Electricity and Magnetism II

Griffiths Chapter 8 Conservation Laws

Clicker Questions

The work energy theorem states:

$$W = \int_i^f \mathbf{F}_{\mathbf{net}} \cdot d\mathbf{l} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

This theorem is valid

A. only for conservative forces.
B. only for non-conservative forces.
C. only for forces which are constant in time
D. only for forces which can be expressed as potential energies
E. for all forces.

Local conservation of electric charge is expressed mathematically by:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{\mathbf{J}}$$

where J is "current density"

J = ρ**v** has units of (charge/sec)/m^2

We are trying to come up with a "conservation of energy" expression:

$$\frac{\partial(\text{energy density})}{\partial t} = -\nabla \cdot (something)$$

What sort of beast is this "something" ?

- Is it a scalar, vector, something else?
- How would you interpret it, what words would you use to try to describe it?
- What are its UNITS?

A) J   B) J/s   C) J/m^2 D) J/(s m^2)

E) Other!

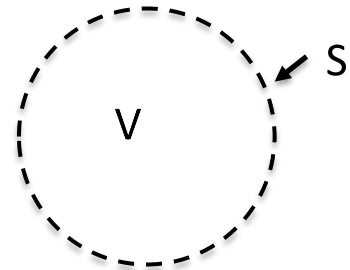
A + and - charge are held a distance R apart and released.

The two particles accelerate toward each other as a result of the Coulomb attraction.

As the particles approach each other, the energy contained in the electric field surrounding the two charges...

A: increases B: decreases C: stays the same D: ??

$U_{em\ (outside\ V)}$ = EM energy in field outside volume V



$$\frac{d}{dt}\left(U_{em\ \text{outside V}}\right) = ?$$

A) $\int_{\text{outside V}} \nabla \cdot \vec{S}\, d\tau$

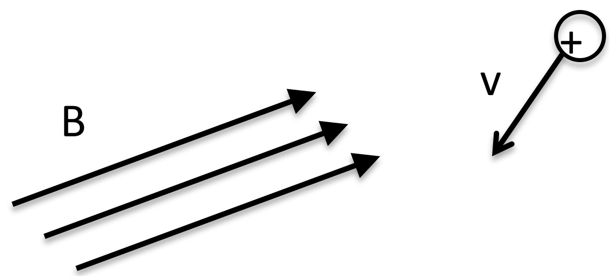
B) $\int_{\text{inside V}} \nabla \cdot \vec{S}\, d\tau$

C) $\oint_S \vec{S} \cdot d\vec{a}$

D) $-\oint_S \vec{S} \cdot d\vec{a}$

E) $\nabla \cdot \vec{S}$

Can the force from a magnetic field do work on a charge?



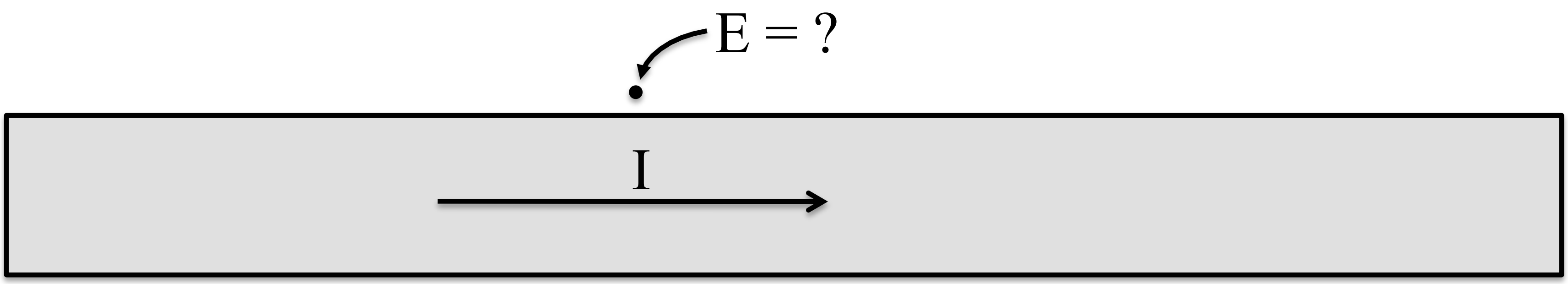
A) Yes, sure

B) Yes, but only if <u>other</u> forces are moving the charge

C) Yes, but only if <u>no</u> other forces are moving the charge

B) No, never

A steady current I flows along a long straight wire.



