

Electricity and Magnetism II

AC Circuits & Complex Numbers

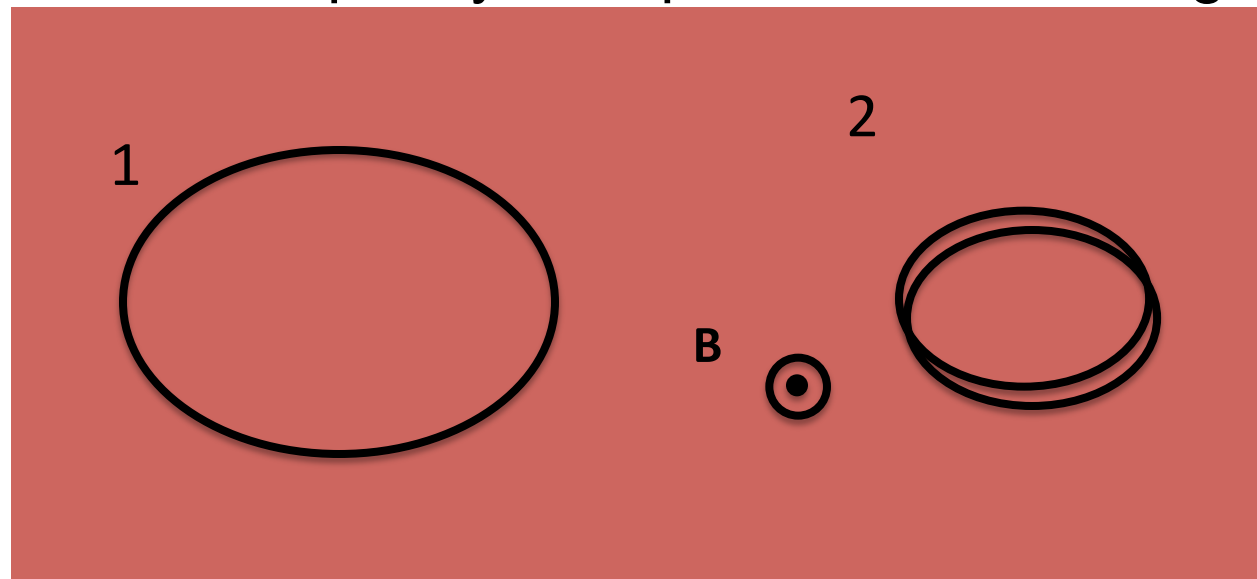
Clicker Questions



Loop 1 sits in a uniform field **B** which is increasing in magnitude. Loop 2 has the SAME LENGTH OF WIRE looped (coiled) to make two (smaller) loops. (The 2 loops are connected appropriately, think of it as the start of a solenoid)

How do the induced EMFs compare?

HINT: Don't answer too quickly, it requires some thinking!

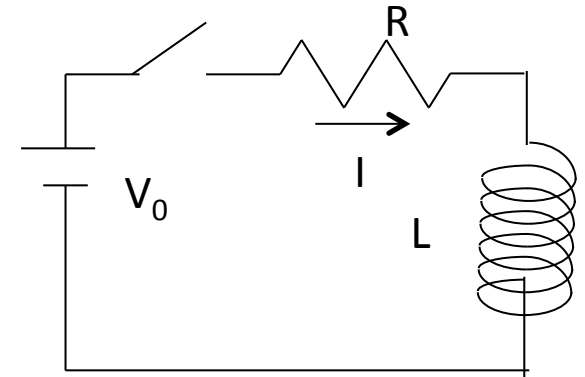


- A) $\text{EMF}(1) = 4 \text{ EMF}(2)$
- B) $\text{EMF}(1) = 2 \text{ EMF}(2)$
- C) They are both the same.
- D) $\text{EMF}(2) = 4 \text{ EMF}(1)$
- E) $\text{EMF}(2) = 2 \text{ EMF}(1)$

The switch is closed at $t=0$.

What can you say about $I(t=0^+)$?

