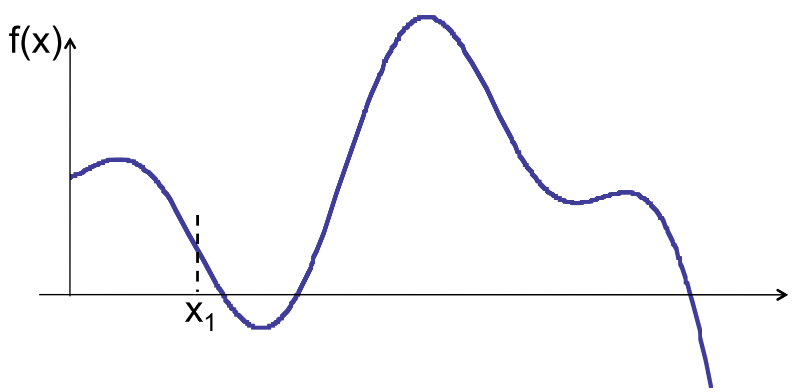
In real life, the answer is shorter than this formula would imply. Why?

3

An object is launched directly upwards with initial speed v_0.
Compare the time t_1 to reach the top to the time t_2 to return back to the starting height.

A) $t_1>t_2$
B) $t_1=t_2$
C) $t_1<t_2$
D) Answer depends on whether v_0 exceeds v_t or not.

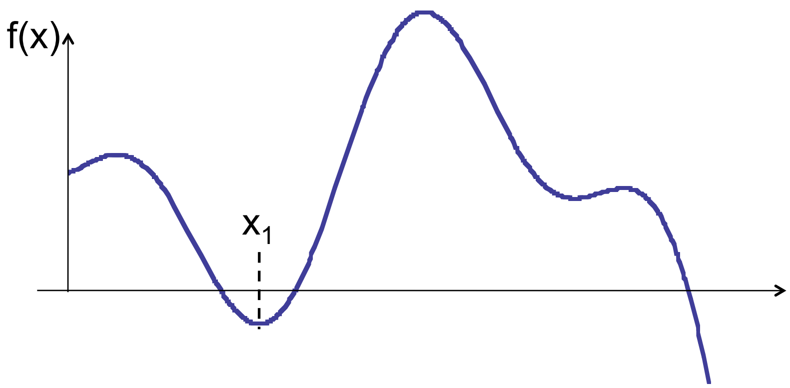
4



If we expand f(x) around point x_1 in a Taylor series,
$f(x) = a_0 + a_1 (x-x_1) + a_2 (x-x_1)^2 + \ldots$
What is your best guess for the signs of a_0 and a_1?

A) a_0 is +, a_1 is +
B) a_0 is +, a_1 is –
D) a_0 is -, a_1 is +
D) a_0 is -, a_1 is –
E) Other! (Something is 0, or it's unclear)

8



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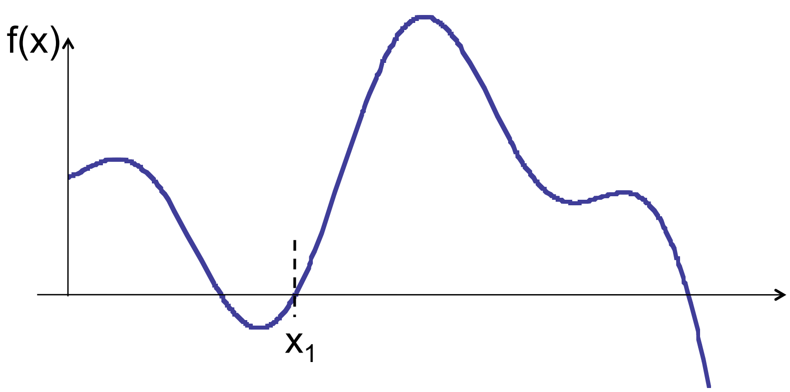

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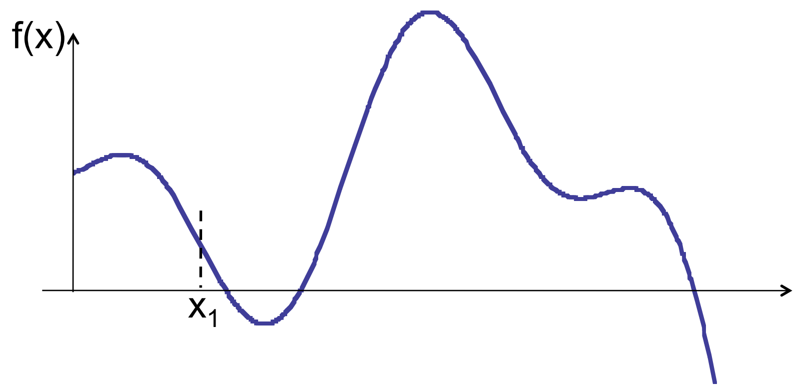
10



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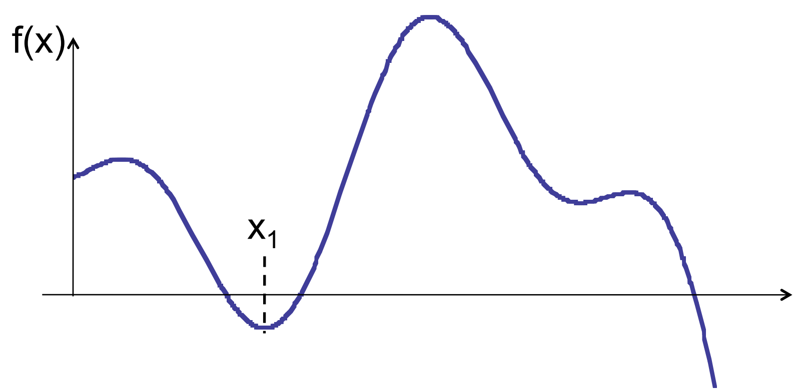
$f(x) = a_0 + a_1 (x - x_1) + a_2 (x - x_1)^2 + \ldots$
What is your best guess for the sign of a_2?

