02 – Ohm's Law INSTRUCTOR'S INFORMATION PAGE

Topics: Ohm's law, continuity equation, boundary conditions on the electric field inside a conductor

Summary: A steady current flowing through a cone-shaped resistor is used as the context for addressing the implications of the microscopic version of Ohm's Law $\mathbf{J} = \sigma\mathbf{E}$. The initial multiple-choice question orients students to the situation by having them consider the current density inside the resistive material. They are then led to make conclusions about the electric field and local charge density inside the resistor by using Ohm's law in conjunction with the continuity equation and Gauss' law. Students are presented with two possible configurations for the electric field inside the conductor, and are asked to identify which aspects of those configurations are allowed, and which are precluded by boundary conditions or conservation of charge/current. The final activity asks them to interpret a graph of the correct field and equipotential lines inside the resistor in terms of the concepts discussed in the previous sections.

Supporting Material: J. D. Romano and R. H. Price, The conical resistor conundrum: A potential solution. *Am. J. Phys.* **64**, 1150-1153 (1996)

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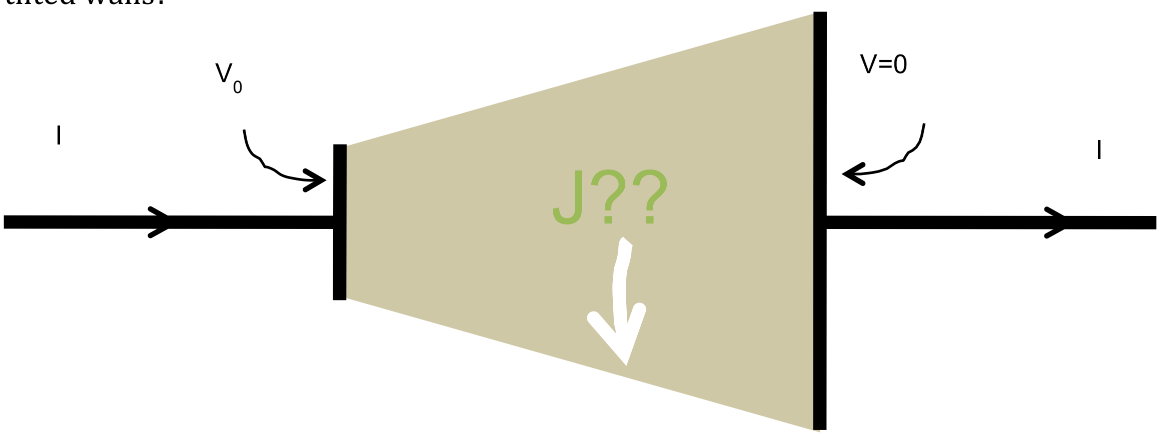
Comments: Most students should be able to complete this activity in less than 50 minutes. (In Fa 13 we had only 25 minutes for it – they did not finish, but we did get through page 1 together) The initial multiple-choice question is used as an in-class concept question, then have them work in small groups on the remaining questions (below). There is an assumption that students have already had some kind of introduction to Ohm's law. We have tried to be as explicit as possible in this first question, but many students may still think they're being asked about the current density *everywhere* inside the resistor, and not just at the edge, as indicated by the arrow. Instructors should be sure that students reconcile their mathematical conclusions ($\nabla \cdot \mathbf{E} = 0$ inside the resistor) and the fact that the correct field lines are spreading outwards (which may *look* to them like a "diverging" field). It is not essential that the field lines drawn by students on the second page are completely correct before moving on – we just want them to develop some kind of expectation for what they ought to look like. In the final challenge question, the electric field at the corners of the resistor must approach zero in order to satisfy incompatible boundary conditions (simultaneously parallel with the side and perpendicular to the metal surface) – some students may anticipate this result before reaching the final page, but many might not realize until asked to interpret the graph that the two boundary conditions are incompatible there.

