

# Electricity and Magnetism II

Griffiths Chapter 9 EM Waves  
Clicker Questions



A function,  $f(x, t)$ , satisfies this PDE:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 f}{\partial t^2}$$

Invent two different functions  $f(x, t)$  that solve this equation. Try to make one of them “boring” and the other “interesting” in some way.

The complex exponential,  $e^{i2\pi}$   
is equal to:

- A) 0    B)  $i$     C) 1    D)  $p$   
E) Something else

$$e^{i2\pi} = 1$$

A function,  $f$ , satisfies the wave equation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

Which of the following functions work?

- A)  $\text{Sin}(k(x - vt))$
- B)  $\text{Exp}(k(-x - vt))$
- C)  $a(x + vt)^3$
- D) All of these.
- E) None of these.

A function,  $f$ , satisfies the wave equation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

Which of the following functions work?

- A)  $\sin(k(x - vt))$
- B)  $\exp(k(-x - vt))$
- C)  $a(x + vt)^3$
- D) All of these.
- E) None of these.

In fact, ANY function  $f(x \pm vt)$  is a good solution!!

Shape travels left (+)  
or right (-).

A “right moving” solution to the wave equation is:

$$f_R(z,t) = A \cos(kz - \omega t + \delta)$$

Which of these do you prefer for a “left moving” soln?

A)  $f_L(z,t) = A \cos(kz + \omega t + \delta)$

B)  $f_L(z,t) = A \cos(kz + \omega t - \delta)$

C)  $f_L(z,t) = A \cos(-kz - \omega t + \delta)$

D)  $f_L(z,t) = A \cos(-kz - \omega t - \delta)$

E) more than one of these!

(Assume  $k$ ,  $\omega$ ,  $\delta$  are positive quantities)

To think about; Is(are) the answer(s) really just “preference” (i.e. human convention) or are we forced into a choice?

Two different functions  $f_1(x,t)$  and  $f_2(x,t)$  are solutions of the wave equation.

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

Is  $(A f_1 + B f_2)$  also a solution of the wave equation?

A) Yes, always

B) No, never.

C) Yes, sometimes, depending of  $f_1$  and  $f_2$  .

Two traveling waves 1 and 2 are described by the equations:

$$y_1(x,t) = 2 \sin(2x - t)$$

$$y_2(x,t) = 4 \sin(x - 0.8 t)$$

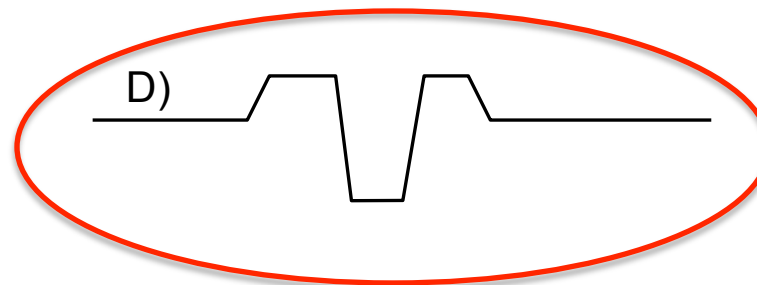
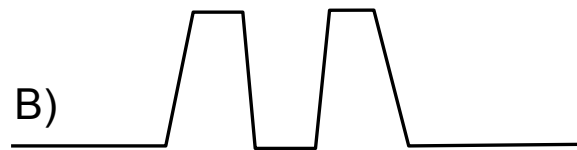
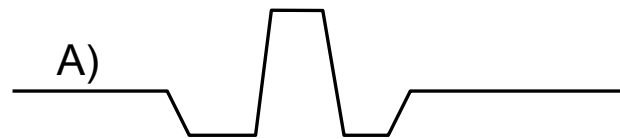
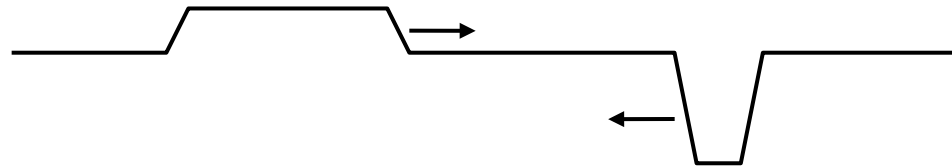
All the numbers are in the appropriate SI (mks) units.

Which wave has the higher speed?

- A) 1    **B) 2**    C) Both have the same speed.



Two impulse waves are approaching each other, as shown. Which picture correctly shows the total wave when the two waves are passing through each other?



A solution to the wave equation is:

$$f(z,t) = A \cos(kz - \omega t + \delta)$$

What is the speed of this wave?

Which way is it moving?

If  $\delta$  is small (and  $>0$ ), is this wave “delayed” or “advanced”?

What is the frequency?

The angular frequency?

The wavelength?

The wave number?

A solution to the wave equation is:

$$f(z,t) = \text{Re}[A e^{(kz - \omega t + \delta)}]$$

What is the speed of this wave?

Which way is it moving?

If  $\delta$  is small (and  $>0$ ), is this wave “delayed” or “advanced”?

What is the frequency?

The angular frequency?

The wavelength?

The wave number?

A complex solution to the wave equation in 3D is:

$$\tilde{f}(\mathbf{r}, t) = \tilde{A} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

What is the speed of this wave?

Which way is it moving?

Why is there no  $\delta$ ?

What is the frequency?

The angular frequency?

The wavelength?

The wave number?

Is "The Wave" at the stadium a transverse wave or a longitudinal wave?

- A) Transverse      B) Longitudinal      C) neither

A wave on a stretched drum head is an example of a

- A) Transverse wave      B) Longitudinal wave  
C) it's not a wave at all

A solution to the wave equation is:

$$f(z,t) = \text{Re}[A e^{i(kz - \omega t + \delta)}]$$

If  $\delta$  is small (and  $>0$ ), is this wave “delayed” or “advanced” compared to  $e^{i(kz - \omega t)}$ ?

A) delayed

B) advanced

C) neither, depends on values of  $z$  and  $t$ .

“delayed”



“advanced”



The electric field for a plane wave is given by:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)}$$

The vector  $\mathbf{k}$  tells you:

- A) The direction of the electric field vector.
- B) The direction of the magnetic field vector.
- C) The direction in which the wave is not varying.
- D) The direction the plane wave moves.
- E) None of these.

The electric field for a plane wave is given by:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)}$$

The vector  $\mathbf{k}$  tells you:

- A) The direction of the electric field vector.
- B) The speed of the traveling wave.
- C) The direction the plane wave moves.
- D) A direction *perpendicular* to the direction the plane wave moves
- E) None of these/MORE than one of these/???



The electric field for a plane wave is given by:

$$\mathbf{E}(\mathbf{r}, t) = \vec{E}_0 e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)}$$

Suppose  $\mathbf{E}_0$  points in the +x direction.

Which direction is this wave moving?

- A) The x direction.
- B) The radial ( $\mathbf{r}$ ) direction
- C) A direction *perpendicular* to both  $\mathbf{k}$  and  $\mathbf{x}$
- D) The  $\mathbf{k}$  direction
- E) None of these/MORE than one of these/???

The 1-D wave equation is  $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$

One particular “traveling wave” solution to this is

$$f_1(z,t) = A_1 \cos(k_1 z - \omega_1 t + \delta_1)$$

This wave has speed  $v = \omega_1/k_1$  (do you see why?)

There are many *other* solutions, including  $f_2(z,t)$  with the SAME functional form, but with higher frequency,  $\omega_2 > \omega_1$ .

What can you say about the *speed* of that new solution?

- A) greater than  $v$       B) less than  $v$       C) equal to  $v$
- D) indeterminate!

By the way:

This wave travels rightward (do you see why?)

This wave has wavelength  $\lambda = 2\pi/k_1$  (do you see why?)

This wave has period  $2\pi/\omega_1$  (do you see why?)

The 3-D wave equation is  $\nabla^2 f(x, y, z, t) = \frac{1}{v^2} \frac{\partial^2 f(x, y, z, t)}{\partial t^2}$

One particular “traveling wave” solution to this is often written

$$\tilde{f}_1(x, y, z, t) = \tilde{A} e^{i(\vec{k} \cdot \vec{r} - \omega t)}, \text{ where } \tilde{A} = A e^{i\delta}.$$

This wave travels in the **k** direction (do you see why?)

This wave has wavelength  $\lambda = 2\pi/|\mathbf{k}|$  (do you see why?)

This wave has period  $2\pi/\omega$  (do you see why?)

This wave has speed  $v = \omega/|\mathbf{k}|$  (do you see why?)

What is the real form of this wave?

- A)  $A \cos(kx - \omega t)$                       B)  $A \cos(kx - \omega t + \delta)$
- C)  $A \cos(\vec{k} \cdot \vec{r} - \omega t)$                       D)  $A \cos(\vec{k} \cdot \vec{r} - \omega t + \delta)$
- E) More than one of these/other/???

A wave is moving in the +z direction:

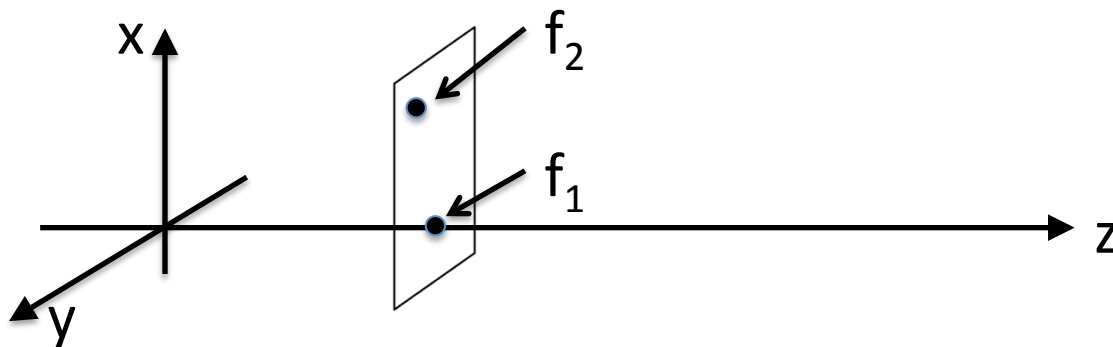
$$f(x, y, z, t) = \text{Re}[A e^{i(kz - \omega t + \delta)}]$$

The value of  $f$  at the point  $(0, 0, z_0, t)$  and the point at  $(x, y, z_0, t)$  are related how?

