Electricity and Magnetism II

Griffiths Chapter 11 Radiation

Clicker Questions

The integrated Poynting flux heading out to infinity is

$$\iint_{\text{large S}} \frac{\vec{E}\times\vec{B}}{\mu_0}\cdot d\vec{a}$$

If the E and B fields are static, with localized sources:
How do E & B fall off with distance?

What does that tell you about the above integral?

In order for a localized source (near the origin) to radiate energy off to infinity, this integral must be non-zero.

$$\oiint_{\text{area}\to\infty} \frac{\vec{E}\times\vec{B}}{\mu_0}\cdot d\vec{a}$$

How must E and B fall off with distance r, in order for the source to radiate energy to infinity? Both E and B must fall off as

A) $1/r$
B) $1/r^2$
C) $1/r^{3/2}$
D) $1/r^3$
E) Something else.

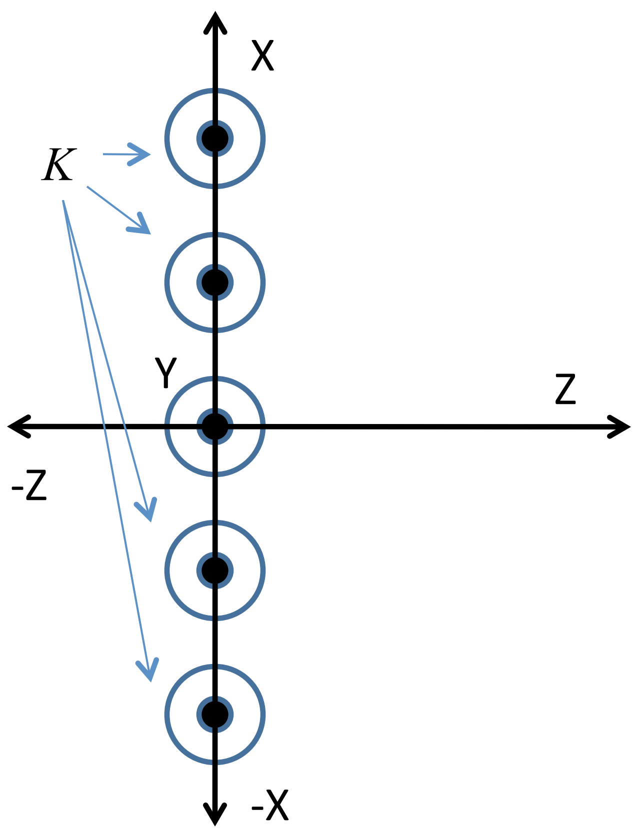
Could E and B fall off as $1/r^{1/2}$ from a localized source?

A) yes   B) no

A charge moves in a straight line with constant velocity. Does it radiate?