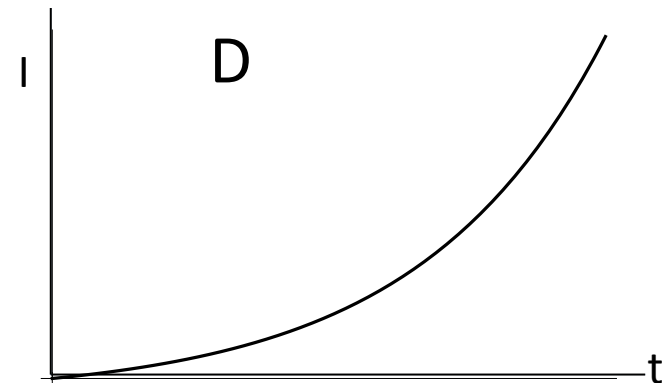
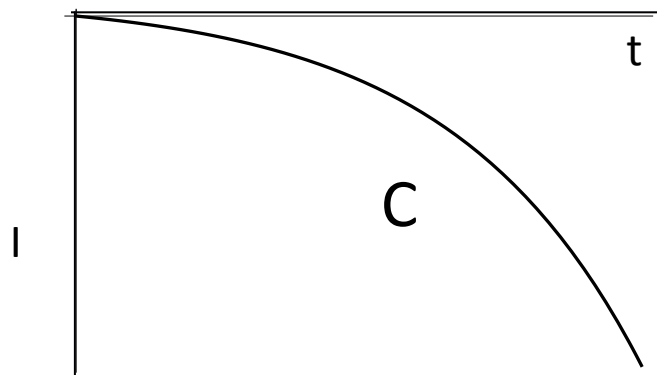
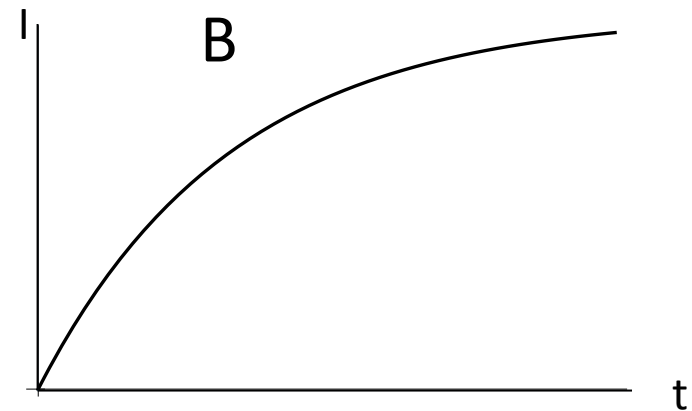
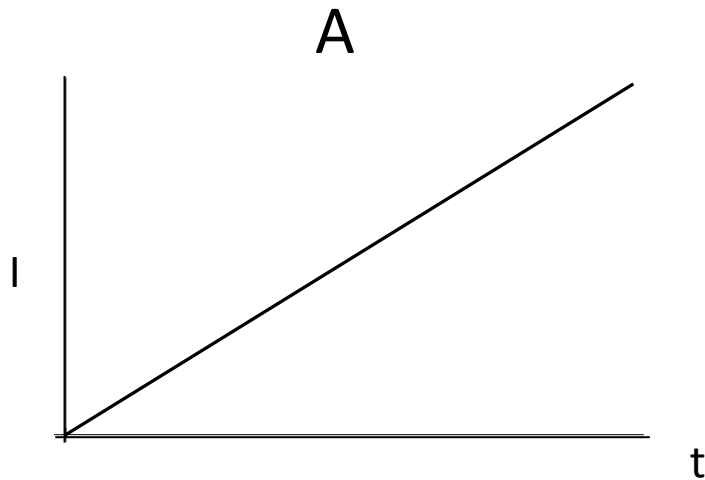
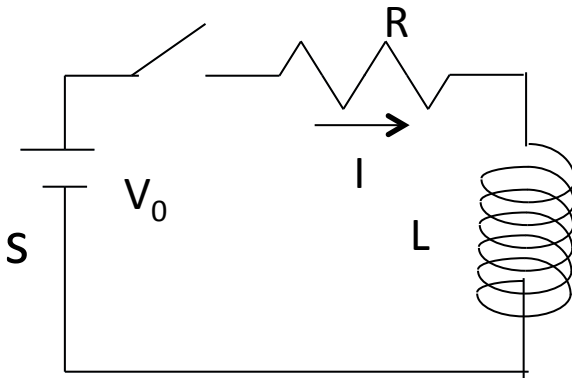
- A) Zero
- B) V_0/R
- C) V_0/L
- D) Something else!
- E) ???



The switch is closed at $t=0$.

Which graph best shows $I(t)$?

E) None of these (they all have a serious error!)



