Phys 2210 Lecture Notes ①

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www.colorado.edu /physics /phys2210

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Texts, grading, schedule, notes, office hrs, lots more
(2)
↳ Tues & Wed.
– See web!

Homework due every Thurs, start of class
(1st one due this week)

Online HW due every Tues AM (10 AM!)

– See web!

Exams: Feb 10, Mar 17 + final
↳ in-class.

Bring clickers to class

Math co-req is APPM 2360 (or MATH 3130)

CLASSICAL MECHANICS & MATHEMATICAL METHODS I

2 sides to this course!

Flesh out 1110 concepts & build math toolbox

→ Problem-solving skills, & background for rest of physics.

Big topics for this term (subject to change...)

	Physics	Math
- Motion + Energy	• Newton's Laws with friction/drag • Potential Energy • Stability • Non-inertial frames	• Ordinary diff eq's, ODE's Curvilinear coords gradient operator
- Harmonic Motion	• with damping, + outside forces • Model for lots of physics.	Fourier series + transform Delta functions
- Heat flow & Gravity	• Model " " " "	Partial diff eq's PDE's Special functions

Overview of CLASSICAL Mechanics (Taylor 1.3–1.5)

Kinematics

Description of Motion

- Coordinate systems
- Vector nature

(+ Rotational Kinematics)

Dynamics

Explanation of Motion

Newton's Laws

1) Object at rest [uniform motion] remains " " " if $\vec{F}_{net, external} = 0$

[Defines inertial reference frame)

2) $\vec{F}_{net, external} = d\,\vec{P}_{system}/dt$

$= m\,\vec{a}_{cm}$ (if mass, m, is constant)

3) $\vec{F}_{AB} = -\vec{F}_{BA}$

(+ Rotational extensions, e.g $\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$)

Conservation Laws

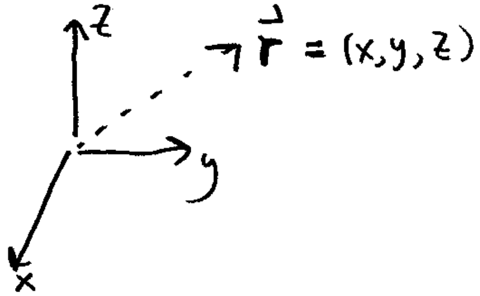
1) If $\vec{F}_{net, ext} = 0$, then $\vec{P}_{system}$ is conserved (And similarly for $\vec{L}$, if $\vec{\tau}_{net} = 0$)

2) If Forces are all conservative, E_{Total} is conserved ($KE + PE$)

Kinematics + coordinate systems (Taylor 1.2)

Laws of physics independent of choice of coord. system (!!)
(Vector notation helps make this clear)

• Cartesian ("rectangular") coordinates



$\vec{r} = $ "position vector"
$= x\hat{i} + y\hat{j} + z\hat{k}$
$\underline{\underline{\text{or}}}\ x\hat{x} + y\hat{y} + z\hat{z}$
$\underline{\underline{\text{or}}}\ x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3$

Unit vectors : $|\hat{x}| = 1$ (unit vector has no units!) no dimensions

It just "points".

Generally, $\vec{A} = \sum_i A_i \hat{e}_i$

$\vec{A} \cdot \vec{B} = \sum_i A_i B_i = |A|\,|B| \cos\theta$ Between $\vec{A}$ & $\vec{B}$

$= A B \cos\theta$

A means $|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}}$

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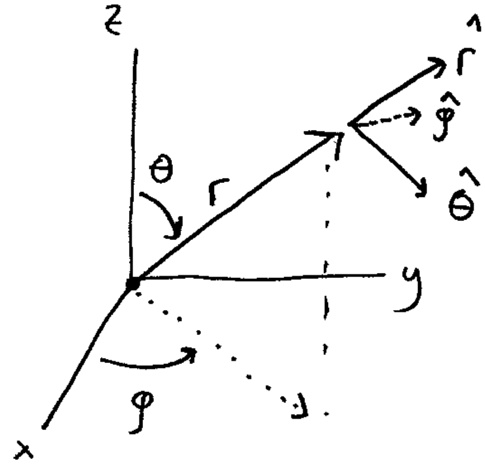
Spherical coords

(Taylor 1.7)



Note! θ, φ are opposite what math uses! This is the physics convention. (Taylor p. 134)

$\hat{r}$, $\hat{\theta}$, and $\hat{\varphi}$ depend on where the point is (!)

Each points in direction of increase in r, (or θ, or φ)

$\vec{r}$ = position vector (Bold in textbook)

$r = |\vec{r}|$ is the distance from the origin.