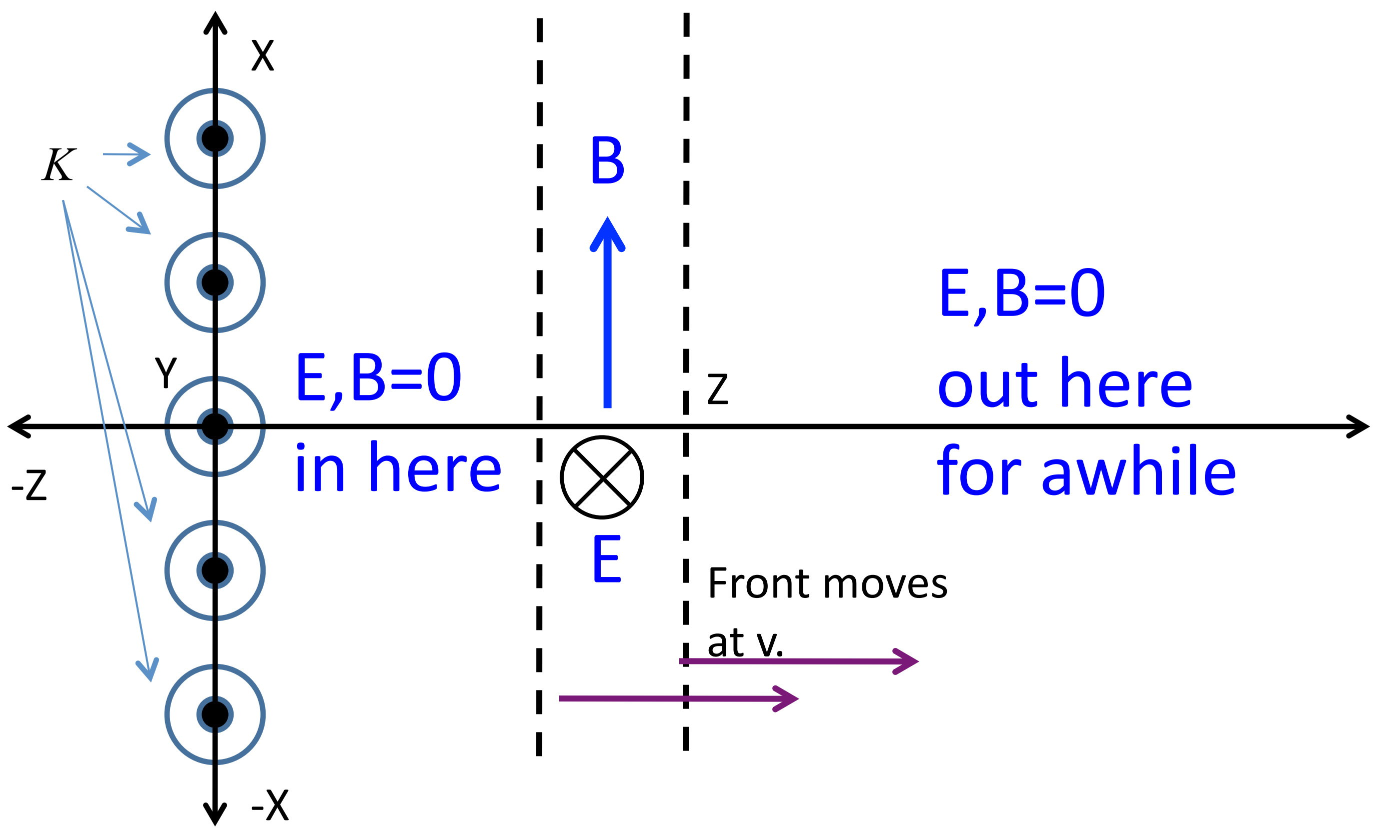
A) Yes, and I can defend my answer
B) Yes, but I cannot explain why I believe this
C) No, and I can defend my answer
D) No, but I cannot explain why I believe this
E) It depends on the reference frame of the observer!

A neutral infinite current sheet, **K**, is turned on at t=0, flows in the x-y plane, in the +y direction.
It is suddenly turned OFF at $t=t_1$.
Describe E and B everywhere in space!



(No clicker question,
just be ready to voice your ideas!)

A neutral infinite current sheet, **K**, is turned on at *t*=0, flows in the *x-y* plane, in the +*y* direction.
It is suddenly turned OFF at t=0.
Describe E and B everywhere in space!



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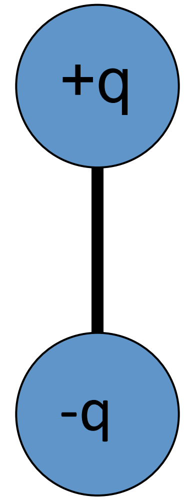
A small oscillating dipole has height d,
and charge q(t) at the ends.
A wire carries the oscillating current back and forth between the two poles.



$=q_0 \cos\omega t$

What is I(t), the current in the wire?

A) $q_0 \cos \omega t$

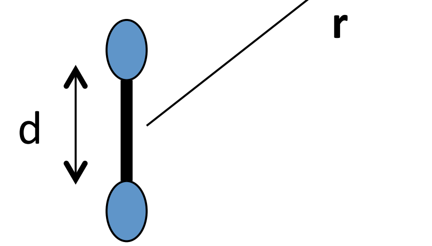
B) $\dfrac{q_0 \cos \omega t}{d}$

C) $-q_0\, \omega \sin \omega t$

D) $-q_0 d\, \omega \sin \omega t$

E) Something else?!

You are FAR from a small oscillating dipole, $I(t) = -q_0 \omega \sin \omega t$ and you want to compute the vector potential:



$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}(\vec{r}\,', t_R)}{|\vec{r} - \vec{r}\,'|} d^3\vec{r}\,', \text{ with } t_R = t - \frac{|\vec{r} - \vec{r}\,'|}{c}$$

What is the leading order approximate expression for **A**(**r**, t) ?