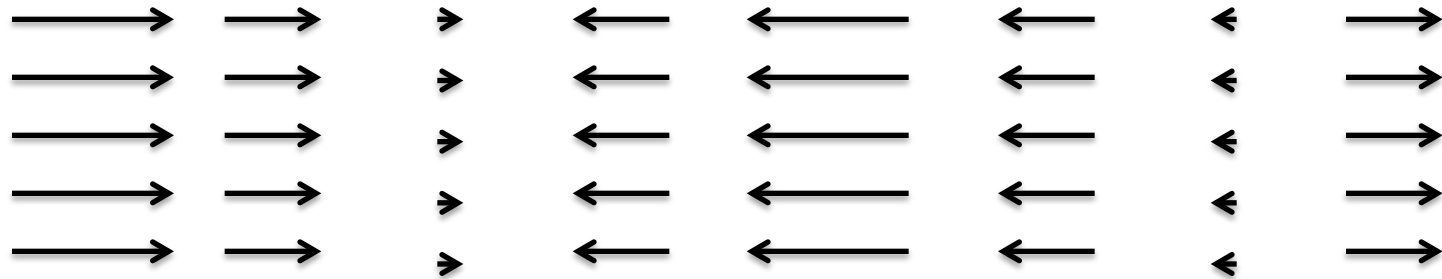
$$f_1 = f(0, 0, z_0, t) \text{ vs. } f_2 = f(x, y, z_0, t)$$

A)  $f_1 = f_2$  always

B)  $f_1 > \text{or } < \text{ or } = f_2$  depending on the value of  $x, y$



Here is a snapshot in time of a longitudinal wave:



The divergence of this field is:

- A) Zero
- B) Non-zero
- C) Impossible to tell without further information

The electric fields of two E/M waves in vacuum are both described by:

$$\vec{E} = E_0 \sin(kx - \omega t) \hat{y}$$

The "wave number"  $k$  of wave 1 is larger than that of wave 2,  $k_1 > k_2$ .

Which wave has the larger frequency  $f$ ?

- A) Wave 1      B) Wave 2      C) impossible to tell

The electric field of an E/M wave is described by

$$\vec{E} = E_0 \sin(kx - \omega t) \hat{y}$$

**k = 0.1 m<sup>-1</sup>, at x=1m and t=0, what is the direction of the B-field?**

**A) +x    B) +y    C) -x    D) +z    E) -z**

You have this solution to Maxwell's equations in vacuum:

$$\tilde{\vec{E}}(x, y, z, t) = \tilde{\vec{E}}_0 \exp\left[i(\vec{k} \cdot \vec{r} - \omega t)\right]$$

If this wave travels in the y direction, is polarized in the x direction, and has a complex phase of 0,

what is the x component of the physical wave?

A)  $E_x = E_0 \cos(kx - \omega t)$

B)  $E_x = E_0 \cos(ky - \omega t)$

C)  $E_x = E_0 \cos(kz - \omega t)$

D)  $E_x = E_0 \cos(k_x x + k_y y - \omega t)$

E) Other!!

To think about: What is the y component?

What would change if the complex phase of  $\vec{E}_0$  was  $90^\circ$ ?  $-90^\circ$ ?



Think about the first of Maxwell's Equations (Gauss's Law) in vacuum:  $\vec{\nabla} \cdot \vec{E} = 0$

Try a complex exponential “linearly polarized plane wave”:

$$\vec{E}(x, y, z, t) = \vec{E}_0 \exp\left[i(\vec{k} \cdot \vec{r} - \omega t)\right]$$

Then, Gauss's Law becomes:

$$\text{A) } i\vec{k} \cdot \vec{E}_0 = 0 \qquad \text{C) } i\vec{k} \times \vec{E}_0 = 0$$

$$\text{B) } \vec{k} |\vec{E}_0| = 0 \qquad \text{D) } |i\vec{k}| |\vec{E}_0| = 0$$

E) None of these.

What does this mean, in words?

Given the wave solutions

$$\vec{E}(x, y, z, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}, \quad \vec{B}(x, y, z, t) = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

What does Faraday's law  $\vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t$  tell us?

$$A) \quad \nabla \times \vec{E}_0 = -\frac{\partial \vec{B}_0}{\partial t}$$

$$B) \quad \nabla \times \vec{E}_0 = i\omega \vec{B}_0$$

$$C) \quad k\vec{E}_0 = \omega |\vec{B}_0|$$

$$D) \quad \vec{k} \times \vec{E}_0 = \omega \vec{B}_0$$

E) None of these!

An electromagnetic plane wave propagates to the right.

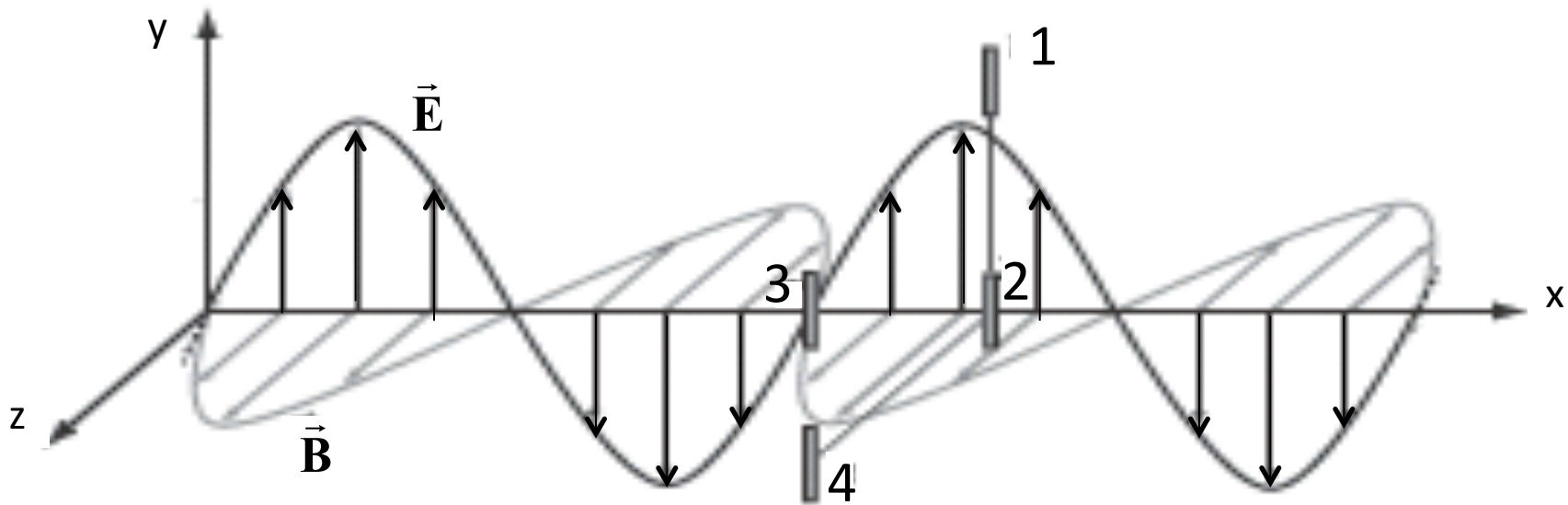
Four vertical antennas are labeled 1-4.

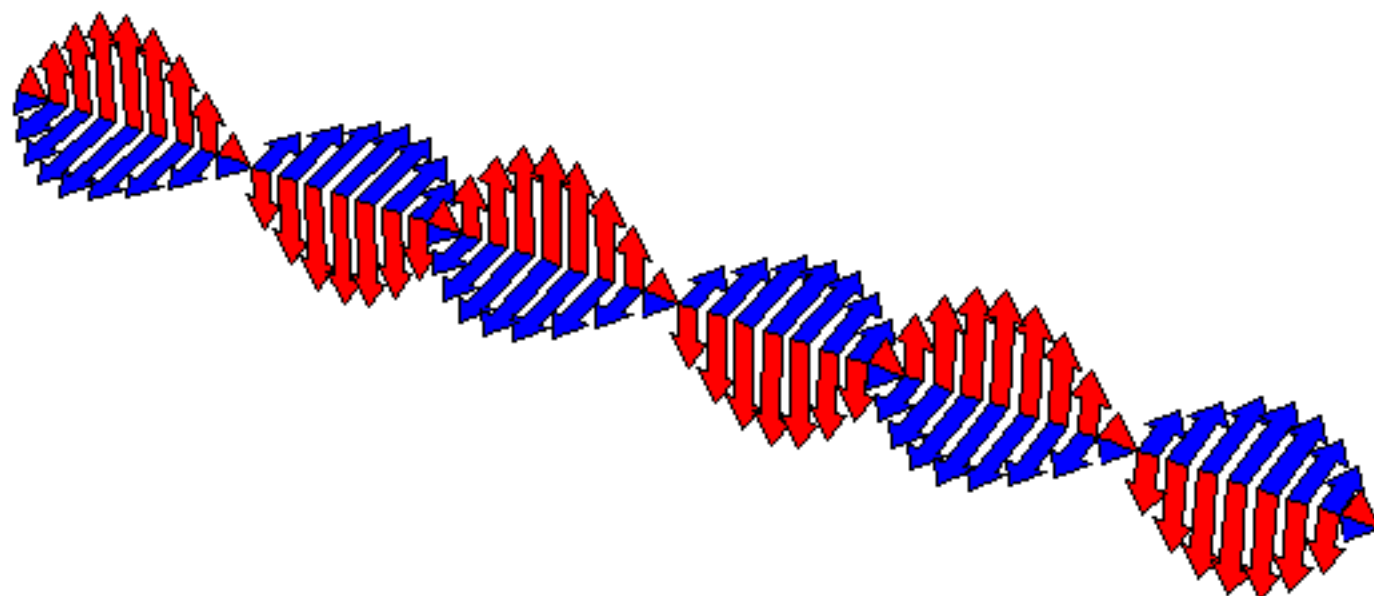
1,2, and 3 lie in the  $x$ - $y$  plane.