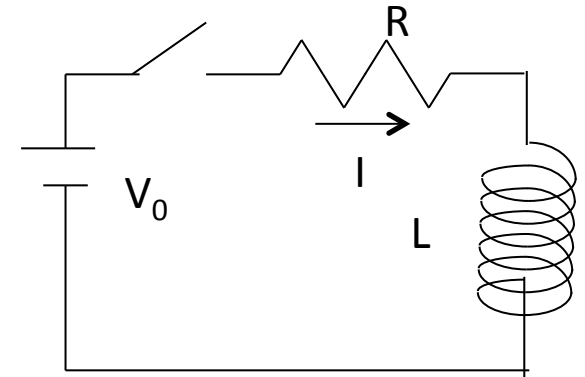
Consider a cubic meter box of uniform magnetic field of 1 Tesla and a cubic meter box of uniform electric field of 1 Volt/meter. Which box contains the most energy?

- A. The box of magnetic field
- B. The box of electric field
- C. They are both the same
- D. Not enough information given

The switch is closed at $t=0$.

What can you say about the magnitude of ΔV (across the inductor) at $(t=0^+)$?

- A) Zero
- B) V_0
- C) L
- D) Something else!
- E) ???



The solution to an ODE is

$$I(t) = a \cos(\omega t) + b \sin(\omega t),$$

(with a and b still undetermined constants) Or equivalently,

$$I(t) = A \cos(\omega t + \phi)$$

(with A and ϕ still undetermined constants)

Which expression connects the constants in these two forms?

- A) $a = A \cos \phi$
- B) $a = A \sin \phi$
- C) I can do this, but it's more complicated than either of the above!
- D) I'm not sure at the moment how to do this.
- E) It's a trick, these two forms are not equivalent!

The solution to an ODE is

$$I(t) = a \cos(\omega t) + b \sin(\omega t),$$

(with a and b still undetermined constants) Or equivalently,

$$I(t) = A \cos(\omega t + \phi)$$

(with A and ϕ still undetermined constants)

Which expression connects the constants in these two forms?

A) $A = a^2 + b^2$

B) $A = \text{Sqrt}[a^2 + b^2]$

C) I can do this, but it's more complicated than either of the above!

D) I'm not sure at the moment how to do this.

The complex exponential: $e^{i\omega t}$

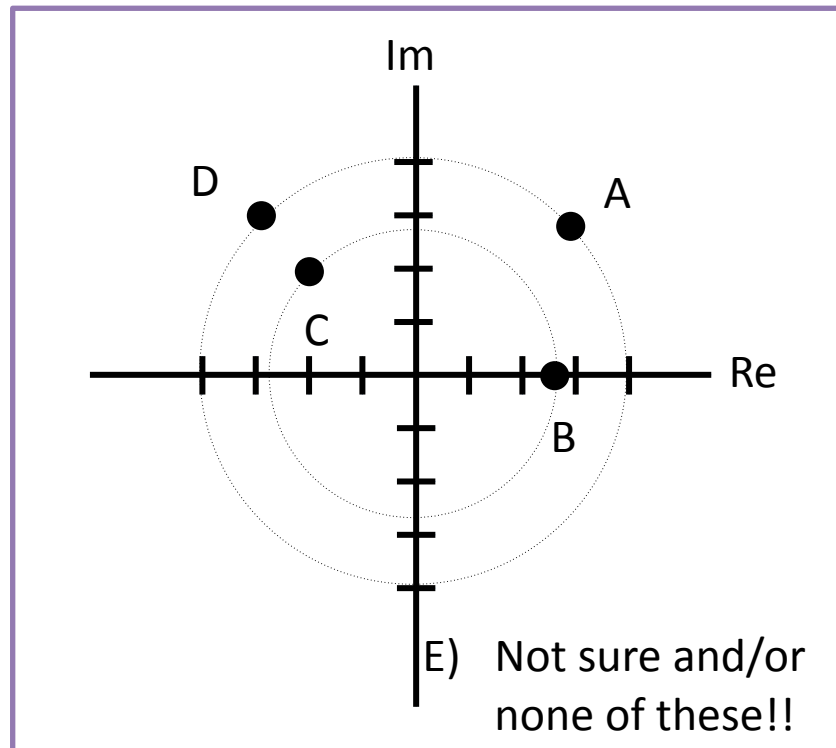
is useful in calculating properties of many time-dependent equations. According to Euler, we can also write this function as:

- A) $\cos(i \omega t) + \sin(i \omega t)$
- B) $\sin(\omega t) + i \cos(\omega t)$
- C) $\cos(\omega t) + i \sin(\omega t)$
- D) MORE than one of these is correct
- E) None of these is correct!

What is $|2+i|$

- A) 1
- B) $\text{Sqrt}[3]$
- C) 5
- D) $\text{Sqrt}[5]$
- E) Something else!

Which point below best represents $4e^{i3\pi/4}$ on the complex plane?