The parallel component of the E-field, $E^{||}$, just outside the surface …

A) must be zero B) must be non-zero

C) might be zero or non-zero depending on details

F_{AB} = F_onA_fromB $\qquad$ F_{BA} = F_onB_fromA

$\mathbf{F}_{AB}$ = [illegible] $\mathbf{F}_{BA}$

Newton's 3rd Law is equivalent to..

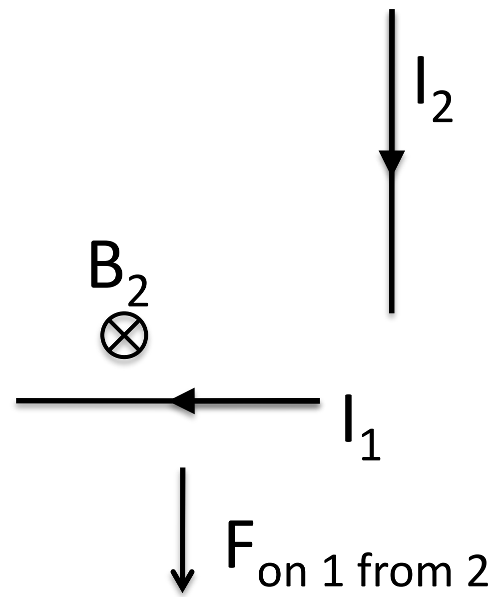
A) Conservation of energy

B) Conservation of linear momentum

C) Conservation of angular momentum

D) None of these. NIII is a separate law of physics.

Two short lengths of wire carry currents as shown.
(The current is supplied by discharging a capacitor.)
The diagram shows the direction of the force on wire 1 due to wire 2.



What is the direction of the force on wire 2 due to wire 1?

A) ⇨

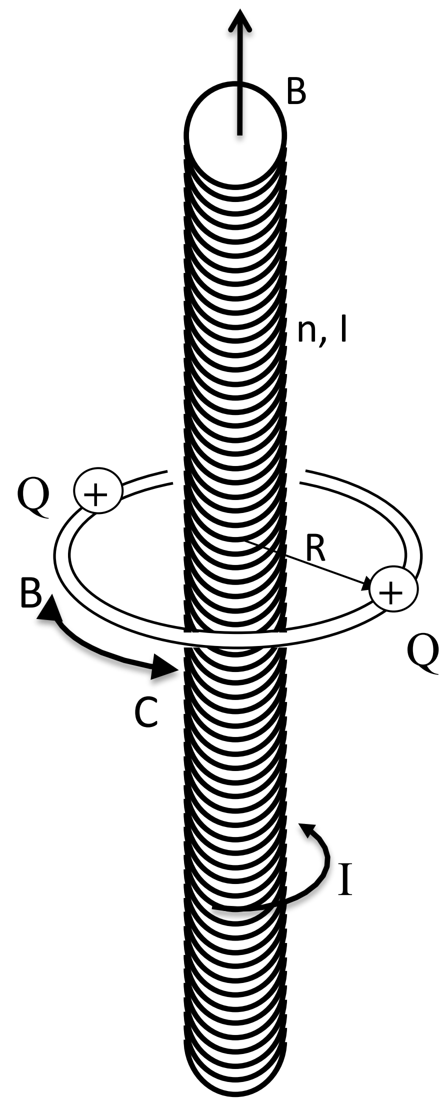
B) ⇦

C) ⇧

D) ⇩

E) None of these

Feynman's Paradox:
Two charged balls are attached to a horizontal ring that can rotate about a vertical axis without friction. A solenoid with current I is on the axis. Initially, everything is at rest.