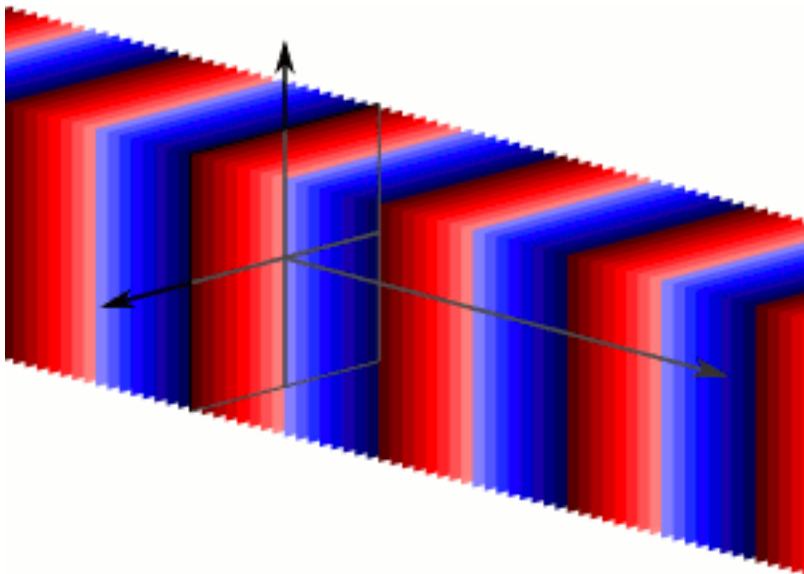
1,2, and 4 have the same  $x$ -coordinate, but antenna 4 is located further out in the  $z$ -direction.

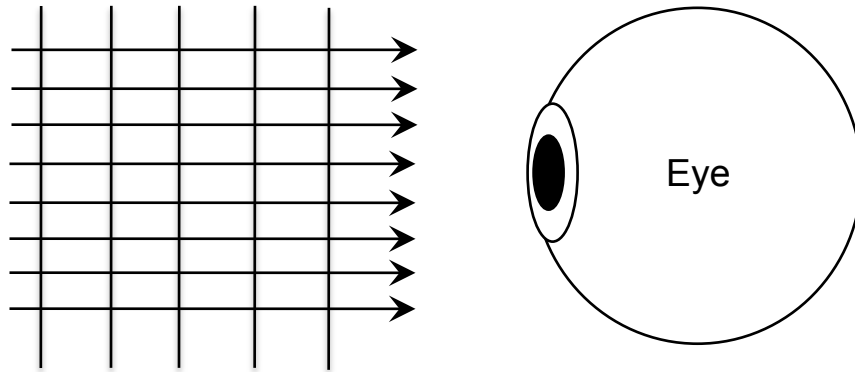
Rank the time-averaged signals received by each antenna.

- A)  $1=2=3>4$     B)  $3>2>1=4$     C)  $1=2=4>3$   
D)  $1=2=3=4$     E)  $3>1=2=4$









A plan wave approaches the eye and some of the light rays in the wave enter the eye's pupil. No other rays enter the eye. What does the eye see?

- A) A single point of light, surrounded by blackness.
- B) A uniformly illuminated wall of light, like a white wall.
- C) Many scattered points of light, like stars in the night sky.
- D) None of these

A point source of radiation emits power  $P_o$  isotropically (uniformly in all directions). A detector of area  $a_d$  is located a distance  $R$  away from the source. What is the power  $p$  **received** by the detector?

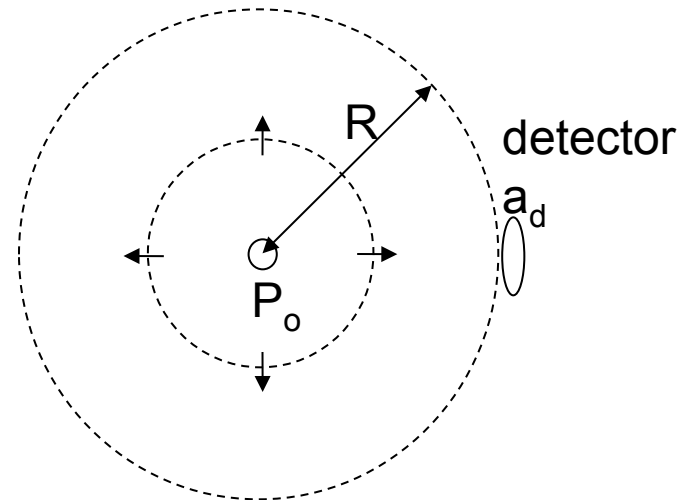
A)  $\frac{P_o}{4\pi R^2} a_d$

B)  $P_o \frac{a_d^2}{R^2}$

C)  $P_o \frac{a_d}{R}$

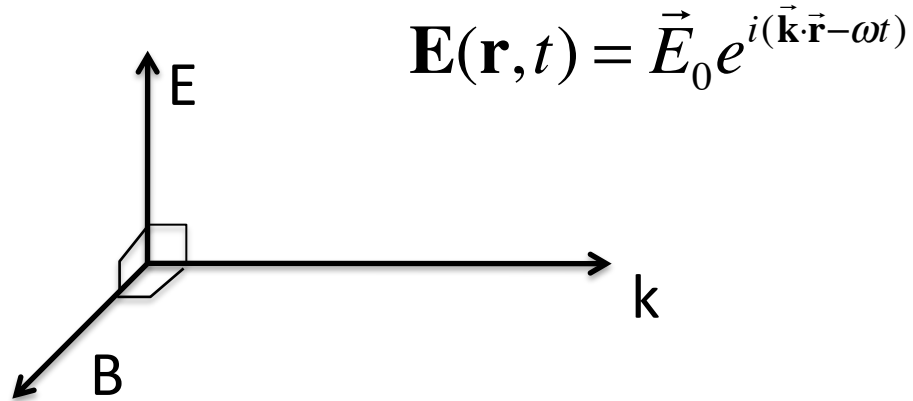
D)  $\frac{P_o}{\pi R^2} a_d$

E) None of these



For a plane electromagnetic wave in vacuum:

- $\mathbf{E} \perp \mathbf{k}$
- $\mathbf{B} \perp \mathbf{k}$
- $\mathbf{E} \perp \mathbf{B}$
- $\mathbf{E}, \mathbf{B}$  in phase
- $B = E/c$



Which of Maxwell's equations requires that  $\mathbf{B} \perp \mathbf{k}$  ?

A)  $\nabla \cdot \vec{E} = \rho / \epsilon_0$       B)  $\nabla \cdot \vec{B} = 0$

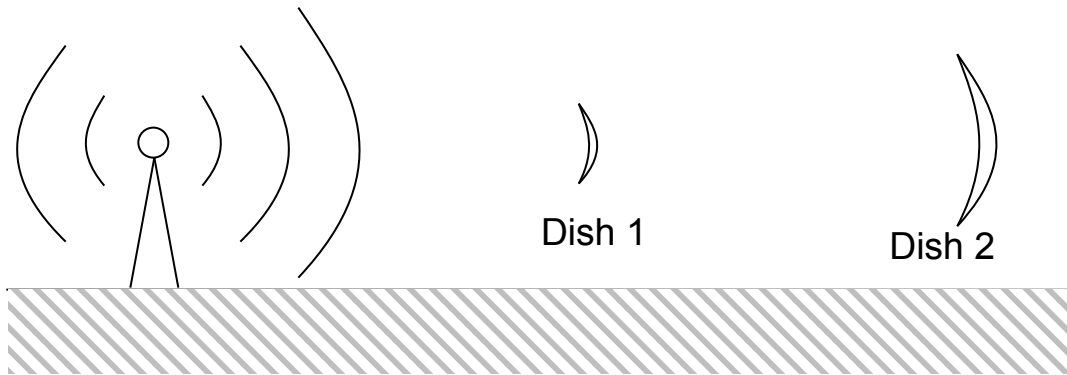
C)  $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$       D)  $\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Which of Maxwell's equations requires that  $B = E/c$  ?



Two radio dishes (shaped like bowls) are receiving signals from a radio station which is sending out radio waves in all directions with power  $P$ . Dish 2 is twice as far away as Dish 1, but has twice the diameter. Which dish receives more power? (Dish 2 is not in the shadow of Dish 1.)