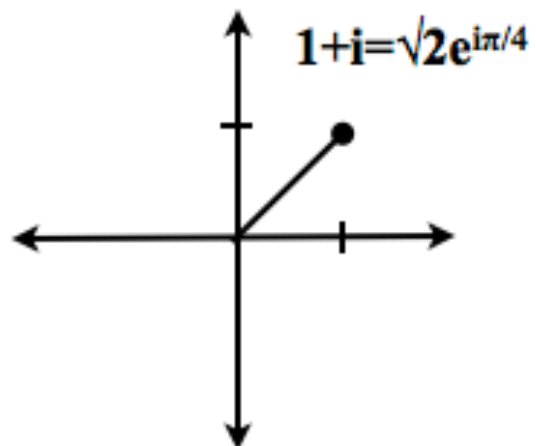
Challenge question: Keeping the general form $Ae^{i\theta}$, do any OTHER values of θ represent the SAME complex number as this? (If so, how many?)

What is $\frac{(1+i)^2}{(1-i)}$

- A) $e^{i\pi/4}$
- B) $\text{Sqrt}[2] e^{i\pi/4}$
- C) $e^{i3\pi/4}$
- D) $\text{Sqrt}[2]e^{i3\pi/4}$
- E) Something else!

There are two obvious methods. 1) multiply it out (“rationalizing” the denominator)
Or 2) First write numerator and denominator in standard $Ae^{i\theta}$ form.
Both work. Try it with method 2b

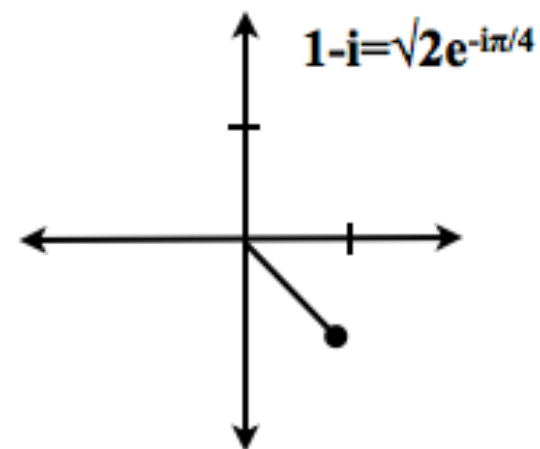
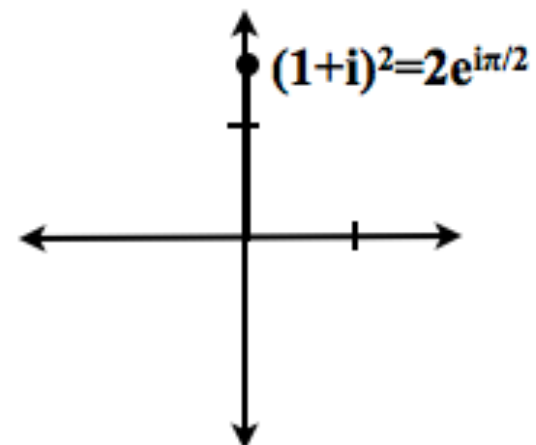
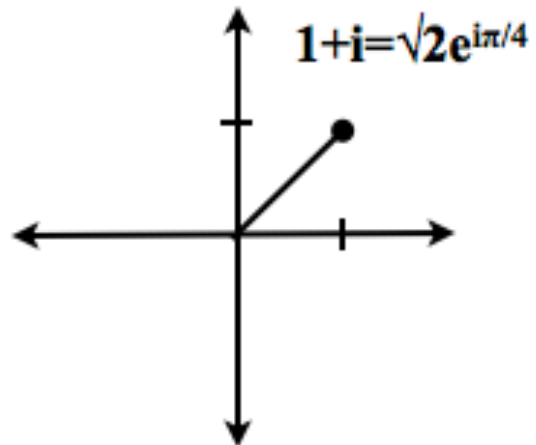
What is $(1+i)^2/(1-i)$