A) $\dfrac{\mu_0}{4\pi} \dfrac{-q_0 \omega \sin \omega t}{r} \hat{z}$

B) $\dfrac{\mu_0}{4\pi} \dfrac{-q_0 \omega \sin \omega (t - r/c)}{r} \hat{z}$

C) $\dfrac{\mu_0}{4\pi} d \dfrac{-q_0 \omega \sin \omega t}{r} \hat{z}$

D) $\dfrac{\mu_0}{4\pi} d \dfrac{-q_0 \omega \sin \omega (t - r/c)}{r} \hat{z}$

E) Something else?!

What is $\hat{z}$ in spherical coordinates?

A) $\cos\theta\,\hat{r}$

B) $\sin\theta\,\hat{r}+\cos\theta\,\hat{\theta}$

C) $\cos\theta\,\hat{r}+\sin\theta\,\hat{\theta}$

D) $\cos\theta\,\hat{r}-\sin\theta\,\hat{\theta}$

E) Something else?!

$$Thus,\quad \vec{\mathrm{A}}(\mathrm{r,t}) = \frac{\mu_0}{4\pi}d\frac{-q_0\omega\sin\omega(t-r/c)}{r}\left(\cos\theta\,\hat{r}-\sin\theta\,\hat{\theta}\right)$$

$$\vec{B}(\mathrm{r,t})=\nabla\times\vec{\mathrm{A}}(\mathrm{r,t}) = \frac{-\mu_0 p_0\omega^2}{4\pi}\sin\theta\frac{\cos\omega(t-r/c)}{rc}\,\hat{\varphi}$$

A function of the form

$$B(r,t) \propto \frac{1}{r}\cos[\omega(t - r/c)] = \frac{1}{r}\cos\left[-\frac{\omega}{c}(r - ct)\right]$$

$$B(r,t) \propto \frac{1}{r}\cos\left[-(kr - \omega t)\right] = \frac{1}{r}\cos(kr - \omega t)$$

represents a ..

A) traveling wave moving in the r-hat direction
B) traveling wave moving in the q-hat direction
C) traveling wave moving in the z-hat direction
D) traveling wave moving in some other direction
E) Something other than a traveling wave

For an oscillating dipole, $p=p_0 \cos(\omega t)$,
we worked out last class (assuming $r >> \lambda >> d$) that:

$$B(r,t)\,\hat{\varphi} = \frac{-\mu_0 p_0 \omega^2}{4\pi} \sin\theta \frac{\cos\omega(t-r/c)}{rc}\hat{\varphi}$$

To think about (be prepared to discuss): In what ways is it like (and not like) our familiar free-space "traveling plane wave"?

Which of the following describes the E field?

A) $\vec{E} = cB\,\hat{\varphi}$ B) $\vec{E} = cB\,\hat{\theta}$ C) $\vec{E} = cB\,\hat{r}$ D) $\vec{E} = cB\,\hat{z}$

E) None of these/something else?

Total power radiated by a small electric dipole is

$$P = \iint \frac{\mu_0 p_0^2 \omega^4}{(16\pi^2)cr^2} \sin^2\theta \cos^2 \omega(t - r/c)\,da$$

$$= \frac{\mu_0 p_0^2 \omega^4}{6\pi c} \cos^2 \omega(t - r/c)$$

What is the time averaged power?

What is the time averaged intensity at distance “r”?

The time averaged Poynting vector (far from a small electric dipole) is approximately:

$$\left\langle \vec{S} \right\rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi c r^2} \sin^2\theta \;\; \hat{r}$$

Describe this energy flow in words, pictures, or graph.