Ohm's Law

We started this activity with a powerpoint question that looks like this:

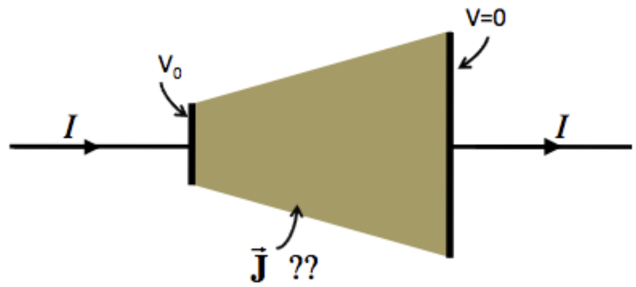
Inside this resistor setup, (in steady state) what is the current density **J** near the tilted walls?



A) Must be exactly parallel to the wall
B) Must be exactly perpendicular to the wall
C) Could have a mix of parallel and perp components
D) No obvious way to decide!?

(Then, we printed out the remaining 4 pages and gave that as the in-class Tutorial)

A. A <u>steady</u> current I flows through conducting wires that are connected to two metal disks that cap the ends of a cone-shaped resistor (made from a material with uniform conductivity σ). There is a potential difference V_0 between the two metal end caps.



In the figure, predict (sketch) the E-field lines inside the resistor.
Don't spend much time on this, your first intuition is fine for now!

i) When there is a steady current flowing, is the time-derivative of the charge density $\partial\rho/\partial t$ *inside* the resistor *zero* or *non-zero*?

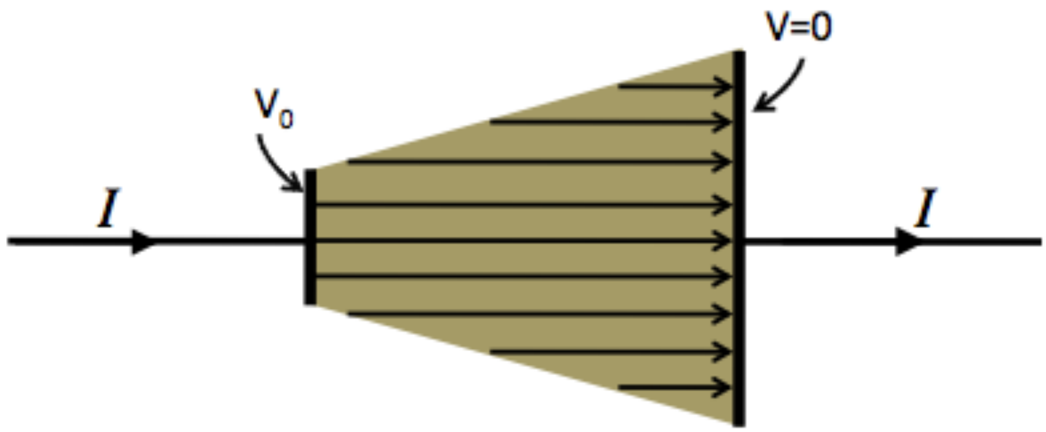
ii) Considering the continuity equation: $\nabla\cdot\mathbf{J} = -\partial\rho/\partial t$, is the divergence of the current density *inside* the resistor *zero* or *non-zero*?

iii) Considering Ohm's law: $\mathbf{J} = \sigma\mathbf{E}$, is the divergence of the electric field *inside* the resistor *zero* or *non-zero*?

iv) Considering Gauss' law: $\nabla\cdot\mathbf{E} = \rho/\varepsilon_0$, is the volume charge density *inside* the resistor *zero* or *non-zero*?

Refer back to all the conclusions on this page for the next parts!