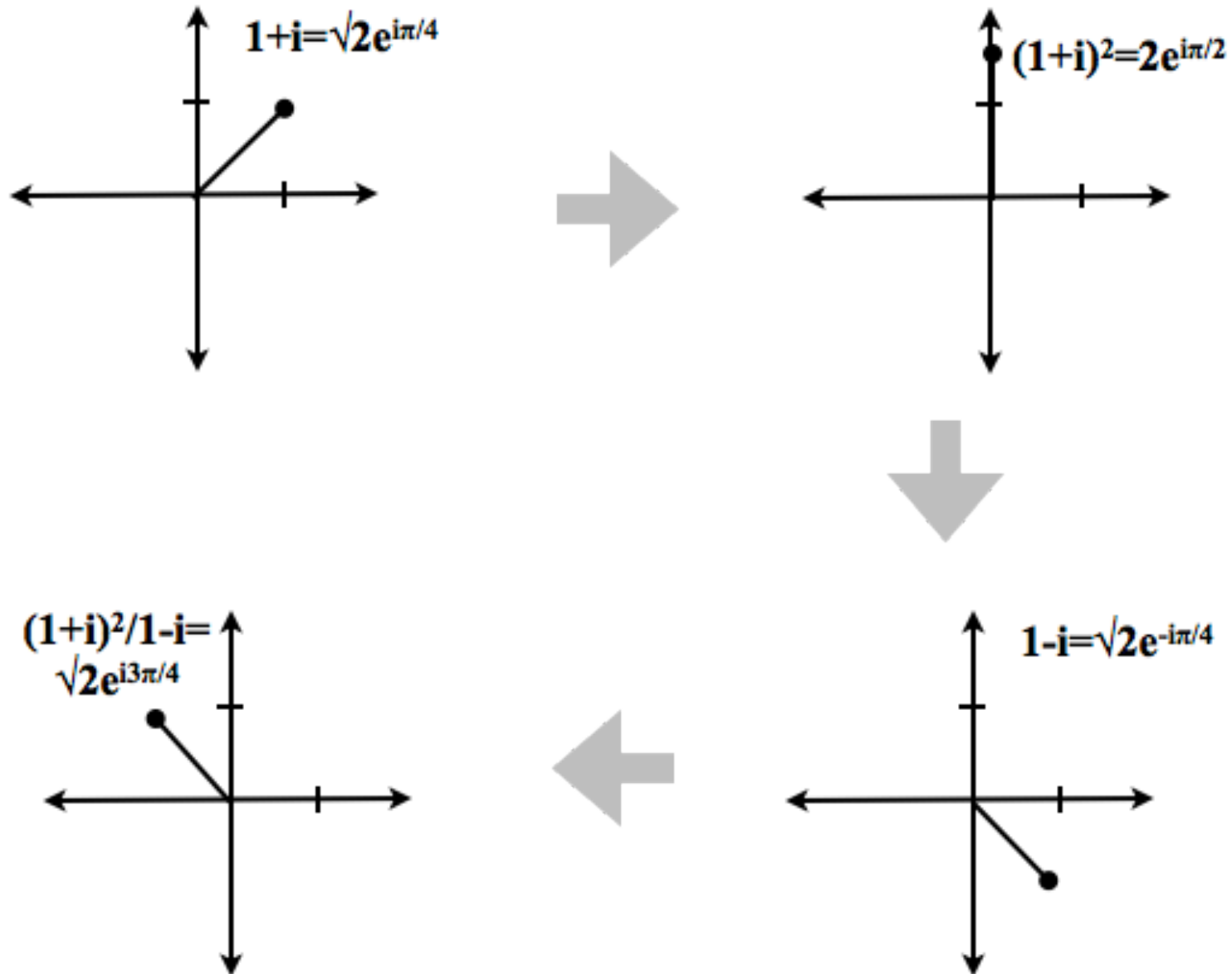
What is $(1+i)^2/(1-i)$



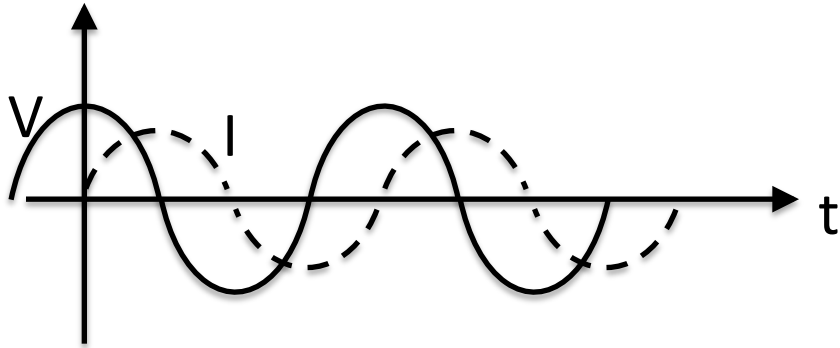
What is $(1+i)^2/(1-i)$



What is $(1+i)^2/(1-i)$



AC voltage V and current I vs time t are as shown:



The graph shows that..