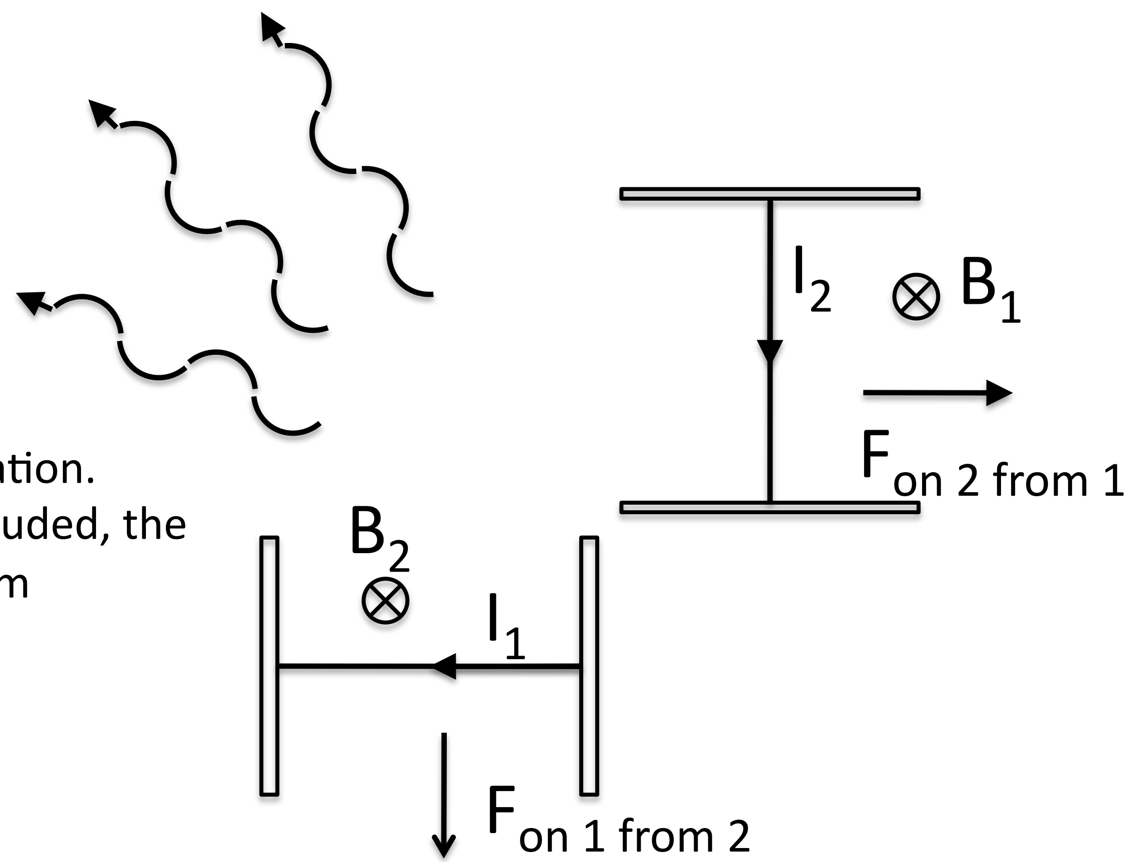


The current in the solenoid is turned off. What happens to the charges?

A) They remain at rest
B) They rotate CW.
C) They rotate CCW.

Does this device violate Conservation of Angular Momentum?
A) Yes
B) No
C) Neither, Cons of Ang Mom does not apply in this case.

Two charged capacitors discharge through wires. The magnetic field forces are not equal and opposite. After the discharge the momentum of the capacitors is to the lower right. What's the resolution of this Newton's Third Law paradox?



Answer:
Momentum in the EM radiation.
When <u>all</u> momentum is included, the center-of-mass of the system remains stationary.

$$\frac{\partial}{\partial t}u_q = -\frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right) - \nabla \cdot \vec{\mathbf{S}}$$

(Where **S**=**E** x **B** /μ_0)

How do you interpret this equation? In particular:

Does the – sign on the first term on the right seem OK?

A) Yup B) It's disconcerting, did we make a mistake? C) ??

$$\frac{\partial}{\partial t}\left(u_q + \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right) = -\nabla \cdot \vec{\mathbf{S}}$$

$$\frac{\partial}{\partial t}(u_q + u_{EM}) = -\nabla \cdot \vec{\mathbf{S}}$$

$$\frac{d}{dt}\iiint (u_q + u_{EM})\,d\tau = -\iiint \nabla \cdot \vec{\mathbf{S}}\,d\tau$$

(Where **S**=**E** x **B** /μ_0)

