2.16

The binomial expansion is:

$$(1+\varepsilon)^n \approx 1 + n\varepsilon + \ldots$$

On Tuesday, a clicker question suggested we might try :

$$(a^2 + z^2)^{-1/2} \stackrel{?}{\approx} a^2 - (1/2)z^2 + \ldots$$

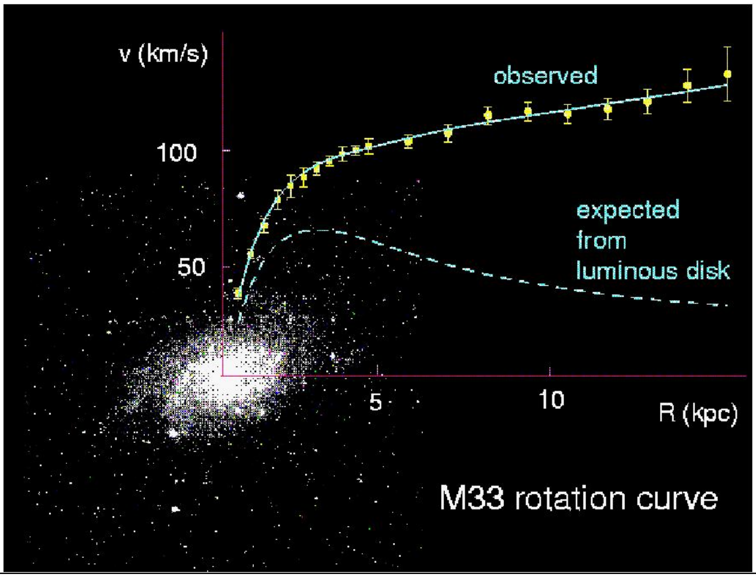
When asked if that was ok, I hesitated, saying I didn't trust it...
So, what do you think? Is that alternate form

A) Correct, but only to leading order, it will fall apart in the next term
B) It's fine, it's correct to all orders, it's the binomial expansion!
C) Utterly false, even to leading order (that someone should really have noticed on Tuesday ☺)

2-

Summary: Gauss' law

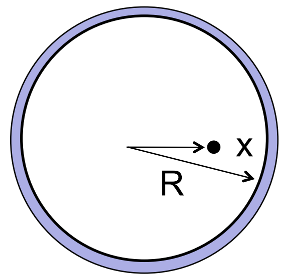
$$\text{For gravity:} \quad \oiint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = -4\pi G M_{enclosed}$$



2- 2

You have a THIN spherical uniform mass shell of radius R, centered on the origin.
What is the g-field at a point "x" near an edge, as shown?

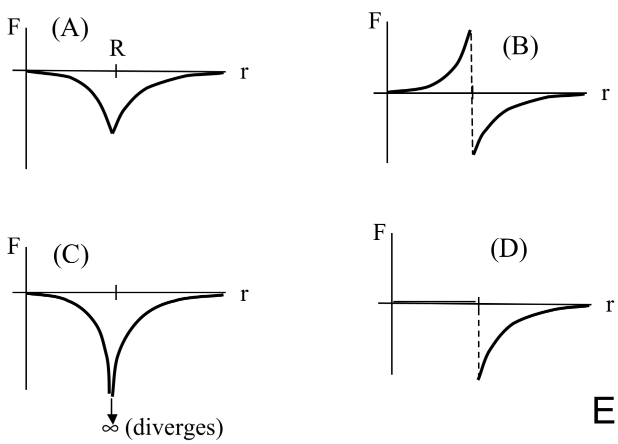
A) zero
B) nonzero, to the right
C) nonzero, to the left
D) Other/it depends!

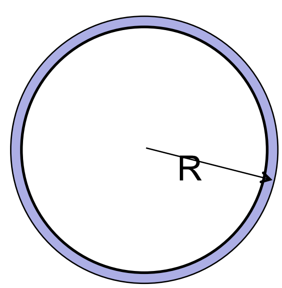


Challenge question: Can you prove/derive your answer by thinking about Newton's law of gravity directly?