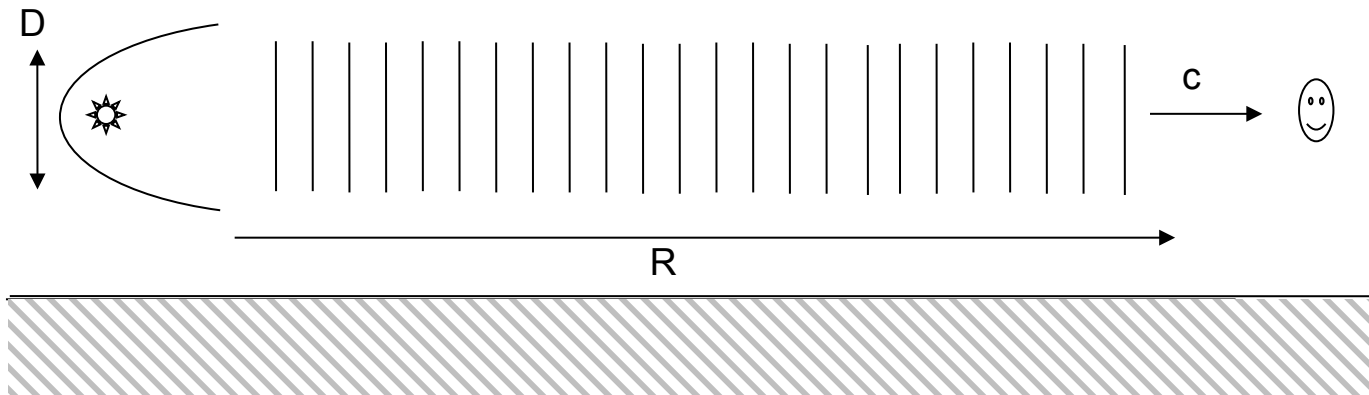
- A: Dish 1                      B: Dish 2  
C: Both receive the same power



A parabolic dish focuses the EM radiation from a source into a beam of constant diameter  $D$  :

The intensity of the light in the beam falls with distance  $R$  as:

- A)  $I \sim 1/R^2$
- B)  $I \sim 1/R$
- C)  $I = \text{constant}$
- D) Something else



When a jet flies faster than the speed of sound, there is...

- A) a sonic boom occurring only at the moment that the jet exceeds the speed of sound.
- B) a continuous sonic booming occurring all the time that the jet is going faster than Mach 1.

In linear dielectrics,  $\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E}$

In a linear dielectric is  $\epsilon > \epsilon_0$  ?

- A) Yes always. B) No, never.
- C) Sometimes yes, sometimes no.  
Depends on details of the dielectric.

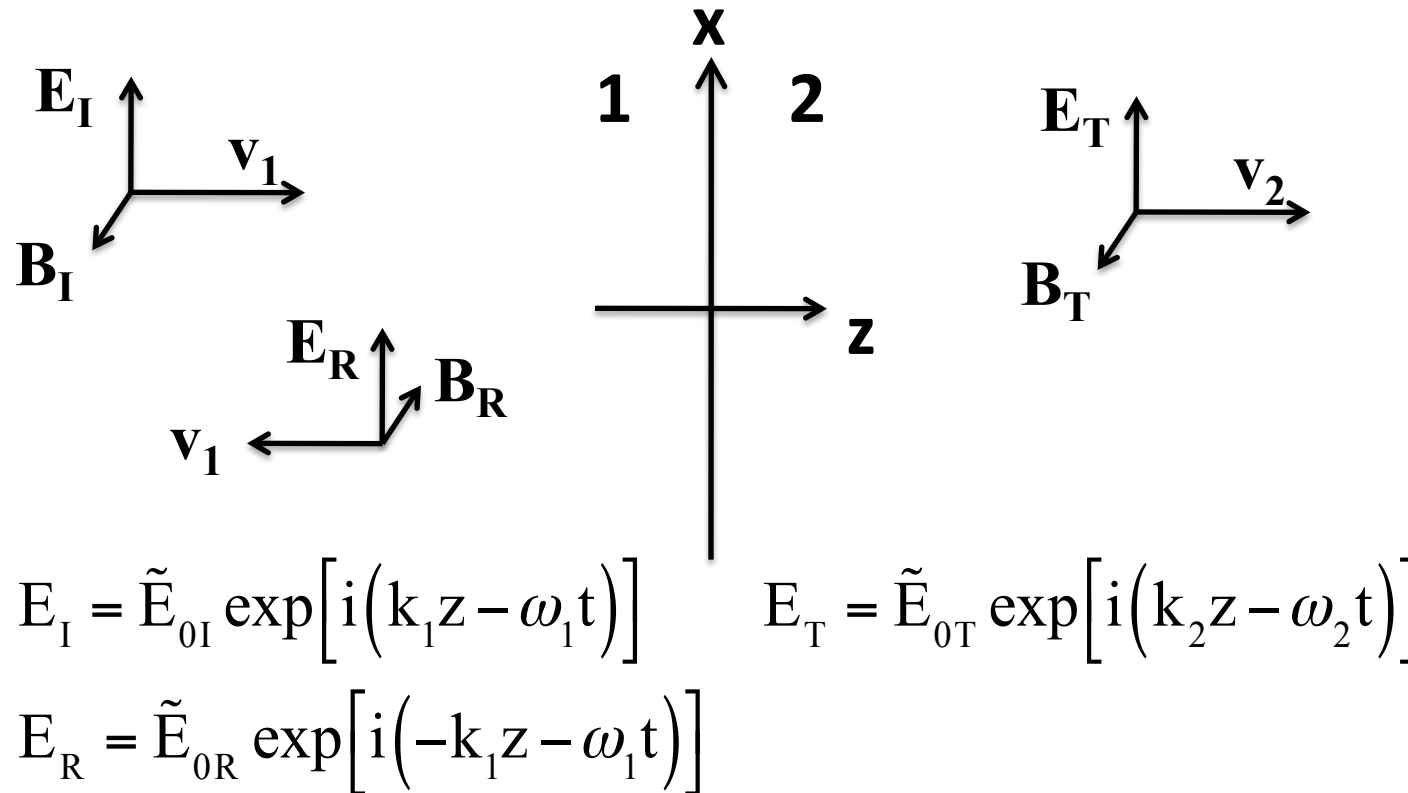
In a non-magnetic, linear dielectric,

$$V_{\text{wave}} = v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \epsilon_r \epsilon_0}} = \frac{c}{\sqrt{\epsilon_r}}$$

How does  $v$  compare to  $c$ ?

- A)  $v > c$  always
- B)  $v < c$  always
- C)  $v > \text{ or } < c$  depending on

A plane wave normally incident on an interface between 2 linear (non-magnetic) dielectrics ( $n_1 \neq n_2$ )



How do  $k_1$  and  $k_2$  compare? How do  $\omega_1$  and  $\omega_2$  compare?

- A)  $k_1 = k_2$ ,  $\omega_1 = \omega_2$       B)  $k_1 \neq k_2$ ,  $\omega_1 \neq \omega_2$   
 C)  $k_1 = k_2$ ,  $\omega_1 \neq \omega_2$       D)  $k_1 \neq k_2$ ,  $\omega_1 = \omega_2$

For an electric plane wave given by  $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$

the magnetic field is required by Faraday's law to be

$$\mathbf{B} = \mathbf{B}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \quad \text{where} \quad \mathbf{B}_0 = (\mathbf{k} \times \mathbf{E}_0) / \omega$$

Which of the following is true at a particular point in space?

- A. The E and B fields are at maximum value at the same time
- B. The B field maximum lags the E field by a quarter period
- C. The B field maximum leads the E field by a quarter period
- D. The E and B field maxima are exactly a half period apart
- E. Not enough information to tell

“Wave on a 1D string”, hitting a boundary between 2 strings of different speeds:

$$\tilde{f} = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & (z < 0) \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & (z > 0) \end{cases}$$

Boundary conditions  
(continuity)  
give the results:

$$\tilde{A}_R = \left( \frac{k_1 - k_2}{k_1 + k_2} \right) \tilde{A}_I, \quad \tilde{A}_T = \left( \frac{2k_1}{k_1 + k_2} \right) \tilde{A}_I$$

Is the transmitted wave in phase with the incident wave?

A) Yes, always    B) No, never    C) Depends

Why? How do you decide?



“Wave on a 1D string”, hitting a boundary between 2 strings of different speeds:

$$\tilde{f} = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & (z < 0) \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & (z > 0) \end{cases}$$

Boundary conditions  
(continuity)  
give the results:

$$\tilde{A}_R = \left( \frac{k_1 - k_2}{k_1 + k_2} \right) \tilde{A}_I, \quad \tilde{A}_T = \left( \frac{2k_1}{k_1 + k_2} \right) \tilde{A}_I$$

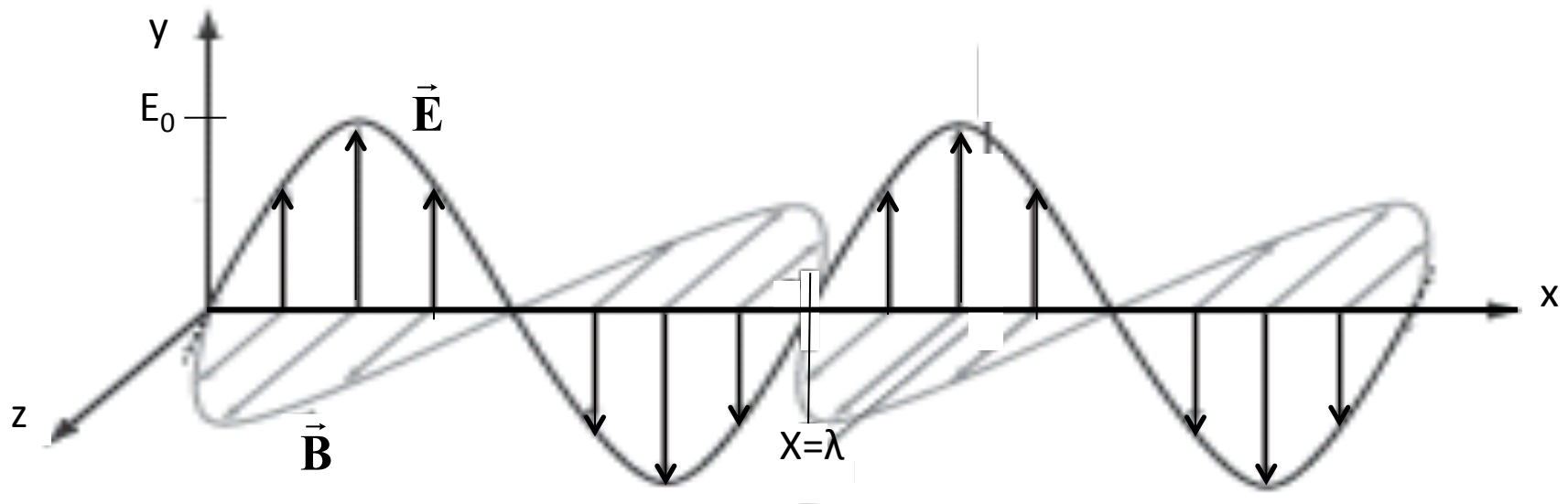
Is the reflected wave in phase with the incident wave?