Would you interpret **S** as the

A) OUTFLOW of energy/area/time or
B) INFLOW of energy/area/time
C) OUTFLOW of energy/volume/time
D) INFLOW of energy/volume/time
E) ???

$$\frac{d}{dt}\iiint (u_q + u_{EM})\, d\tau = -\iiint \nabla \cdot \vec{\mathbf{S}}\, d\tau$$

(Where **S**=**E** x **B** /μ_0)

$$= -\oiint \vec{\mathbf{S}} \cdot d\vec{\mathbf{a}}$$

Would you interpret **S** as the

A) OUTFLOW of energy/area/time or

B) INFLOW of energy/area/time

C) OUTFLOW of energy/volume/time

D) INFLOW of energy/volume/time

E) ???

What are the SI metric units of the combination of fields
E x **B** /μ_0 ?

A) J B) J/s C) J/m^2 D) J/(m^2 s)
E) Other!

The fields can change the total energies of charged particles by:

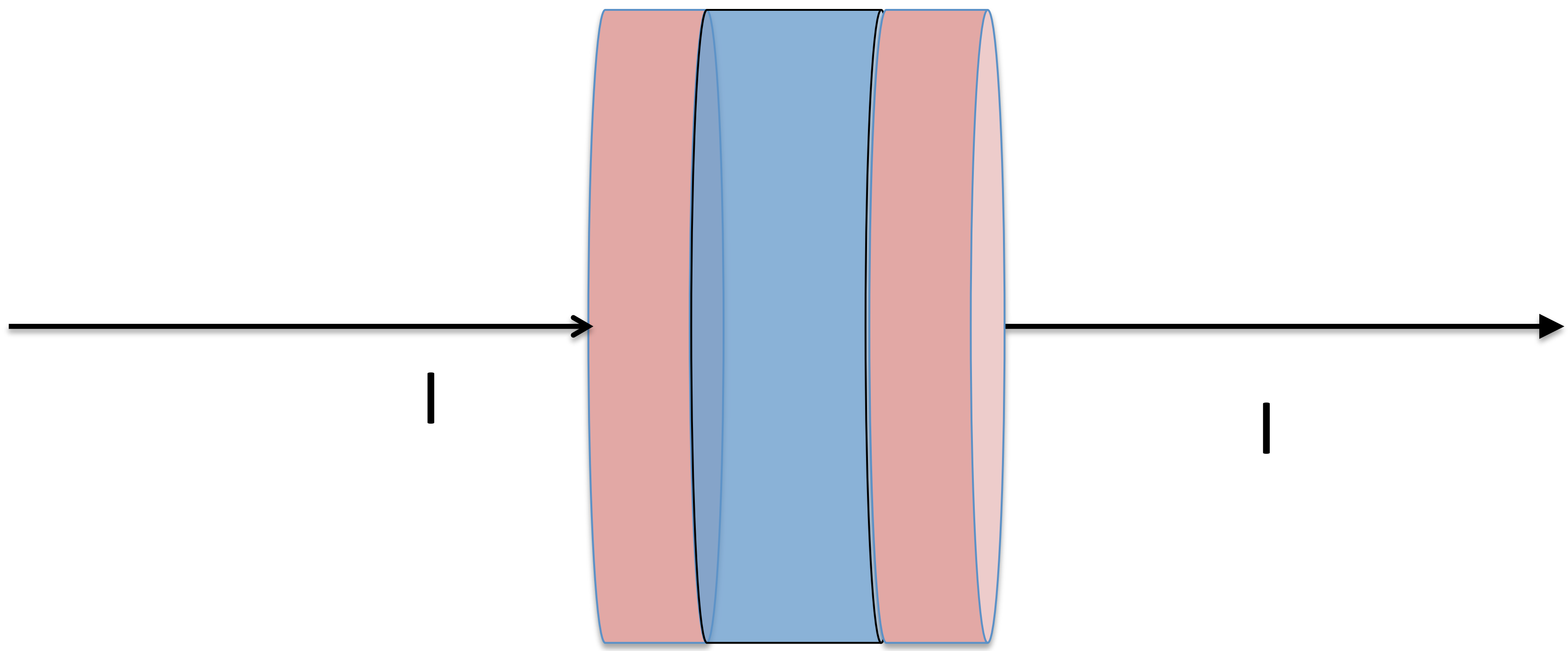
A) Doing work on the particles
B) Changing the potential energies only
C) Changing the kinetic energies only
D) Applying forces only perpendicular to the particle motion.
E) None of the above.