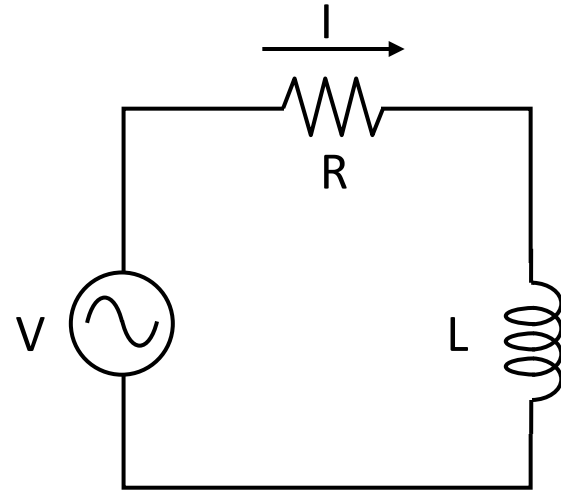
- A) I leads V (I peaks before V peaks)
- B) I lags V (I peaks after V peaks)
- C) Neither

I leads V = I peaks before V peaks

I lags V = I peaks after V peaks

Suppose $\hat{V} = V_0 e^{j\omega t}$ and \hat{I} are complex solutions of this equation:

$$\hat{V} = \hat{I}R + L \frac{d\hat{I}}{dt}$$



Is it always true that the real parts of these complex variables are solutions of the equation?

$$\text{Re}[\hat{V}] \stackrel{?}{=} \text{Re}[\hat{I}]R + L \frac{d}{dt} \text{Re}[\hat{I}]$$

A) Yes, always B) No, not always

$$I_0 e^{i\delta} = \frac{V_0}{i\omega L} = \frac{V_0}{\omega L} e^{i\delta}$$

The phase angle $\delta =$

A) 0

B) $+\pi/2$

C) $-\pi/2$

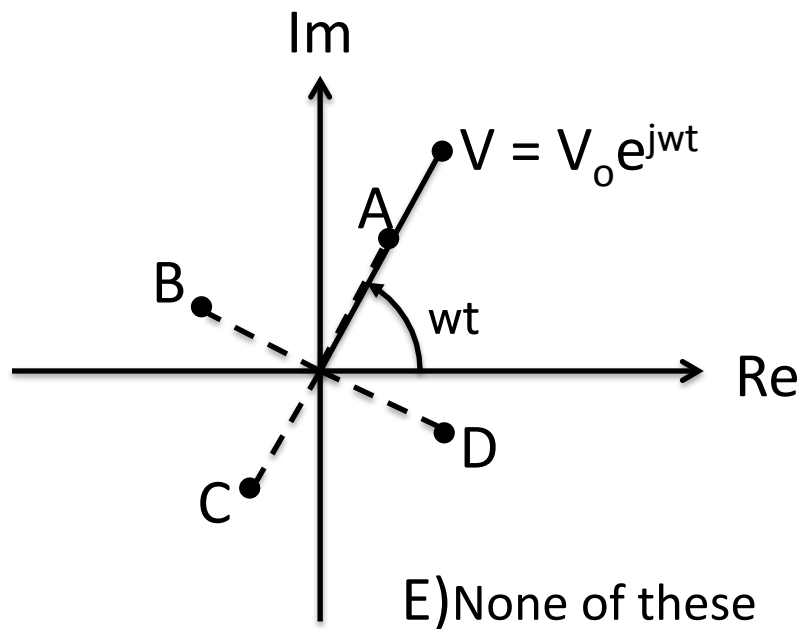
D) $+\pi$

E) $-\pi$

$$V = IZ = I |Z| e^{+j\pi/2}$$

$$I = \frac{V}{Z} = \frac{V}{|Z|} e^{-j\pi/2}$$

Which is the correct current phasor?



What is the total impedance of this circuit?

$Z_{\text{total}} =$

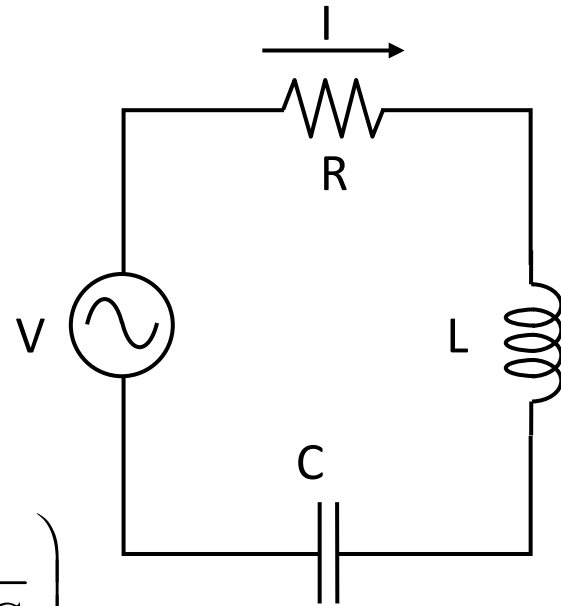
A) $R + j\left(\omega L + \frac{1}{\omega C}\right)$