2- 3

A test mass m moves along a straight line toward the origin, passing through a THIN spherical mass shell of radius R, centered on the origin.
Which graph correctly shows the force F on the test mass vs. position r ?



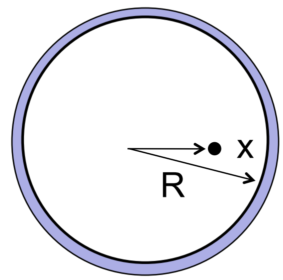


E) Other!

2- 5

Assuming that $U(\infty)=0$, what can you say about the potential at point x?

A) It is zero (everywhere inside the sphere)
B) It is positive and constant everywhere inside the sphere
C) It is negative, and constant everywhere inside the sphere
D) It varies within the sphere
E) Other/not determined



Challenge question: What is the formula for $U(0)-U(\infty)$?

Oscillations

How many initial conditions are required
to fully determine the general solution
to a 2nd order linear differential equation?

2- 8

Since cos(ωt) and cos(-ωt) are both solutions of

$$\ddot{x}(t) = -\omega^2 x(t)$$

can we express the general solution as
$y(t)=C_1\cos(\omega t) + C_2\cos(-\omega t)$?

A) yes
B) no
C) ???/depends

2- 9

What is the general solution to the ODE $\ddot{x}(t) = -\omega^2 x(t)$ where ω is some (known) constant?

A) $x(t) = A\cos\omega t + B\sin\omega t$

B) $x(t) = Ce^{i\omega t} + De^{-i\omega t}$

C) $x(t) = A\cos\omega t + B\sin\omega t + Ce^{i\omega t} + De^{-i\omega t}$

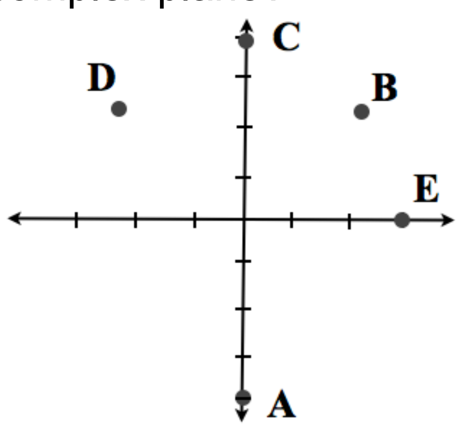
D) None of these is fully general!

E) More than one of these is fine

Challenge question: Is there any OTHER general form for this solution?

2- 10

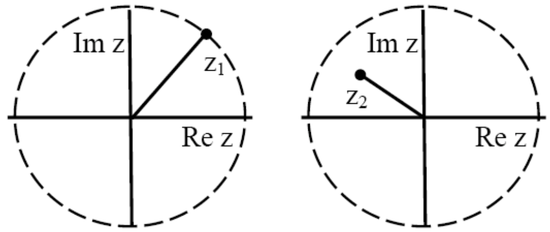
Which point below best represents $4e^{i3\pi/4}$ on the complex plane?



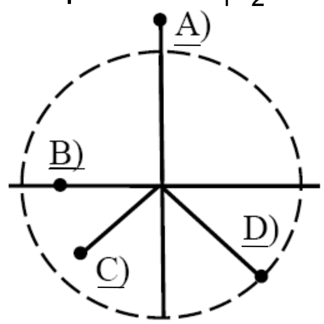
Challenge question: Keeping the general form $Ae^{i\theta}$, do any OTHER values of θ represent the SAME complex number as this? (If so, how many?)

2- 11

Consider two complex numbers, z_1 and z_2.
The dotted circle shows the unit circle, where $|z|=1$.



Which shows the product $z_1 z_2$?