Consider the cylindrical volume of space bounded by the capacitor plates. Compute $\mathbf{S} = \mathbf{E} \times \mathbf{B} / \mu_0$ at the outside (cylindrical, curved) surface of that volume.

Which WAY does it point?

A) Always inward B) Always outward

C) ???



Given a quantity with units of (Joules/m^3), you can convert it to a quantity with units of Joules/(m^2 * seconds) by multiplying by:

A) a length

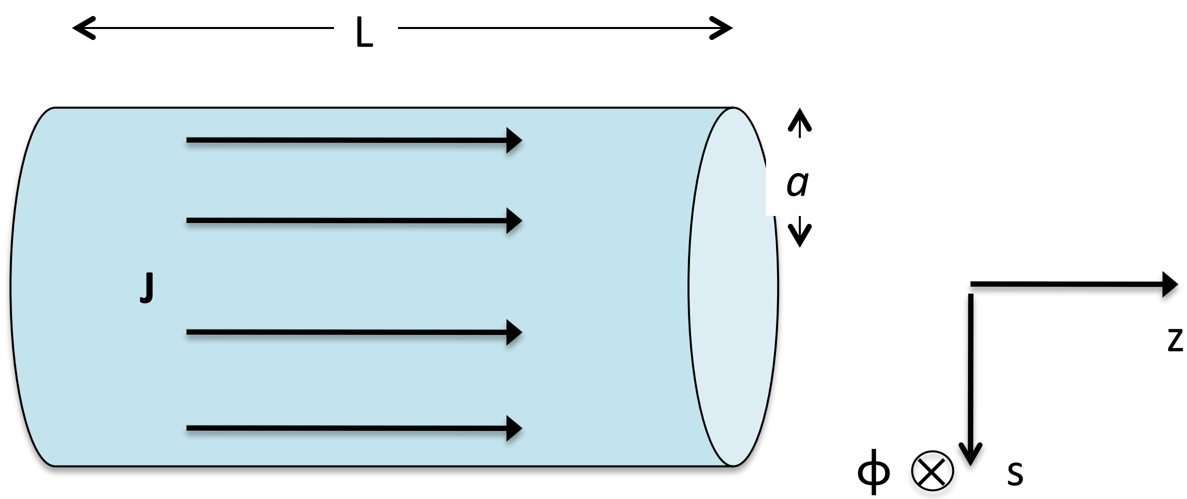
B) a frequency

C) a speed

D) an acceleration

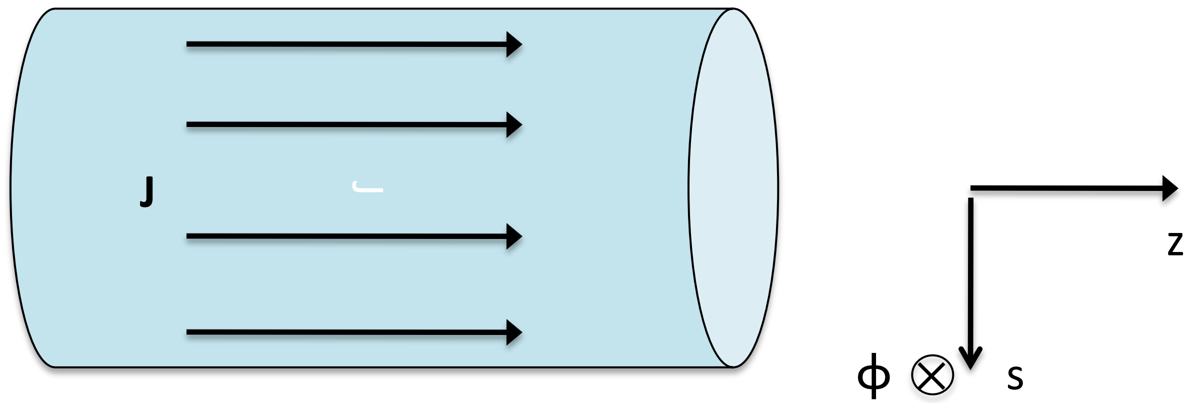
E) None of the above

Consider a current I flowing through a cylindrical resistor of length L and radius *a* with voltage V applied. What is the E field inside the resistor?