B) $R + j\left(\omega L - \frac{1}{\omega C}\right)$

C) $\frac{1}{R} + \frac{1}{j\omega L} + j\omega C$

D) $\frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C}$

E) None of these



What is $\operatorname{Re}\left[\frac{e^{i\omega t}}{1+i}\right]$

A) $\frac{1}{\sqrt{2}}\cos(\omega t + \pi / 4)$

B) $\frac{1}{\sqrt{2}}\cos(\omega t - \pi / 4)$

C) $\frac{1}{2}\cos(\omega t + \pi / 4)$

D) $\frac{1}{2}\cos(\omega t - \pi / 4)$

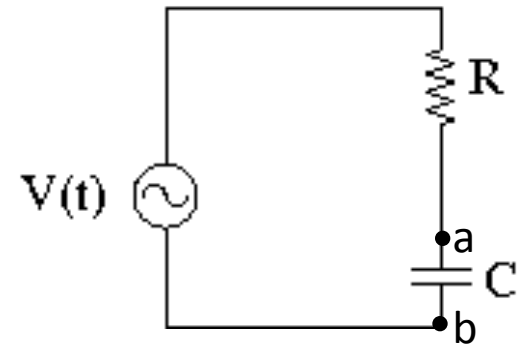
E) Not sure/ something entirely different!

Suppose you have a circuit driven by a voltage $V(t) = V_0 \cos(\omega t)$,
and you observe the resulting current is $I(t) = I_0 \cos(\omega t - \pi/4)$.

Would you say the current is
A) leading
B) lagging
the voltage by 45 degrees?

A simple RC circuit is driven by an AC power supply with an emf described by

$$V(t) = \begin{cases} 0, & t < 0 \\ V_0 \cos \omega t, & t > 0 \end{cases}$$

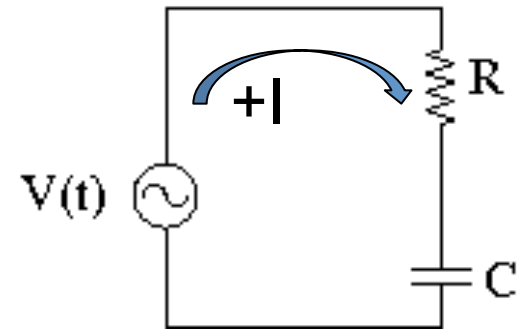


The voltage across the capacitor ($V_a - V_b$) just after $t=0$ is

- A. 0
- B. V_0
- C. $-V_0$
- D. Not enough information given

A simple RC circuit is driven by an AC power supply with an emf described by

$$V(t) = \begin{cases} 0, & t < 0 \\ V_0 \cos \omega t, & t > 0 \end{cases}$$



The current through the capacitor just after $t=0$ is

- A. 0
- B. V_0/R
- C. $-V_0/R$
- D. Not enough information given

Given a capacitance, C , and a resistance, R , the units of the product, RC , are:

- A) Amps
- B) Volts*seconds
- C) seconds
- D) 1/seconds.
- E) I do know the answer, but can't prove it in the 60 seconds I'm being given here...

The ac impedance of a RESISTOR is:

- A) Dependent on voltage drop across the resistor.
- B) Dependent on current flowing into the resistor.
- C) Both A) and B)
- D) None of the above.
- E) ???