A) Yes, always   B) No, never   C) Depends

Why? How do you decide?

Below is an idealized picture of a traveling EM plane wave. It is a snapshot at  $t=0$ . (Wavelength  $\lambda$  is given, as is  $E_0$ .)

Write down a pair of mathematical formulae which describe this wave (in complex form) for all times. (One for  $\mathbf{E}$  and  $\mathbf{B}$ )



With spatially varying  $\epsilon(\mathbf{r})$ , plane EM waves of the form,

$$\vec{E}(x, y, z, t) = \vec{E}_0 \exp\left[i(k_x x + k_y y + k_z z - \omega t)\right]$$

$$\vec{B}(x, y, z, t) = \vec{B}_0 \exp\left[i(k_x x + k_y y + k_z z - \omega t)\right]$$

are:

- A) No longer good solutions to Maxwell's Eqs.
- B) No longer have the same  $\omega$ .
- C) Are unchanged from the vacuum case.
- D) Are no longer transverse.
- E) None of the above.

In matter, we have

$$\left\{ \begin{array}{l} \nabla \cdot \vec{D} = \rho_F \\ \nabla \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{B} = \vec{J}_F + \frac{\partial \vec{D}}{\partial t} \end{array} \right.$$

$$\left\{ \begin{array}{l} \vec{D} = \epsilon_0 \vec{E} + \vec{P} \\ \vec{H} = \vec{B}/\mu_0 - \vec{M} \end{array} \right.$$

If there are no free charges or currents, can we argue

$$\nabla \cdot \vec{E} = 0 ?$$

- A) Yes, always
- B) Yes, under certain conditions (what are they?)
- C) No, in general this will NOT be true!
- D) ??

In matter with  
no free charges  
or currents,  
we have:

$$\left\{ \begin{array}{l} \text{i) } \nabla \cdot \vec{D} = 0 \\ \text{ii) } \nabla \cdot \vec{B} = 0 \\ \text{iii) } \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \text{iv) } \vec{\nabla} \times \vec{H} = +\frac{\partial \vec{D}}{\partial t} \end{array} \right.$$

To figure out formulas for boundary conditions,  
match the picture to the PDE(s) above.

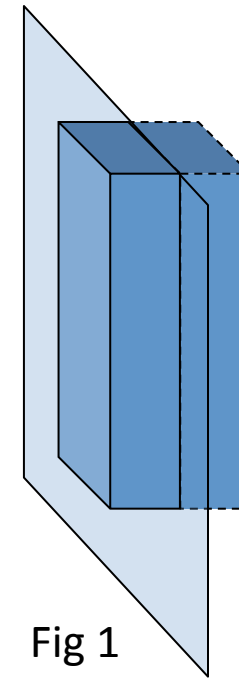


Fig 1

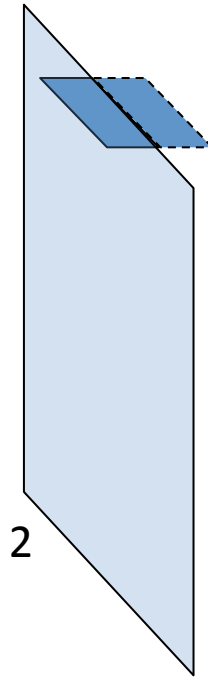
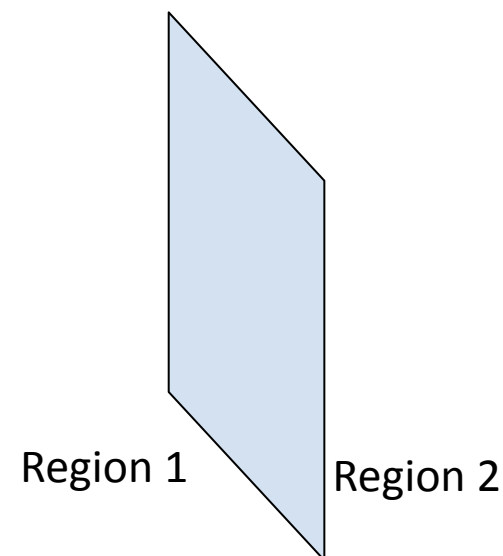


Fig 2

- A) Fig 1 goes with i and iii (i.e. the ones involving  $D$  and  $E$ ),  
Fig 2 goes with ii and iv (""  $B$  and  $H$ )
- B) Fig 1 goes with i and ii ("" *div*),  
Fig 2 goes with iii and iv ("" *curl*)
- C) Fig 1 goes with ii only (""  $B$  field),  
Fig 2 goes with iii only (""  $E$  field)
- D) Something else!      E) Frankly, I don't really understand this question.

$$\left\{ \begin{array}{l} \text{i) } \nabla \cdot \vec{D} = 0 \\ \text{ii) } \nabla \cdot \vec{B} = 0 \\ \text{iii) } \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \text{iv) } \vec{\nabla} \times \vec{H} = +\frac{\partial \vec{D}}{\partial t} \end{array} \right.$$

$$\left\{ \begin{array}{l} 1) B_1^\perp = B_2^\perp \\ 2) E_1^\perp = E_2^\perp \\ 3) \mathbf{B}_1^{\prime\prime} = \mathbf{B}_2^{\prime\prime} \\ 4) \mathbf{E}_1^{\prime\prime} = \mathbf{E}_2^{\prime\prime} \end{array} \right.$$



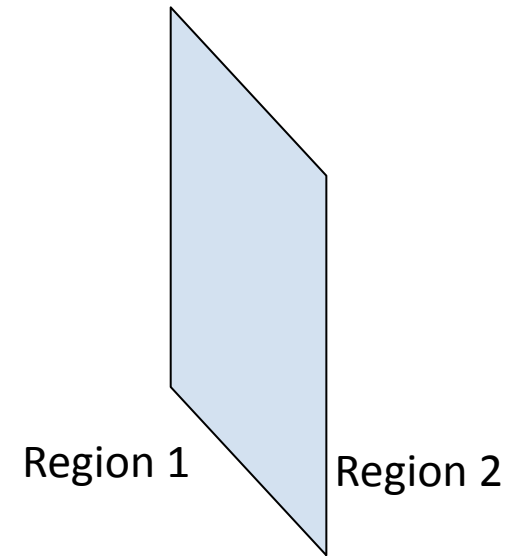
Match *the* Maxwell equation (i-iv) in matter (no free charges) with the corresponding *boundary condition* (1-4) it generates:

- A) i  $\rightarrow$  4, ii  $\rightarrow$  3    iii  $\rightarrow$  2    iv  $\rightarrow$  1
- B) i  $\rightarrow$  2, ii  $\rightarrow$  1    iii  $\rightarrow$  4    iv  $\rightarrow$  3
- C) Wait, only SOME of the BC's on the right are correct!
- D) Wait, NONE of the BC's on the right are correct!
- E) Frankly, I don't really understand this question.

$$\left\{ \begin{array}{l} \text{i) } \nabla \cdot \vec{D} = 0 \\ \text{ii) } \nabla \cdot \vec{B} = 0 \\ \text{iii) } \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \text{iv) } \vec{\nabla} \times \vec{H} = +\frac{\partial \vec{D}}{\partial t} \end{array} \right.$$

$$\left\{ \begin{array}{l} 1) B_1^\perp = B_2^\perp \\ 2) \epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp \\ 3) \mathbf{B}_1'' / \mu_1 = \mathbf{B}_2'' / \mu_2 \\ 4) \mathbf{E}_1'' = \mathbf{E}_2'' \end{array} \right.$$

(For linear materials)



Match *the* Maxwell equation (i-iv) in matter (no free charges) with the corresponding *boundary condition* (1-4) it generates:

B) i  $\rightarrow$  2, ii  $\rightarrow$  1 iii  $\rightarrow$  4 iv  $\rightarrow$  3

An EM plane wave in free space comes from the left towards an interface.

Which statement is true?

- A) Only certain frequencies are allowed.
- B) You are free to choose the wave speed.
- C) A compensating wave must travel towards the interface from the right too.
- D) You may independently select the frequency and the k-vector.
- E) None of the above.



An EM plane wave in free space comes from the left towards an interface.

Which statement is true?

- A) Only certain wave speeds are allowed.
- B) You are free to choose  $\mathbf{k}$ .
- C) A reflected wave on the left and a transmitted wave on the right may travel away from the interface too.
- D) All of the above.
- E) None of the above.

For linear materials, we found that the index of refraction  $n$  is given by:

$$n = \sqrt{(1 + \chi_E)(1 + \chi_M)}$$

Can  $n$  be less than one?