A. (V/*a*) z-hat
B. (V/*a*) ϕ-hat
C. (V/*a*) s-hat
D. (Vs/a^2) z-hat
E. None of the above

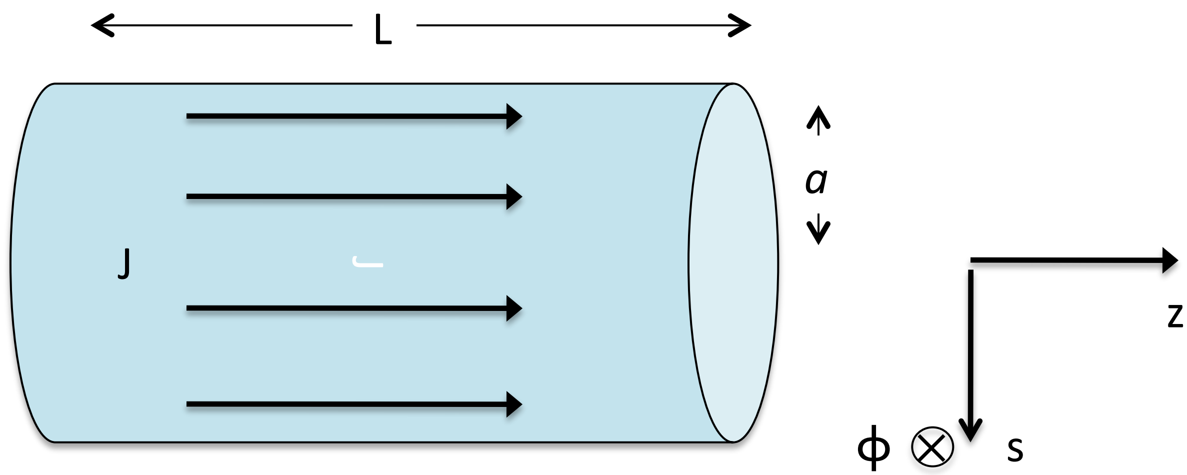
Consider a current I flowing through a cylindrical resistor of length L and radius a with voltage V applied. What is the B field inside the resistor?



A. $(I\mu_0/2\pi s)$ $\hat{\phi}$-hat
B. $(I\mu_0 s/2\pi a^2)$ ϕ-hat
C. $(I\mu_0/2\pi a)$ ϕ-hat
D. $-(I\mu_0/2\pi a)$ ϕ-hat
E. None of the above

Consider a current I flowing through a cylindrical resistor of length L and radius a with voltage V applied.

What is the direction of the **S** vector on the outer curved surface of the resistor?



A. ±ϕ-hat
B. ± s-hat
C. ± z-hat
D. ???

And, is it + or -?

The energies stored in the electric and magnetic fields are:

A) individually conserved for both E and B, and cannot change.
B) conserved only if you sum the E and B energies together.
C) are not conserved at all.
D) ???

The fields can change the total momentum of charged particles by:

A) Fields cannot change particle momentum
B) Applying a net force to the particles
C) Changing only the potential energy
D) Only if they do net work on the particles.
E) None of the above.

Consider two point charges, each moving with constant velocity v, charge 1 along the +x axis and charge 2 along the +y axis. They are equidistant from the origin.

What is the direction of the magnetic force on charge 1 from charge 2? (You'll need to sketch this! Don't do it in your head!)

A. +x
B. +y
C. +z
D. More than one of the above
E. None of the above

Consider two point charges, each moving with constant velocity v, charge 1 along the +x axis and charge 2 along the +y axis. They are equidistant from the origin.

What is the direction of the magnetic force on charge 2 from charge 1? (You'll need to sketch this! Don't do it in your head!)

A. Equal to the answer of the previous question
B. Equal but opposite to the answer of the previous question
C. Something *different* than either of the above.