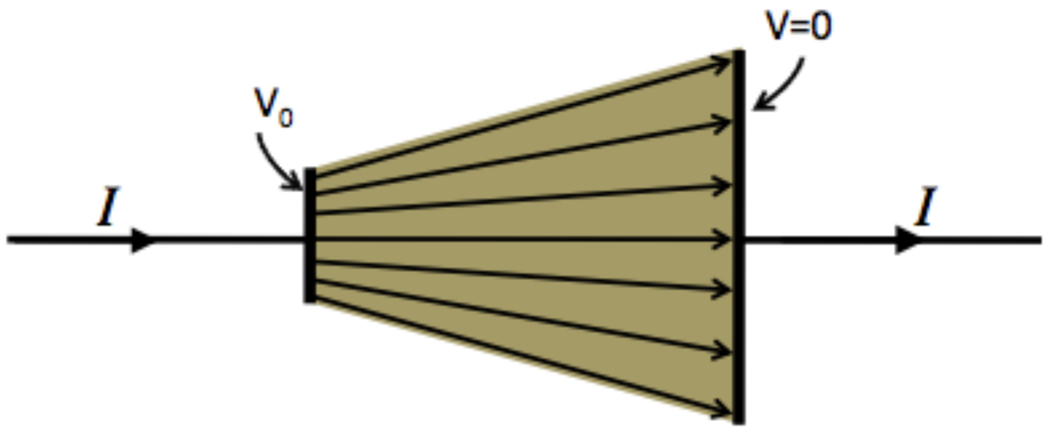
Is there anything about the sketch you started off with that you want to fix up, yet? Again, don't spend too much time on this, we'll come back to it.



B. For this same situation, the diagram now shows one way that a student has drawn the electric field lines inside the resistor.

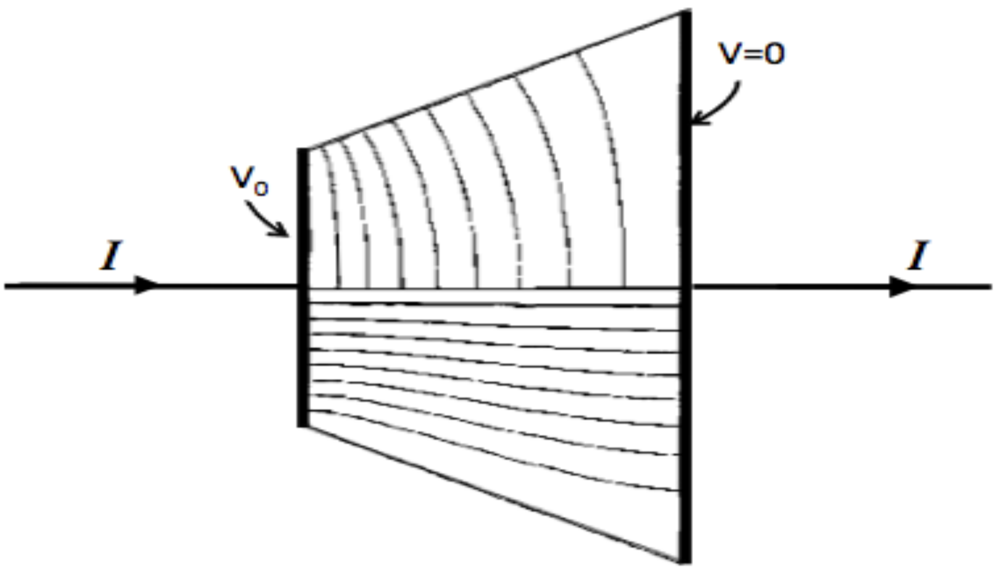
Which aspects of this drawing are correct, and which are incorrect? List as many as you can *in favor* of this drawing, and as many as you can *against* it.

[Hint: Try to answer for yourself questions like the following: Is charge conserved? Is the total current flowing constant? Is the divergence of the electric field inside the resistor correct? Are the boundary conditions satisfied? Are there other things that need to be considered?]



C. The diagram now shows another way that a student has drawn the electric field lines inside the resistor.

Which aspects of this drawing are correct, and which are incorrect? List as many as you can *in favor* of this drawing, and as many as you can *against* it.



D. The diagram now shows the <u>correct</u> *equipotential* lines (upper half) and <u>correct</u> *E-field* lines (lower half) inside this cone-shaped resistor.[1]

Explain how both of these sets of lines are consistent with the conditions that needed to be satisfied by the electric field inside the resistor, as you discussed on the previous pages.

Challenge Question: What can you conclude about the magnitude of the electric field as you get closer and closer to the bottom right-hand corner of the resistor? Explain your answer in terms of boundary conditions.

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[1] Credit to J. D. Romano and R. H. Price for finding this solution numerically.