# phase space DIAGRAMS: damped harmonic motion

## Underdamped oscillator

Suppose that a simple harmonic oscillator were subject to a retarding force that is proportional to the velocity of the oscillator.   
The phase space plot shown below corresponds to a motion of the oscillator in the undamped case.

### On the diagram, sketch the (approximate) phase space trajectory for the situation described. Discuss your reasoning with your partners.

**Starts at point *P;* amplitude decreases**

**by a factor of 2 with each oscillation**



### For a damped oscillator, is it correct for the phase space trajectory to cross the vertical *(**)* axis at right angles? Explain why or why not. (*Hint:* In this case, is the net force exerted on the oscillator equal to zero when it passes through *x* = 0?)

For a damped oscillator, is it correct for the phase space trajectory to cross the horizontal *(x)* axis at right angles? Explain why or why not.

### Is the phase space trajectory that you sketched in part A consistent with your answers in part B? If not, resolve the inconsistencies.

**✓ STOP HERE** and check your results with an instructor before proceeding to the next section.

## Critically damped oscillator

Now suppose that the oscillator were critically damped, *i.e.,* suppose that the damping factor *(β)* for the retarding force were now equal to the angular frequency of the undamped oscillator *(β*= *o).* In this case, the position *x(t)* of the critically damped oscillator is given by:

*x(t)* = *(At* + *B)* *e*- *βt*

where *A* and *B* are arbitrary constants.

### Differentiate the above expression for *x(t)* and show that the parametrized equation *(x*, *t)* can be written:

 = – *βx* + *A* *e*- *βt*

Your result above suggests that the asymptotic behavior (as *t* → ∞) of the critically damped oscillator can be represented by a *straight line* on a phase space diagram. What is the equation for this line?

### Each phase space plot shown below corresponds to a motion of the oscillator in the undamped case.

#### On each diagram, *accurately* sketch the line that would describe the asymptotic behavior of the oscillator in the critically damped case. Explain your reasoning.

 

#### For each of the starting points *(R* and *T)* shown in the diagrams above, draw a qualitatively correct phase space trajectory for the subsequent motion of the critically damped oscillator. Discuss your results with your partners.

**✓ STOP HERE** and check your results with an instructor.