We seek a local conservation law that relates the time change in momentum density (units of momentum/m^3), to the divergence of a current density, "T", with units of:

A) Newtons/m^2

B) kg*m/(m^2*second^2)

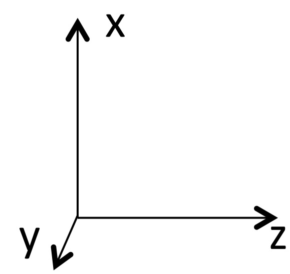
C) Joules/m^3

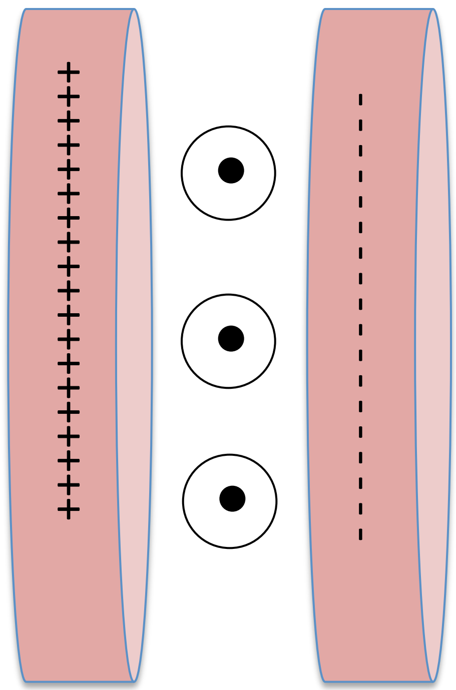
D) More than one of the above

E) None of the above

Momentum in the fields: $\vec{p}_{EM}\,/\,volume = \mu_0 \varepsilon_0 \vec{\mathbf{S}}$

Consider a charged capacitor placed in a uniform B field in the +y direction



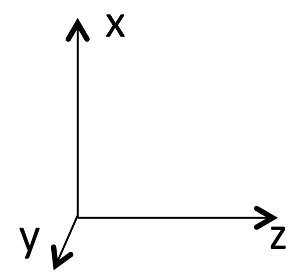


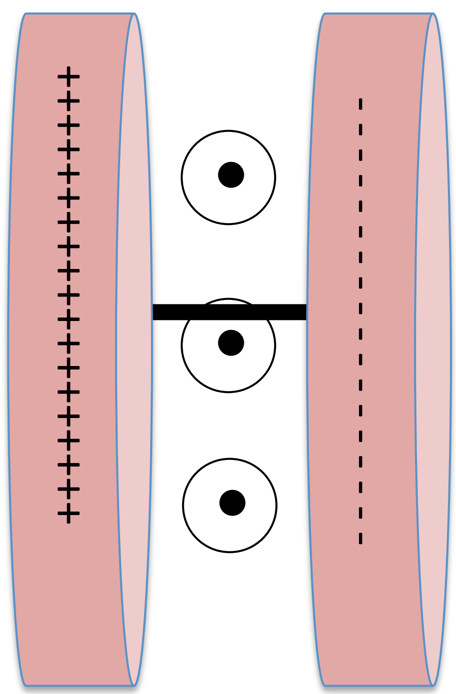
Which way does the stored field Momentum in this system point?

A) +/- x
B) +/- y
C) +/- z
D) Zero!
E) Other/???

Momentum in the fields: $\vec{p}_{EM}\,/\,volume = \mu_0 \varepsilon_0 \vec{\mathbf{S}}$

Now “short out” this capacitor with a small wire.
As the current flows, (while the capacitor is discharging)…





which way does the magnetic force push the wire (and thus, the system)?

A) +/- x
B) +/- y
C) +/- z
D) Zero!
E) Other/???

Is your answer consistent with “conservation of momentum”?

What units should a momentum density have?

A. N s/m^3
B. J s/m^3
C. kg/(s m^2)
D. More than one of the above
E. None of the above

What units should a momentum flux density have?

A. N/m^3
B. N/m^2
C. $kg/(s\ m)$
D. More than one of the above
E. None of the above

The Maxwell stress tensor is given by:

$$T_{ij} = \varepsilon_0 (E_i E_j - \tfrac{1}{2}\delta_{ij} E^2) + \frac{1}{\mu_0}(B_i B_j - \tfrac{1}{2}\delta_{ij} B^2)$$

What is the E field part of the T_{zx} term?