A. Yes

B. No

For our reflected and transmitted waves, how many unknowns have we introduced?

$$\mathbf{E}_R = \tilde{E}_{0R} e^{i(k_R z - \omega_R t)} \hat{n}_R$$

$$\mathbf{E}_T = \tilde{E}_{0T} e^{i(k_T z - \omega_T t)} \hat{n}_T$$

- A. 2
- B. 4
- C. 8
- D. 12
- E. None of the above

For our reflected and transmitted waves, how many unknowns have we introduced?

$$\mathbf{E}_R = \tilde{E}_{0R} e^{i(-k_I z - \omega_I t)} \hat{n}_I$$

$$\mathbf{E}_T = \tilde{E}_{0T} e^{i(k_T z - \omega_I t)} \hat{n}_I$$

- A. 2
- B. 4
- C. 8
- D. 12
- E. None of the above

An EM plane wave comes from the left towards an interface. Reflected and transmitted waves leave. Therefore, the total E-field is like:

$$\vec{E}_1 = \vec{E}_I e^{[i(\vec{k}_I \cdot \vec{r} - \omega_I t)]} + \vec{E}_R e^{[i(\vec{k}_R \cdot \vec{r} - \omega_R t)]} \quad (\text{region 1})$$

$$\vec{E}_2 = \vec{E}_T e^{[i(\vec{k}_T \cdot \vec{r} - \omega_T t)]} \quad (\text{region 2})$$

- A) True.
- B) False.
- C) Don't understand and have questions
- D) Too confused for questions...

An EM plane wave comes from the left towards an interface. Reflected and transmitted waves leave. Therefore, at the interface we expect to match the waves with something like:

$$(\text{?})e^{[i(\vec{k}_I \cdot \vec{r} - \omega t)]} + (\text{?})e^{[i(\vec{k}_R \cdot \vec{r} - \omega t)]} = (\text{?})e^{[i(\vec{k}_T \cdot \vec{r} - \omega t)]}$$

- A) No. This type of condition cannot work.
- B) No. This is mathematically possible, but physically incorrect.
- C) Ah, therefore I see a necessary condition...
- D) None of these.

A EM plane wave comes from the left. Reflected and transmitted waves leave. Therefore, at the interface we expect to match the waves with something like:

$$(\text{?})e^{[i(\vec{k}_I \cdot \vec{r} - \omega t)]} + (\text{?})e^{[i(\vec{k}_R \cdot \vec{r} - \omega t)]} = (\text{?})e^{[i(\vec{k}_T \cdot \vec{r} - \omega t)]}$$

at the interface,

- A)  $\vec{k}_I \cdot \vec{r} = 0$
- B)  $\vec{k}_I \cdot \vec{r} = k_{I,Y}y + k_{I,Z}z$
- C)  $\vec{k}_I \cdot \vec{r} = k_{I,X}x$
- D) None of these.

Do plane waves in matter (see above) represent general solutions of Maxwell's Equations?

- A) Yes! Fixed  $k$ -vector and frequency are predicted via Fourier superposition.
- B) No. In some cases you would need complex  $k$ -vectors, which is unphysical.
- C) No. In some cases you would need frequency dependent  $\epsilon$  or  $m$ . Then plane waves don't work.
- D) No. In spatially varying material, you might need non-transverse plane waves and that's unphysical.
- E) None of the above.



In matter without any free charge density, I can conclude that

- A. Divergence of **D** and **E** are zero
- B. Divergence of **D** is zero and divergence of **E** may be zero
- C. Divergence of **D** and **E** are not zero
- D. Divergence of **D** may be zero and divergence **E** is zero
- E. Not enough information to tell

In the case where medium 1 had a very slow wave velocity and medium 2 had a much higher wave velocity,  $\mu_2 v_2 \gg \mu_1 v_1$ . Assuming that the permeabilities ( $\mu$ 's) are essentially equal to the permeability of the vacuum, what is the relation between  $\epsilon_1$  and  $\epsilon_2$ ?

- A.  $\epsilon_1 \cong \epsilon_2$
- B.  $\epsilon_1 > \epsilon_2$
- C.  $\epsilon_1 < \epsilon_2$
- D. Not enough information to tell

$$\mathbf{E}_R = \tilde{E}_{0R} e^{i(k_R z - \omega_R t)} \hat{n}_R$$

$$\mathbf{E}_T = \tilde{E}_{0T} e^{i(k_T z - \omega_T t)} \hat{n}_T$$

For our reflected and transmitted waves, we found that  $\omega_R = \omega_T = \omega_I$ . What can we now conclude about the wavelengths of the transmitted and reflected waves?

- A.  $\lambda_R = \lambda_T = \lambda_I$
- B.  $\lambda_R = \lambda_T \neq \lambda_I$
- C.  $\lambda_R \neq \lambda_T = \lambda_I$
- D.  $\lambda_R = \lambda_I \neq \lambda_T$
- E. Need more information

$$\left\{ \begin{array}{l} 1) B_1^\perp = B_2^\perp \\ 2) \epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp \\ 3) \mathbf{B}_1'' / \mu_1 = \mathbf{B}_2'' / \mu_2 \\ 4) \mathbf{E}_1'' = \mathbf{E}_2'' \end{array} \right.$$



For light at normal incidence, we found:

$$R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}, \quad T = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

(For linear materials)

What gives a large transmission of light at normal incidence?

- A) When  $v_1 \gg v_2$
- B) When  $v_2 \gg v_1$
- C) When  $v$  is very *different* in the two media
- D) When  $v$  is nearly the *same* in the two media
- E) None of these/other/I'm confused/...

For an electric plane wave given by  $\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$

The contribution from the E field to the (real) energy density  $u_{EM}$  is

A.  $\frac{1}{2} \epsilon_0 (\tilde{\mathbf{E}})^2$

B.  $\frac{1}{2} \epsilon_0 \text{Re}\{(\tilde{\mathbf{E}})^2\}$

C.  $\frac{1}{2} \epsilon_0 (\text{Re}\{\tilde{\mathbf{E}}\})^2$

D.  $\frac{1}{2} \epsilon_0 \tilde{\mathbf{E}}^* \cdot \tilde{\mathbf{E}} = \frac{1}{2} \epsilon_0 |\tilde{\mathbf{E}}|^2$

E. More than one of the above is true

For an electric plane wave given by  $\mathbf{E} = \tilde{\mathbf{E}}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} = E_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta)} \hat{n}$

The Poynting vector is given by  $\vec{S} = \frac{1}{\mu} \vec{E} \times \vec{B}$

(And,  $v^2 = \frac{1}{\epsilon \mu}$ )

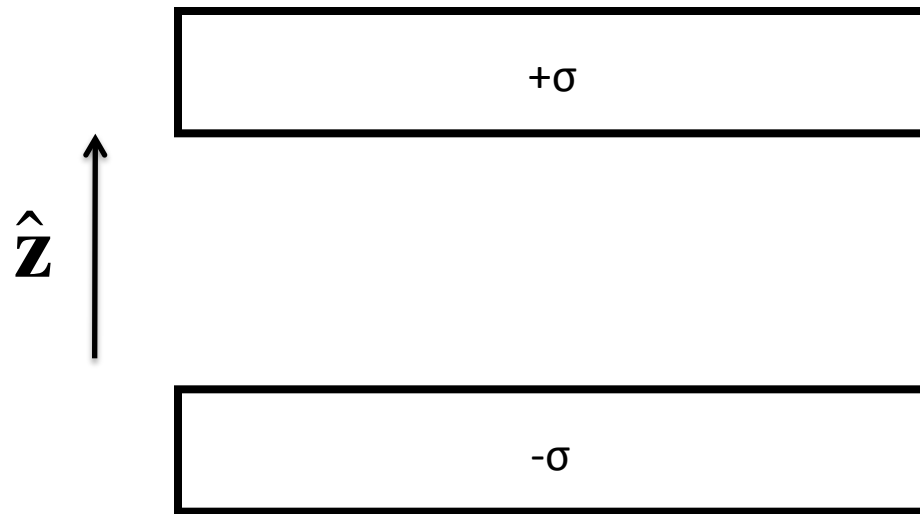
$$\text{So, } S = \epsilon v E_0^2 \cos^2(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta) = \epsilon v [\text{Re}(\tilde{E})]^2$$

$$I = \langle S \rangle = \frac{1}{2} \epsilon v E_0^2 = \frac{1}{2} \epsilon v |\tilde{E}|^2$$

(Surprise?! It's *not*  $\sim [\text{Re}(E)]^2$  !)

An ideal (large) capacitor has surface charge density  $+\sigma$  on its top plate and  $-\sigma$  on its bottom plate. There is only vacuum between the plates.

**What is  $\mathbf{E}$  inside the capacitor ?**

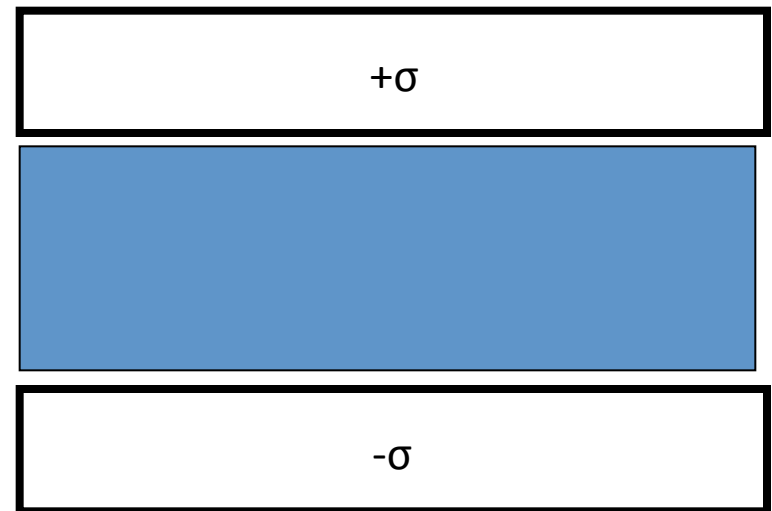


- A)  $\mathbf{E} = \sigma/\epsilon_0 \hat{\mathbf{z}}$
- B)  $\mathbf{E} = -\sigma/\epsilon_0 \hat{\mathbf{z}}$
- C)  $\mathbf{E} = \sigma/2\epsilon_0 \hat{\mathbf{z}}$
- D)  $\mathbf{E} = -\sigma/2\epsilon_0 \hat{\mathbf{z}}$
- E) None of the above

An ideal (large) capacitor has surface charge density  $+\sigma$  on its top plate and  $-\sigma$  on its bottom plate. Now a linear dielectric with permittivity  $\epsilon$  is inserted between the plates.