$\hat{r}$ = unit vector pointing along $\vec{r}$, so $\hat{r} = \dfrac{\vec{r}}{r}$

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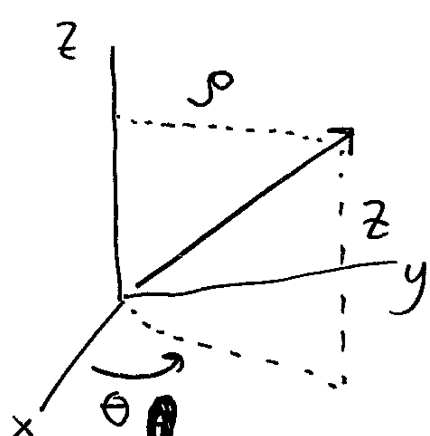
cylindrical coordinates:

ρ = distance from z - axis

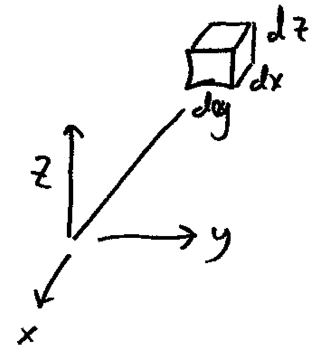
Now it is θ, not φ



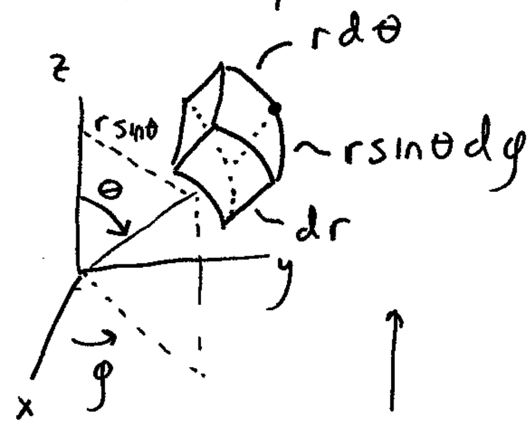
(Except sometimes we stick with φ here too, ack!)

When integrating (over surfaces or volumes), draw a picture! (See. Boas ch 5.4)

e.g $d(\text{Volume}) = dx\, dy\, dz$ ⟶



But, in spherical coords:



$$dV = r^2 \sin\theta\, d\theta\, d\varphi\, dr$$

↑ convince yourself!

↑ Note, $\theta \leftrightarrow \varphi$ swap from math texts.

Life is 3-D! $v = dx/dt$ is too simple, really

position $\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$

$\hat{x}, \hat{y}, \hat{z}$ are all constants in magnitude + direction, so

velocity $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\,\hat{x} + \dfrac{dy}{dt}\,\hat{y} + \dfrac{dz}{dt}\,\hat{z}$

$$= V_x\,\hat{x} + V_y\,\hat{y} + V_z\,\hat{z}$$

Could write this as $\vec{v} = \dot{\vec{r}}$ = "time derivative of $\vec{r}$"

$$= (\dot{x}, \dot{y}, \dot{z})$$

[Boas will use ′ (prime) for derivatives, so e.g

- if $f(x)$ is a function of x, then f' means $\dfrac{d\,f(x)}{dx}$]

acceleration $\vec{a} = \dfrac{d\vec{v}}{dt} = \dot{\vec{v}}$

so in Cartesian coords, $\vec{a} = (\ddot{x}, \ddot{y}, \ddot{z})$

Comments on Newton's Laws, p.3 of these notes
(Taylor 1.4)

- They apply in inertial reference frames, non-accelerating.
  (We'll talk about physics in non-inertial frames later!)

- They apply only if $v \ll c$ (non-relativistic speeds)
  and for "macroscopic objects" (non-Quantum mechanical)

- Newton's 3rd law even has issues in E&M with $\vec{v}$ dependent forces.
  It is saved if you consider EM fields too, as part of your system. This is upper-division E&M stuff!

Newton's Laws are empirical, experimental facts

They are meaningless without operational definitions of the kinematic variables, force, and mass

Force is a measure of interactions, the "push or pull" on an object.

$\vec{a} = \vec{F}/m$ is the "equation of motion" of an object.

Given $\vec{x}$ & $\vec{v}$ of all particles, $\vec{F}$ can be computed, + thus $\vec{a}$ which gives $\vec{x}$ + $\vec{v}$ at all future times.

Cornerstone of mechanics: predictive power!