A. $\varepsilon_0(E_z E_x - ½(E_x^2 + E_z^2))$
B. $\varepsilon_0(E_z E_x - ½E_y^2)$
C. $\varepsilon_0(E_z E_x - ½(E_x^2 + E_y^2 + E_z^2))$
D. $\varepsilon_0(E_z E_x)$
E. None of the above

The Maxwell stress tensor is given by:

$$T_{ij} = \varepsilon_0 (E_i E_j - \tfrac{1}{2}\delta_{ij} E^2) + \frac{1}{\mu_0}(B_i B_j - \tfrac{1}{2}\delta_{ij} B^2)$$

What is the E field part of the T_{zz} term?

½

A. $\varepsilon_0(E_z^2-(E_x^2+E_y^2))/2$
B. $\varepsilon_0(E_z^2-½(E_x^2+E_y^2)$
C. $-\varepsilon_0(E_x^2+E_y^2)$
D. $\varepsilon_0(E_z^2)$
E. None of the above

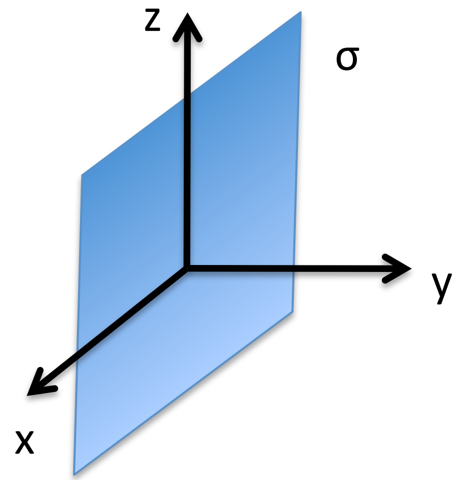
Given a general Maxwell Stress tensor with all elements non-zero, what is the net force on a small isolated area element
$d\mathbf{a} = (dx\ dy)\ \mathbf{z}$?

$$\begin{matrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{matrix}$$

$$\vec{F} = \iint \overleftrightarrow{T} \cdot d\vec{A}$$

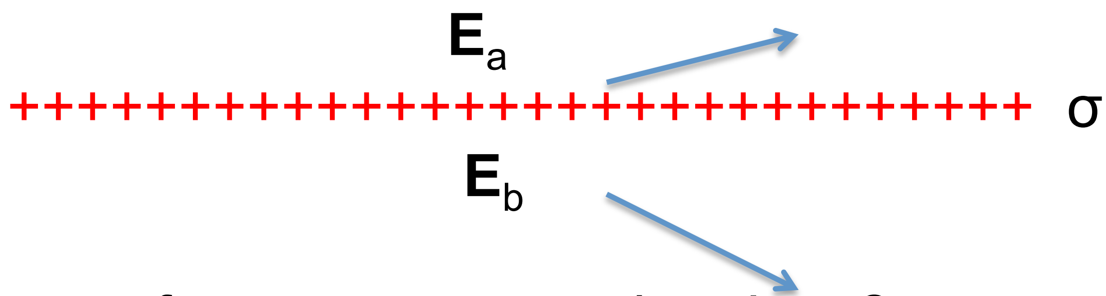
A. $T_{xz}\ dx\ dy\ \mathbf{z}$
B. $T_{yz}\ dx\ dy\ \mathbf{z}$
C. $T_{xz}\ dx\ dy\ \mathbf{z}$
D. $(T_{xz}\ \mathbf{x} + T_{yz}\ \mathbf{y} + T_{zz}\ \mathbf{z})\ dx\ dy$
E. Something else!

An infinite plane of surface charge σ lies in the *xz* plane. In the region *y* > 0, which element(s) of the stress tensor is(are) non-zero?



A. Tyy
B. Txx, Tyy, Tzz
C. Txy,Tyx,Tyy,Tyz,Tzy
D. Txy,Tyx,Tyy,Tyz,Tzy,Txx,Tyy
E. None of the above

Suppose we have a plane of surface charge σ with electric field $\mathbf{E}_a$ above the plane and $\mathbf{E}_b$ below the plane.



What is the net force per area on the plane?

A. $(\mathbf{E}_a + \mathbf{E}_b)\,\sigma$
B. $(\mathbf{E}_a - \mathbf{E}_b)\,\sigma$
C. $(\mathbf{E}_a + \mathbf{E}_b)\,\sigma/2$
D. $(\mathbf{E}_a - \mathbf{E}_b)\,\sigma/2$
E. None of the above