How does  $\mathbf{E}$  inside the dielectric compare with the case with no dielectric?

- A)  $\mathbf{E}$  is smaller
- B)  $\mathbf{E}$  is the same
- C)  $\mathbf{E}$  is larger
- D) Depends on the  $\epsilon$  of the dielectric
- E) Not enough information to tell





In the case where medium 1 had a very slow wave velocity and medium 2 had a much higher wave velocity, we found that  $R \rightarrow 1$  and  $T \rightarrow 0$ . In the opposite case, where the wave velocity in medium 1 is much higher than that in 2, we expect

- A.  $R \rightarrow 1, T \rightarrow 0$
- B.  $R \rightarrow 0, T \rightarrow 1$
- C.  $R \rightarrow 1/2, T \rightarrow 1/2$
- D. Not enough information to tell

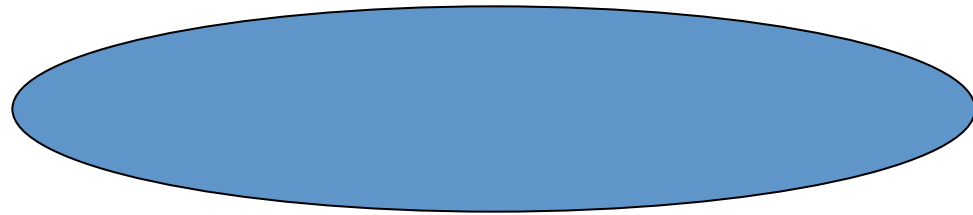
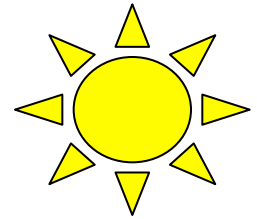
$$\left\{ \begin{array}{l} 1) \ B_1^\perp = B_2^\perp \\ 2) \ \epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp \\ 3) \ \mathbf{B}_1'' / \mu_1 = \mathbf{B}_2'' / \mu_2 \\ 4) \ \mathbf{E}_1'' = \mathbf{E}_2'' \end{array} \right. \quad \longrightarrow \quad \begin{array}{l} \tilde{E}_{0R} = \left( \frac{\alpha - \beta}{\alpha + \beta} \right) \tilde{E}_{0I} \\ \tilde{E}_{0T} = \left( \frac{2}{\alpha + \beta} \right) \tilde{E}_{0I} \end{array} \quad \text{where}$$

$$\begin{array}{l} \alpha \equiv \cos \theta_T / \cos \theta_I \\ \beta \equiv \mu_1 v_1 / \mu_2 v_2 \\ \approx n_2 / n_1 \end{array}$$

(For linear materials)

What is the relative phase angle between the incident wave and the transmitted wave?

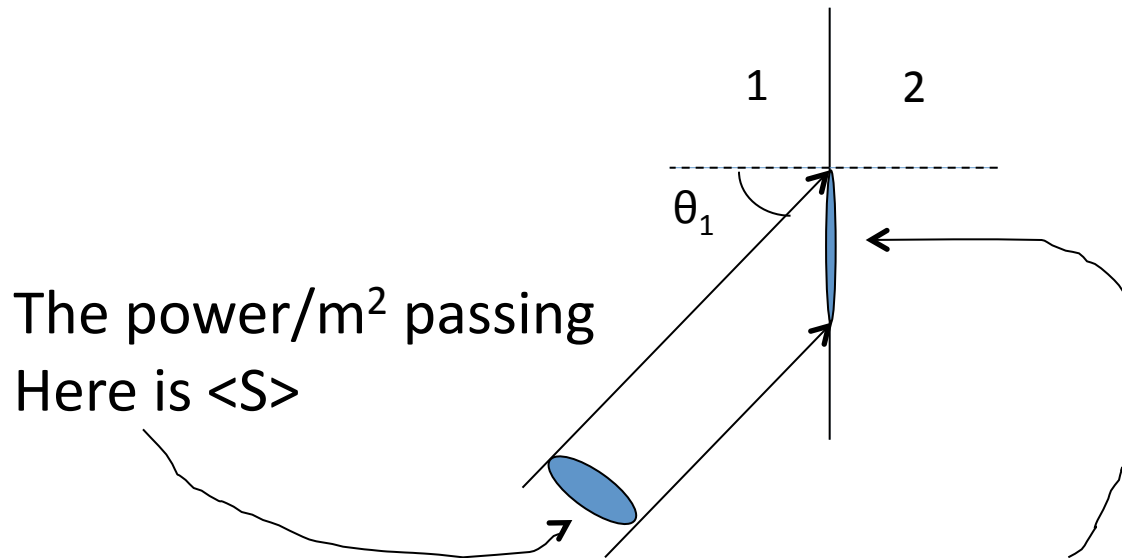
- A. 0
- B. 90
- C. 180
- D. Can be more than one of the above
- E. Depends on the incident angle and dielectric properties



To reduce glare off the lake, should the polarization axis of these sunglasses be

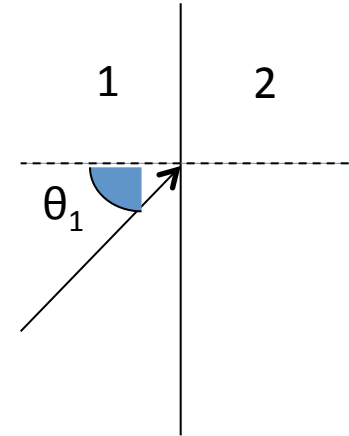
- A) Vertical      B) Horizontal      C) Makes no difference      D) ????





What is the power/m<sup>2</sup> striking the boundary wall?

- A) Still  $\langle S \rangle$       B)  $\langle S \rangle \cos \theta_1$       C)  $\langle S \rangle / \cos \theta_1$   
 D) Something else! ( $\sin \theta_1$  or  $\cos^2 \theta_1$  or  $\cos \theta_2$ , or ...)



Our general solution for the transmitted wave is

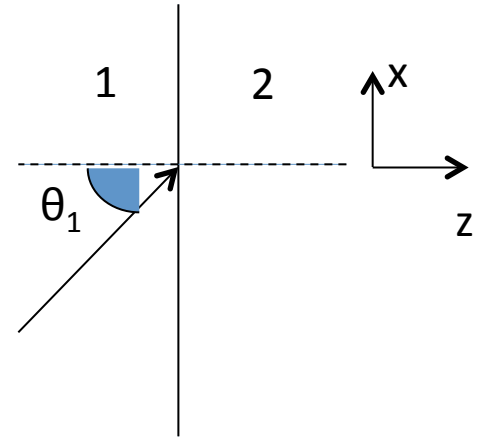
$$\vec{E}_T(\vec{r}, t) = \vec{E}_{0T} e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)}$$

Snell's law tells us  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

If  $n_2 < n_1$ , there is a critical angle,  $\sin \theta_{1,C} = n_2 / n_1$

beyond which there is no real solution for  $\theta_2$ .

How should we interpret this lack of solution *physically*?



$$\vec{E}_T(\vec{r}, t) = \vec{E}_{0T} e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)}$$

If we are pigheaded, we can proceed with  $\cos\theta_2$  *imaginary*, so

$$\vec{k}_2 \cdot \vec{r} = (\text{real}) * x + (\text{imaginary}) * z$$

Can we interpret this?

The square root of  $a + bi$  is

A)  $\sqrt{a} + i\sqrt{b}$

B)  $\sqrt{a^2 + b^2}$

C)  $\sqrt{a^2 - b^2} + i\sqrt{2ab}$

D. It is not defined

E. Something else (this is harder than it looks)



We have a traveling wave solution satisfies  $\vec{\tilde{E}}(\vec{r}, t) = \vec{\tilde{E}}_0 e^{i(\tilde{k} z - \omega t)}$

where the (complex) wave vector

$$\tilde{k}^2 = \omega^2 \mu \epsilon + i(\omega \mu \sigma)$$

True (A) or False (B): This traveling wave is “transverse”.

(Or C) I have no good idea what that means)

The magnetic field amplitude in a metal associated with a linearly polarized electric EM wave is

$$\tilde{B}_0 = \left( \frac{k_R + ik_{\text{Im}}}{\omega} \right) \tilde{E}_0$$

True (A) or False (B): The B field is in phase with the E field.

(C) It depends!

The magnetic field amplitude in a highly conductive metal ( $\sigma \gg \epsilon\omega$ ) associated with a linearly polarized electric EM wave is

$$\begin{aligned}\tilde{B}_0 &= \sqrt{\frac{\mu\sigma}{\omega}} \frac{1+i}{\sqrt{2}} \tilde{E}_0 \\ &= \sqrt{\frac{\sigma}{\epsilon_0\omega}} \frac{1+i}{\sqrt{2}} \frac{\tilde{E}_0}{c}\end{aligned}$$

True (A) or False (B): The B field is in phase with the E field.

(C) It depends!

For a good conductor,  $\tilde{B}_0 = (...)e^{i\pi/4}\tilde{E}_0$

Is the B field

A) Leading    B) Lagging    C) Matching    D) It depends!

the E field.