A) +(?) B) -(?) C) 0 (?)
D) ????? How could we possibly tell this without a formula?

11



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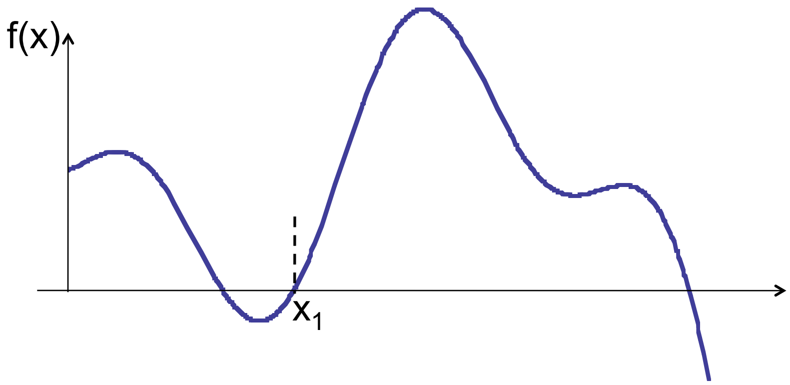
12



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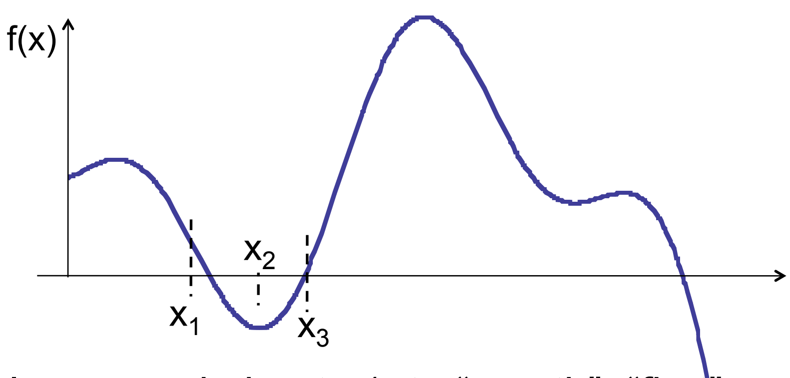
13



Taylor expand about x1, to “zeroth”, “first”, and “second” orders. In each case, SKETCH (on top of the real curve) what your “Taylor approximation” looks like

Repeat for the other two points.

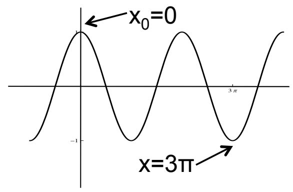
14

f(x)=cos(x).

If x_0=0, will the full Maclaurin series expansion produce the exact value for cos(3π)=-1?

$$\cos(3\pi) \stackrel{?}{=} \cos(0) + \left.\frac{d\cos(x)}{dx}\right|_{x=0}(3\pi) + \frac{1}{2!}\left.\frac{d^2\cos(x)}{dx^2}\right|_{0}(3\pi)^2 + \ldots$$

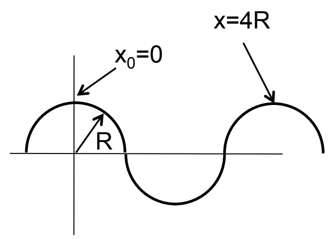
A) Yes
B) No, not even close
C) Close, but not exact



15

Consider f(x), composed of an infinite series of semicircles.

If x_0=0, will the Maclaurin series expansion produce the correct value for f(4R)=+R?



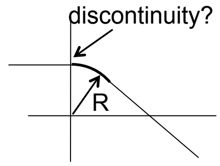
A) Yes
B) No, not even close
C) Close, but not exact

16

Consider f(x), which is a quarter-circle joining two straight lines. Near x=0:

$$f(x) = \begin{cases} R, & x < 0 \\ \sqrt{R^2 - x^2}, & x > 0 \end{cases}$$

A) f(x) is discontin. at x=0
B) f'(x) is discontin. at x=0
C) f''(x) is discontin. at x=0
D) Some higher deriv is discontin. at x=0
E) f and all higher derivs are continuous at x=0.



Does the Taylor Series expansion
$\cos(\theta) = 1 - \theta^2/2! + \theta^4/4! + \ldots$
apply for θ measured in

A) degrees
B) radians
C) either
D) neither

What are the first few terms of the Taylor Series expansion for e^x?

A) $e^x = 1 + x^2/2! + x^4/4! + \dots$

B) $e^x = 1 - x^2/2! + x^4/4! + \dots$

C) $e^x = 1 + x + x^2/2! + x^3/3! + \dots$

D) $e^x = 1 - x + x^2/2! - x^3/3! + \dots$

19

What is the next term in the binomial expansion for $\sqrt{1+\varepsilon} \approx 1 + \varepsilon/2 + \dots$?

A) ε^2

B) $\varepsilon^2/2$

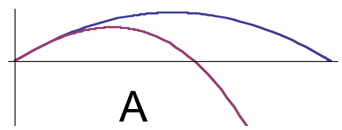
C) $\varepsilon^2/4$

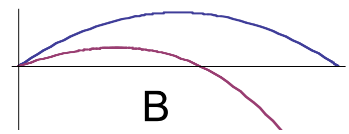
D) $\varepsilon^2/8$

E) Something else

20

Which of these two plots seems more realistic for a tennis ball trajectory, comparing (in each graph) the path with (purple), and without (blue), drag.





C) Neither of these is remotely correct!