In 3-D Cartesian coordinates, $\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$

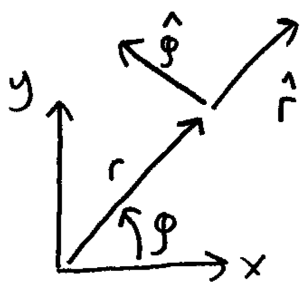
$$\ddot{\vec{r}} = \ddot{x}\,\hat{x} + \ddot{y}\,\hat{y} + \ddot{z}\,\hat{z}$$

so
$$F_x = m\ddot{x}$$
$$F_y = m\ddot{y}$$
$$F_z = m\ddot{z}$$

Separate eq'ns! Handy, and easy enough if you know $\vec{F}$. But sometimes we need to work in other coord systems!

Newton's Law in Polar-coordinates

(so, this is 2-D, basically same as 3-D cylindrical)



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

Look at figure + convince yourself:

$$\hat{r} = \cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}$$
$$\hat{\varphi} = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}$$

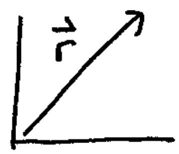
(Taylor 1.7, Boas 6.4)

Force $\vec{F} = F_r\,\hat{r} + F_\varphi\,\hat{\varphi}$.

So... does $F_r \underset{\times}{=} m\ddot{r}$? does $F_\varphi \underset{\times}{=} m\ddot{\varphi}$?? No!!

why not? Because $\vec{r}$ is $\underline{\underline{\text{not}}}$ $r\,\hat{r} + \varphi\,\hat{\varphi}$!!

(This can't be right, units don't even make sense.)

Look again  $\vec{r} = \underbrace{r}_{\text{distance}}\ \underbrace{\hat{r}}_{\text{direction}}$

now $\vec{v} = \dot{\vec{r}}$ is <u>not</u> $\dot{r}\,\hat{r}$! Why not? Because $\hat{r}$ is itself a function of where the object is, so <u>it</u> has a derivative. Taylor gives a "graphical" derivation, but I prefer a direct calculation:

<u>Chain rule</u> says $\dfrac{d}{dt}(\underset{\substack{\uparrow\\ \text{scalar}\\ \text{function}}}{f} \cdot \underset{\substack{\uparrow\\ \text{vector}\\ \text{function}}}{\vec{g}}) = f\dfrac{d\vec{g}}{dt} + \dfrac{df}{dt}\vec{g}$

$$\text{so}\quad \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\,\hat{r}) = \dot{r}\,\hat{r} + \underbrace{r\frac{d\hat{r}}{dt}}_{\text{Something extra!}}$$

what's $d\hat{r}/dt$? Go back to prev. page:

$$\hat{r} = \cos\varphi\,\hat{x} + \sin\varphi\,\hat{y} \quad \leftarrow \hat{x}, \hat{y} \text{ have no time dep}$$

$$\text{so}\quad d\hat{r}/dt = -(\sin\varphi)\,\dot{\varphi}\,\hat{x} + (\cos\varphi)\,\dot{\varphi}\,\hat{y}$$

$$= \dot{\varphi}\,(-\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y})$$

$$= \dot{\varphi}\,\hat{\varphi}$$

Cool! ($\hat{r}$ never changes length, but <u>direction</u> can change (if φ varies with time))

Put it together, using our new identity $\dot{\hat{r}} = \dot{\varphi}\,\hat{\varphi}$

$$\vec{v} = \dot{\vec{r}} = \frac{d}{dt}(r\,\hat{r}) \overset{\text{chain rule}}{=} \underbrace{\dot{r}\,\hat{r}} + \underbrace{r\,\dot{\varphi}\,\hat{\varphi}}$$

The 1st term is just "radial velocity"

The 2nd term is maybe familiar from 1110, where we used to call angular velocity "ω", (now it's $\dot{\varphi}$).

Remember $V_{\text{tangential}} = \omega\, r$?

That's our new term! $V_\varphi = \dot{\varphi}\, r$

$$\vec{a} = \dot{\vec{V}} = \frac{d}{dt}\left(\underbrace{\dot{r}\,\hat{r}} + \underbrace{r\,\dot{\varphi}\,\hat{\varphi}}\right) \rightarrow \text{Chain rule gives 3 terms!}$$

$$= \overbrace{\ddot{r}\,\hat{r} + \dot{r}\,\dot{\hat{r}}} + \overbrace{\dot{r}\,\dot{\varphi}\,\hat{\varphi} + r\,\ddot{\varphi}\,\hat{\varphi} + r\,\dot{\varphi}\,\dot{\hat{\varphi}}}$$

we need $\dot{\hat{\varphi}} = \frac{d}{dt}\left(-\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}\right) = \dot{\varphi}\left(-\cos\varphi\,\hat{x} - \sin\varphi\,\hat{y}\right)$

$= -\dot{\varphi}\,\hat{r} \leftarrow$ (Look back at p. 9)

so $\vec{a} = \ddot{r}\,\hat{r} + \dot{r}\,\dot{\varphi}\,\hat{\varphi} + \dot{r}\,\dot{\varphi}\,\hat{\varphi} + r\,\ddot{\varphi}\,\hat{\varphi} - r\,\dot{\varphi}^2\,\hat{r}$

$$\vec{a} = \hat{r}\left(\ddot{r} - r\dot{\varphi}^2\right) + \hat{\varphi}\left(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}\right)$$

Let's make sense of all these pieces!

$$\vec{a} = \hat{r}(\ddot{r} - r\dot{\varphi}^2) + \hat{\varphi}(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})$$

New!