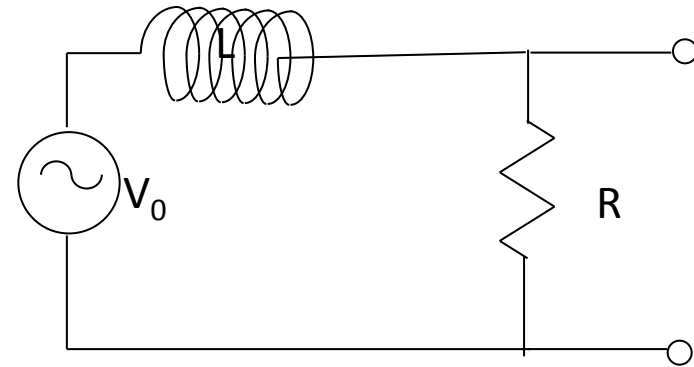
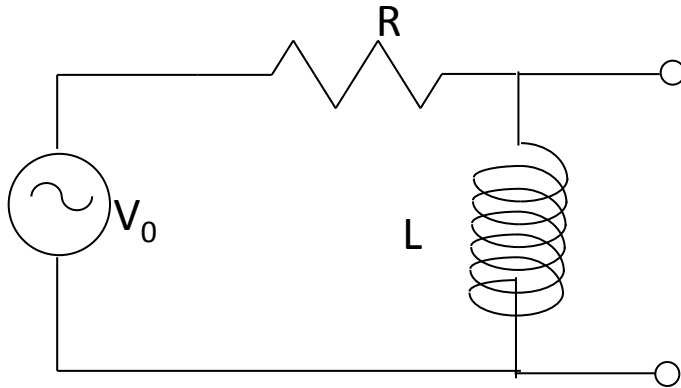
The ac impedance of a capacitor is:

- A) Dependent on the magnitude of the voltage drop across the capacitor.
- B) Dependent on the magnitude of the current flowing into the capacitor.
- C) Both A) and B)
- D) None of the above.
- E) ??

The ac impedance of an inductor is:

- A) Dependent on voltage drop across and/or current through the inductor.
- B) $Z_L = i\omega L$
- C) $Z_L = 1/i\omega L$
- D) None of the above.

Two LR circuits driven by an AC power supply are shown below.

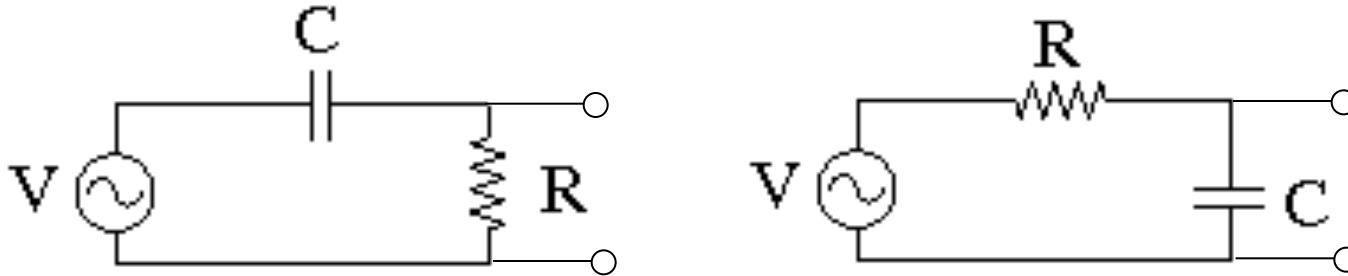


Which circuit is a low pass filter?

("Low pass" means low freq. inputs yield strong output, but high frequency input is "blocked", you get no output. So "low pass" filters reduce high frequencies, and passes the low frequencies...)

- A. The left circuit B) The right circuit C) Both circuits
D) Neither circuit E) ???

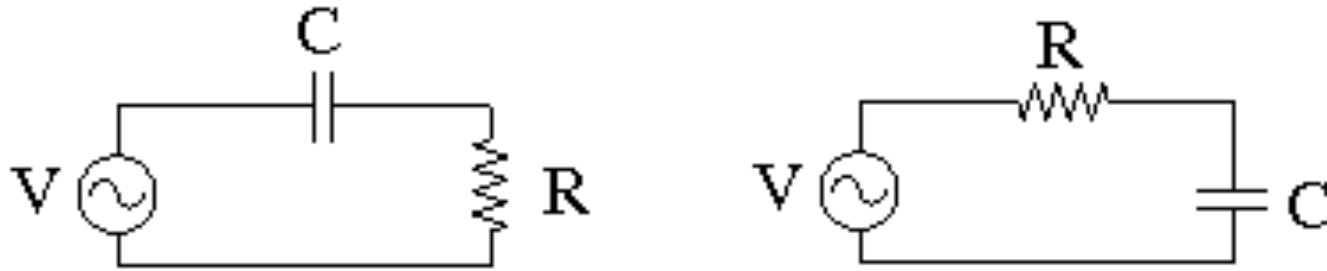
Two RC circuits driven by an AC power supply are shown below.



Which circuit is a high pass filter?

- A. The left circuit
- B. The right circuit
- C. Both circuits
- D. Neither circuit
- E. Not enough information given

Two RC circuits driven by an AC power supply are shown below.



Which circuit is a high pass filter?

- A. The left circuit
- B. The right circuit
- C. Both circuits
- D. Neither circuit