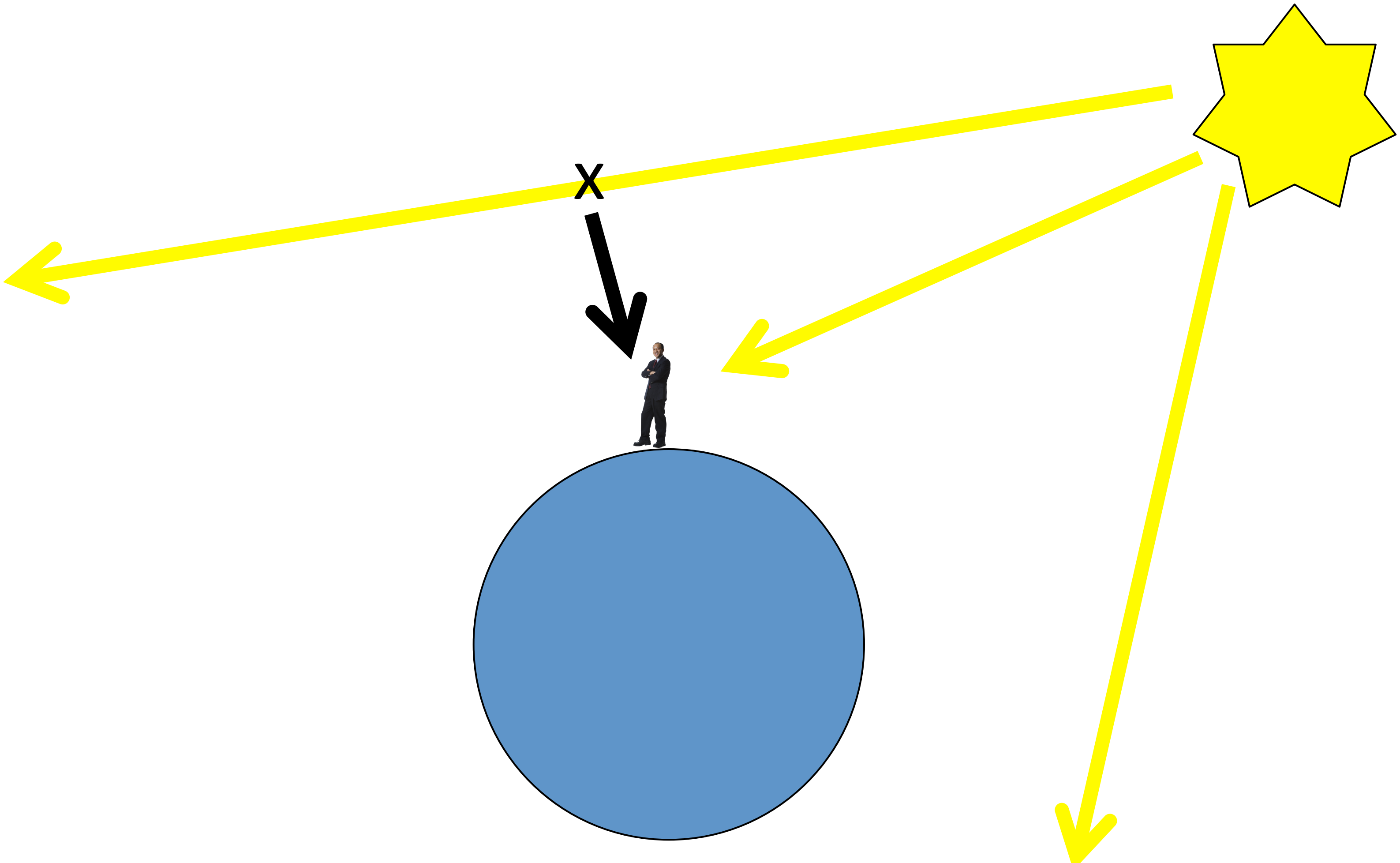
$$R_{rad} \equiv \frac{P_{ave}}{I^2_{rms}}$$

Recall, we found $I = -q_0\omega \cos(\omega t)$.
So what is I_{rms}?

A) $q_0\omega$ B) $q_0\omega / 2$ C) $q_0\omega / \sqrt{2}$ D) $\sqrt{2}q_0\omega$

E) None of these/something else?

$$R_{rad} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{2\pi}{3}(d/\lambda)^2$$



If light scatters from point “x” and heads towards the observer,

What color is it likely to be?

Is the scattered light polarized? If so, which way?

We're interested in power radiated by a wiggling charge.

1) What physics variables might this power possibly depend on? (Come up with a complete, but not OVERcomplete list)

2) If your list of variables was v_1, v_2, etc…, we're saying
$P = v_1^a\, v_2^{\,b} \ldots$
Look at the SI UNITS of all quantities involved. I claim you should be able to uniquely figure out those powers (a,b, …) ! Try it.

Hint: My list of variables is q, a, c, and μ_o

The TOTAL power of an accelerating (non-relativistic) charge is called **Larmor's formula**.
It depends on c, $\mu_{0,}$ a (acceleration) and q (charge).

So I presume that means $P = c^A \mu_0^B a^C q^D$
(!? It's at least a plausible guess…)

Figure out the *constants* A-D in that formula, without using any physics beyond units! (This is *dimensional analysis)*

Note: $[P]$ = Watts = kg m^2/s^3,
$[\mu_0]$= N/A^2 = kg m/C^2