Pure "radial accel"

this is $-r\omega^2$ in 1110 language,

$= -v_{Tang}^2/r$, centripetal accel!

this looks like "$r\alpha$" is 1110 language, angular accel

Newton's 2nd law in Polar coords, then:

$$F_r = m(\ddot{r} - r\dot{\varphi}^2)$$

$$F_\varphi = m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})$$

This is N-2, or "eq'n of motion" for problems with "cylindrical symmetry".

Example Swing a rock on a string of length R

so $\dot{r} = \ddot{r} = 0$, and

$F_r = -mR\dot{\varphi}^2$ (ah, that's just "$F_{radial} = -\frac{mv^2}{R}$"!)

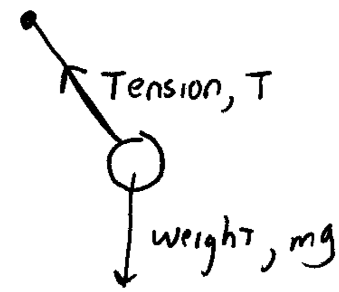
$F_\varphi = mR\ddot{\varphi}$ (ah, that's just "$F_{tangent} = mR\alpha$"!)

Example φ is fixed (no rotation)

$F_r = m\ddot{r}$ Basically just Newton's law in 1-D!

$F_\varphi = 0$

Phys 2210 (13)

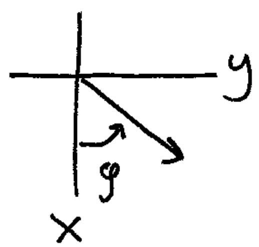


Example: Pendulum

We need to pick a coord. system

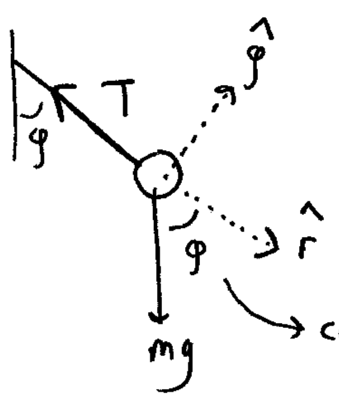
Polar seems natural!

But origin is down, I'd like that to be $\varphi = 0$, so let's tilt our head by $90°$ from previous pictures



Now
$$F_r = m(\ddot{r} - r\dot{\varphi}^2)$$
$$F_\varphi = m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})$$

and r is fixed, it's R, so $\dot{r} = \ddot{r} = 0$



convince yourself this is also φ!

Look at picture: convince yourself!
$$\begin{cases} T_r = -T \\ T_\varphi = 0 \end{cases}$$

$$(\text{Weight})_r = +mg\cos\varphi$$
$$(\text{Weight})_\varphi = -mg\sin\varphi$$

N-2 says $\vec{F}_{net} = m\vec{a}$, or

$$(-T + mg\cos\varphi) = -mR\dot{\varphi}^2 \quad \leftarrow \text{this is } \Sigma F_r = m(\ddot{r} - r\dot{\varphi}^2)$$

$$-mg\sin\varphi = +mr\ddot{\varphi} \quad \leftarrow \text{this is } \Sigma F_\varphi = m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})$$

Phys 2210 (14)

The eq'n of motion for φ is

$$\ddot{\varphi} = -\frac{g}{r}\sin\varphi$$

Familiar! If $\varphi > 0$, $\ddot{\varphi} < 0$, so 

If $\varphi < 0$, $\ddot{\varphi} > 0$, so 

Makes sense

If $\varphi = 0$, $\ddot{\varphi} = 0$. Stable equilibrium at bottom

If $\varphi = 180^\circ$, $\ddot{\varphi} = 0 \rightarrow$ (Unstable) equilibrium, (more later!)

$\leftarrow$ more soon!

For small φ, $\sin\varphi \simeq \varphi$

$$\text{so} \quad \ddot{\varphi} = -\frac{g}{r}\varphi = -\omega^2\varphi \quad \text{with } \varphi \equiv \sqrt{g/r}$$

This is an ordinary diff. eq with general sol'n

$\varphi(t) = A\sin\omega t + B\cos\omega t$. Simple Harmonic motion.

More soon!