E) I have no idea

What is $(1+i)^2/(1-i)$

A) $e^{i\,\pi/4}$

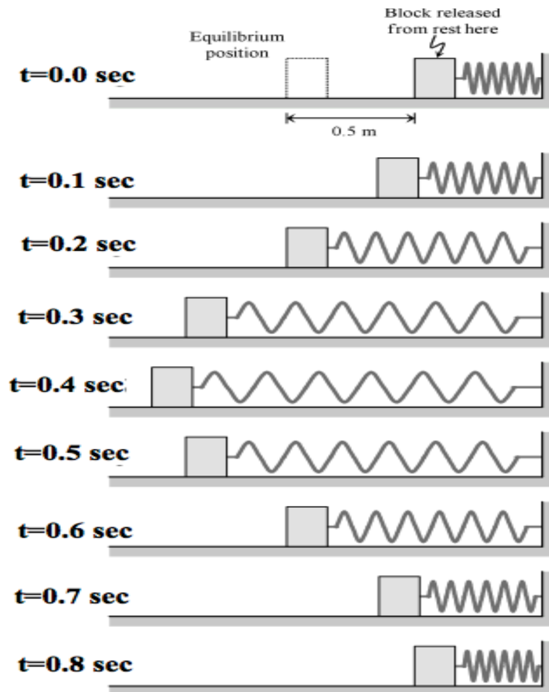
B) Sqrt[2] $e^{i\,\pi/4}$

C) $e^{i\,3\pi/4}$

D) Sqrt[2]$e^{i\,3\pi/4}$

E) Something else!

Based on the pictures what is the period of motion of the block?



A) .2 s
B) .4 s
C) .6 s
D) .8 s
E) None of these/ not enough info!

For the previous situation, what happens to the period of motion if the spring constant is increased?

A) Increases
B) decreases
C) unchanged

2- 21

For the previous situation, what happens to the period of motion if the mass is increased by 4?

A) Increases by 2x
B) Increases by 4x
C) unchanged
D) Decreases by 2x
E) Decreases by 4x

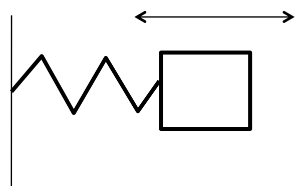
2- 22

For the previous situation, what happens to the period of motion if the initial displacement is increased by 4?

A) Increases by 2x
B) Increases by 4x
C) unchanged
D) Decreases by 2x
E) Decreases by 4x

2- 23

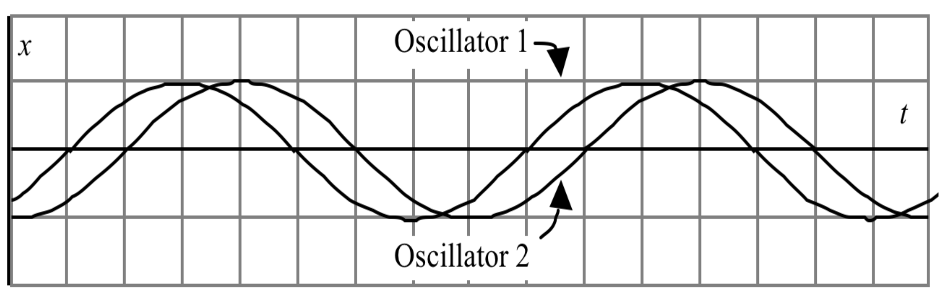
A mass m oscillates at the end of a spring (constant k)
It moves between x=.1 m to x=.5 m.
The block is at x=0.3 m at t=0 sec, moves out to x=0.5 m and returns to x=0.3 m at t=2 sec.



Write the motion in the form $x(t)=x_0+A\cos(\boldsymbol{\omega} t+\varphi)$, and find numerical values for x_0, A, $\boldsymbol{\omega}$, and φ

Write the motion in the form $x(t)=x'_0+A'\sin(\boldsymbol{\omega'} t+\varphi')$, and find numerical values for x'_0, A', $\boldsymbol{\omega'}$, and φ'

2- 24



Oscillators have $x_i(t)=A_i\cos(\omega_i t+\varphi_i)$ (for i=1, 2)
Which parameters are *different*?

What is the difference between φ_1 and φ_2?
Which is larger (more positive?)

2- 25