



## Traveling waves

**Topics:** Traveling waves: why is  $\cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$  a plane wave in  $\mathbf{k}$  direction?

**Summary:** Many students memorize the fact that  $\cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$  represents a plane wave in  $\mathbf{k}$  direction without understanding why. This short group activity is designed to help them picture the geometry and “see” why these waves are planes, why they travel in the  $\mathbf{k}$  direction, and why wavelength is inverse with  $|\mathbf{k}|$ .

On the board (or screen) is the question:

“How do we interpret the wave  $f(x,y,z,t) = \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$ ”

Allow a brief discussion (some may say plane wave, some may say spherical wave)  
With the “grid” handout (I copy onto both sides so they can do it twice if they want)

- Each student picks a  $\mathbf{k} = (a, b, 0)$ , where  $a$  and  $b$  are small positive or negative integers or 0, e.g.  $(1,0,0)$  or  $(0, -2,0)$  or  $(1,2,0)$  They choose their own  $\mathbf{k}$ : but do not choose  $(0,0,0)$ ! (Tell them that it’s easier if they pick  $\mathbf{k}$  purely in  $x$ , or purely in  $y$ , but if they want a more interesting example to choose both components nonzero. This seems to self-select students w.r.t. the speed at which they complete the activity!

- First draw your  $\mathbf{k}$  on your grid.

- Next, start at the origin and go to each black dot, one at a time, and compute  $\mathbf{k} \cdot \mathbf{r}$  at that point, and label the point with the (integer!) value of  $\mathbf{k} \cdot \mathbf{r}$ . So e.g. the origin is 0 for everyone, and the next point to the right, which represents  $\mathbf{r} = (1,0)$  will just be the “ $a$ ” in their  $\mathbf{k}$  vector. Have them label points until they clearly see the “pattern”

- Finally, connect the dots with common values of  $\mathbf{k} \cdot \mathbf{r}$

Now – wrap up with questions to ponder: go back to the “how do you interpret the wave  $\cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$ . At  $t=0$ , look at your grid, what do those lines represent? Picture the physical wave! Why are these called “plane waves”, what’s “planar” here? *(They are “constant phase” lines, or constant values of the wave. They are the PLANES of constant value if you extend into  $z$ -direction )*

- As time goes by, which way do the waves travel?

*(In the  $\mathbf{k}$  direction)*

- If you chose a larger magnitude  $\mathbf{k}$ , what does that do to your picture? (Compare with neighbors who did this and see for yourself!)

*( $\lambda = 2\pi / |\mathbf{k}|$ , so larger  $k$  means the constant phase lines are closer together, smaller wavelength waves)*

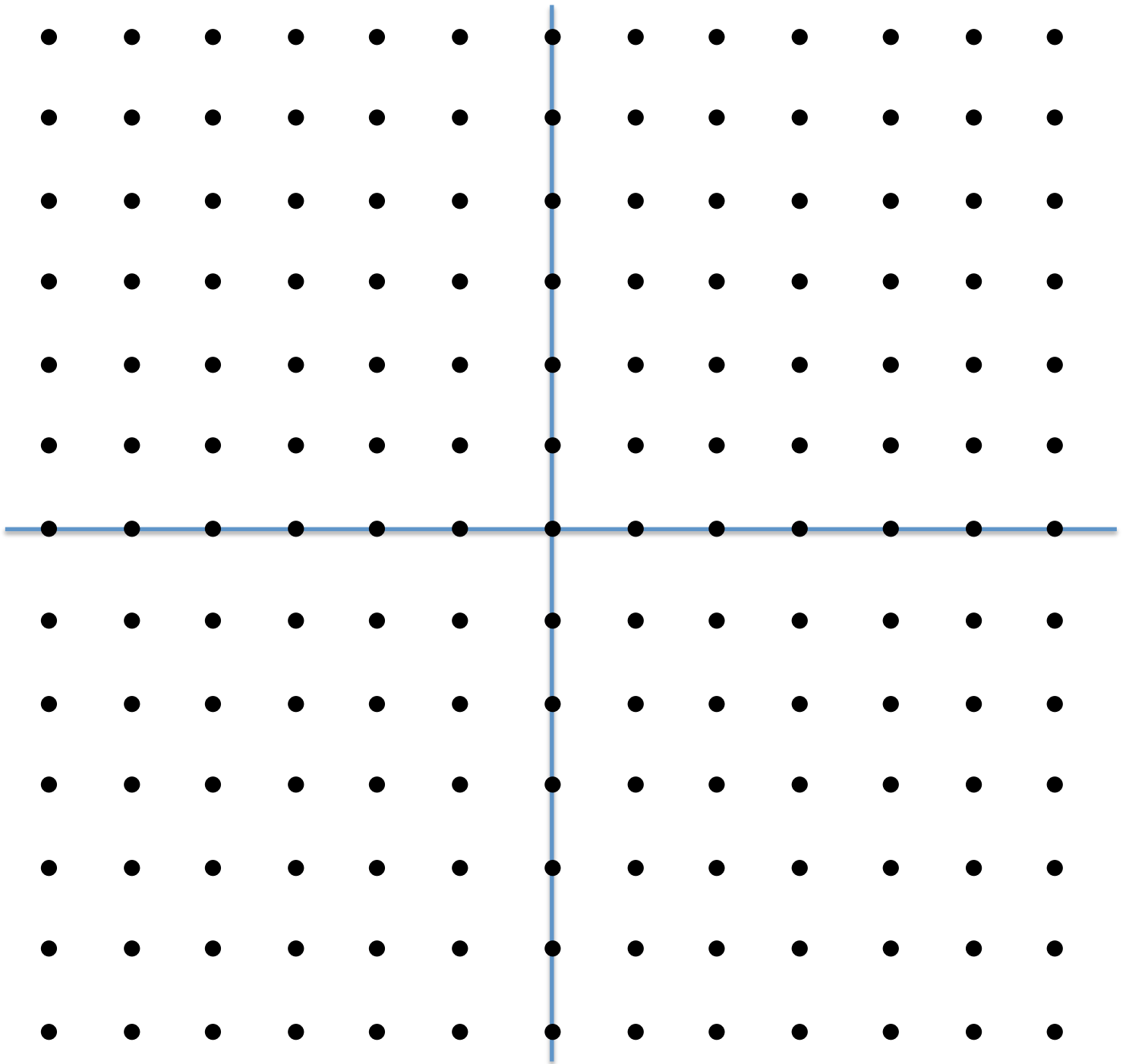
**Written by:** Steven Pollock, but this originates from an Oregon state activity:

[physics.oregonstate.edu/portfolioswiki/doku.php?id=activities:main&file=emplane wave](http://physics.oregonstate.edu/portfolioswiki/doku.php?id=activities:main&file=emplane_wave)

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**Comments:** We took about 5+ minutes for this, it was quick and seemed effective. Some students are just a little confused about the task (labeling dots with value of  $\mathbf{k} \cdot \mathbf{r}$ ), so make sure they are all getting started ok.

k . r exercise



k.r exercise