If  $E_x(x,y,z,t) = E_0 \exp[i(kz - \omega t)]$ , and you have a free charge  $q$  which responds to this  $E$  field, ( $F = qE$ )

what can you say about the relative phase of  $v(t)$  and  $E(t)$  at any point in space?

- A) They are in phase
- B) They are  $90^\circ$  out of phase
- C) They are  $180^\circ$  out of phase
- D) I don't really know what this means
- E) Something else...

The Fresnel equation (for normal incidence) is  $\tilde{E}_{0R} = \left( \frac{n_1 - n_2}{n_1 + n_2} \right) \tilde{E}_{0I}$

If region 2 is a conductor,  $n_2 = (c/\omega) k_2$  is *complex*!

But, recall that for a “good conductor”,  $\omega/k_R \ll c$ .

What do you conclude?

A)  $\tilde{E}_{0R} \approx \tilde{E}_{0I}$       B)  $\tilde{E}_{0R} \approx -\tilde{E}_{0I}$       C)  $\tilde{E}_{0R} \approx 0$

D) Something more complicated (literally, complex!)

E) ???

Using what you know about conduction in a metal, estimate the order of magnitude of the average time between collisions of electrons with the lattice of metal ions. The typical electron speed in a metal is around  $10^6$  m/s.

- A.  $10^{-10}$  s
- B.  $10^{-13}$  s
- C.  $10^{-16}$  s
- D.  $10^{-19}$  s
- E. Not sure how to estimate this

From last class: If you *model* a dielectric as “charges on springs” (with damping) a traveling EM wave will drive those charges, resulting in polarized molecules.

If  $x(t)$ =displacement of charge  $q$ , and  $N$  = molecules/volume, **what is the volume polarization,  $P$ ?**

- A)  $P = x(t)$       B)  $P = qx(t)$       C)  $P = Nx(t)$   
D)  $P = Nq x(t)$     E) Something else!?



$$\nabla^2 \vec{E} = \epsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2},$$

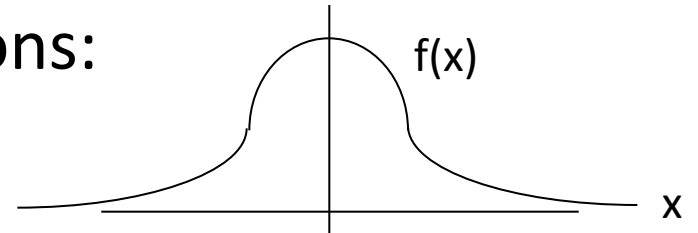
$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

The index of refraction is  $n = ck / \omega = c\sqrt{\epsilon\mu}$

- 1) What does it mean if  $n$  is complex?
- 2) What does it mean if  $n$  depends on  $\omega$ ?

Fourier tells us that we can write a “pulse” by summing up sinusoidal functions:

$$f(x) = \int_{-\infty}^{\infty} a(k) e^{ikx} dk$$



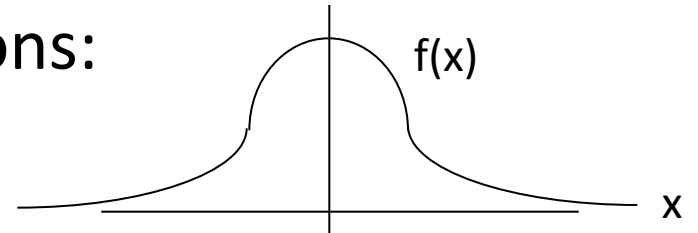
If you were to compute  
(with  $v$  a known constant)  
what would that give?

$$\int_{-\infty}^{\infty} a(k) e^{i(k(x-vt))} dk$$

- A)  $f(x)$       B)  $f(vt)$       C)  $f(x-vt)$   
 D) Something complicated!  
 E) ???

Fourier tells us that we can write a “pulse” by summing up sinusoidal functions:

$$f(x) = \int_{-\infty}^{\infty} a(k) e^{ikx} dk$$



If you were to compute  $\int_{-\infty}^{\infty} a(k) e^{i(k(x - v(k)t))} dk$

what would that give?

- A)  $f(x)$       B)  $f(vt)$       C)  $f(x - vt)$
- D) Something complicated!
- E) ???

Summary of last class: dispersion in a dilute dielectric:

Step1: Incoming  $E_{\text{ext}}$  polarizes electrons, so use Newton's law (with  $qE_{\text{ext}} e^{-i\omega t}$  “driver”) to solve for  $x(t)$  of the electron.

Step 2: Compute the polarization density ( $p=qx$ , then  $P=Np$ )(summing over different possible electron states)

Step 3: Use  $D=\epsilon_0 E+P$  (always), and  $D=\epsilon E$  (if linear), to extract  $\epsilon$ . Done!  
We have  $\epsilon$  (and thus  $n$ ) for this material.

In which step did we assume the material was “dilute”?

- A) Step 1    B) Step 2    C) Step 3    D) More than 1 step  
E) Never! The result is quite general for this “charged-spring” model

For our atomic model of permittivity we found  $\epsilon$  to be

$$\tilde{\epsilon} = \epsilon_0 \left( 1 + \frac{Nq^2}{\epsilon_0 m} \sum_i \frac{f_i}{(\omega_i^2 - \omega^2) - i\gamma_i \omega} \right)$$

We also know  $\frac{n}{c} = \frac{\tilde{k}}{\omega} = \sqrt{\tilde{\epsilon}\mu}$

- i) Find (and simplify) a formula for  $n$ , assuming the term adding to “1” above is *small*.
- ii) In that limit, find  $k_R$  and  $k_{Im}$ .  
What does each one tell you, physically?
- iii) Sketch both of these as functions of  $\omega$  (assuming that only one term in that sum “dominates”)

(Under what condition on  $\omega$  *should* one term in that sum dominate?)

For our atomic model of permittivity we found the absorption coefficient to be:

$$\alpha = \frac{Nq^2\omega^2}{mc\epsilon_0} \sum_i \frac{f_i\gamma_i}{(\omega_i^2 - \omega^2)^2 + \gamma_i^2\omega^2}$$

In the limit of large  $\omega$ ,  $\alpha$

- A. Goes to zero
- B. Approaches a constant
- C. Goes to infinity
- D. I don't know!

For our atomic model of permittivity we found the absorption coefficient to be:

$$\alpha = \frac{Nq^2\omega^2}{mc\epsilon_0} \sum_i \frac{f_i\gamma_i}{(\omega_i^2 - \omega^2)^2 + \gamma_i^2\omega^2}$$

In the limit of no damping, absorption

- A. Goes to zero
- B. Approaches a constant
- C. Goes to infinity
- D. Other, (it's not so simple).
- E. I don't know!

# Waveguides

The following questions are from Ed Kinney's  
Sp11 E&M II course

Waveguides were not covered in  
Fa11 or Sp12 at CU



What is the electric field in a *perfect* conductor if an EM wave is incident on the surface?

- A. Zero
- B. Depends on the angle of incidence
- C. Depends on whether the EM wave is in a dielectric or vacuum
- D. Perfectly in phase with the incident wave but decreasing with distance into the conductor
- E. Not enough information to say

What is the magnetic field in a *perfect* conductor if an EM wave is incident on the surface?

- A. Zero
- B. Depends on the angle of incidence
- C. Depends on whether the EM wave is in a dielectric or vacuum
- D. Out of phase with the incident wave and decreasing with distance into the conductor
- E. Not enough information to say

Which of Maxwell's equations, when applied to regular plane wave solutions

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

required that the electric field was transverse?

- A. Gauss' Law for the electric field
- B. Gauss' Law for the magnetic field
- C. Faraday's Law
- D. Ampere's Law
- E. More than one Maxwell equation was required

The separation of variables technique gives us 2 ODEs of the form

$$\frac{d^2 X}{dx^2} = \pm c^2 X$$

On what basis can we determine whether to use a positive or a negative separation constant  $c^2$ ?

- A. It doesn't matter which we choose, either choice will work
- B. The wave equation itself determines the signs
- C. The boundary conditions at the walls determine the signs
- D. None of the above
- E. More than one of the above

For  $\text{TE}_{mn}$  modes we found the general dispersion relation

$$k = \frac{1}{c} \sqrt{\omega^2 - \omega_{mn}^2} \quad \text{with} \quad \omega_{mn}^2 = \left( \frac{cm\pi}{a} \right)^2 + \left( \frac{cn\pi}{b} \right)^2$$

For the case  $a = 2b$ , waves with  $\omega = 1.01c\pi/b$  are input to the waveguide. Which  $\text{TE}_{mn}$  modes can be used to transport the input wave?

- A. Only  $\text{TE}_{10}$
- B. All modes, except  $\text{TE}_{10}$  and  $\text{TE}_{01}$
- C. Only  $\text{TE}_{10}$  and  $\text{TE}_{01}$
- D. None of the above
- E. Not enough information

For TE<sub>mn</sub> modes we found the general dispersion relation

$$k = \frac{1}{c} \sqrt{\omega^2 - \omega_{mn}^2} \quad \text{with} \quad \omega_{mn}^2 = \left( \frac{cm\pi}{a} \right)^2 + \left( \frac{cn\pi}{b} \right)^2$$

What is the phase velocity of the waves in the waveguide?

- A.  $\omega_{mn}/k$
- B.  $\omega/k$
- C.  $c$
- D.  $\sqrt{\omega^2 - \omega_{mn}^2} / k$
- E. None of the above

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How does the wavelength a TE mode compare to that of a plane wave in vacuum with the same frequency?

- A. The TE mode wavelength is larger than the plane wave's.
- B. The TE mode wavelength is smaller than the plane wave's.
- C. The TE mode wavelength is the same as the plane wave's.
- D. The answer depends on the actual value of the frequency.
- E. The answer depends on the actual values of the side lengths.