15B – Inelastic Collision

Topics: Special relativity, 4-momentum, relativistic collisions.

Summary: Students use conservation of relativistic 4-momentum to find the final mass of an object resulting from the merging of two colliding particles.

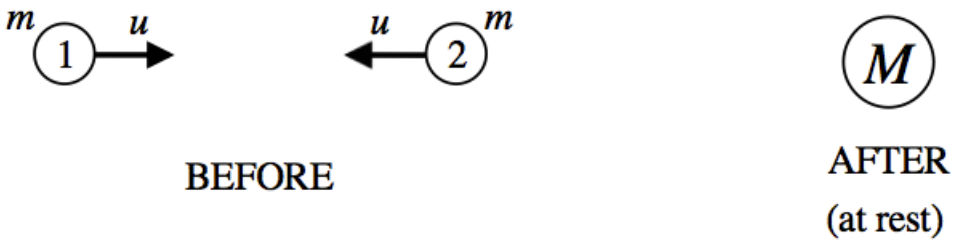
Written by: Michael Dubson, Charles Baily and Steven Pollock.

Contact: Steven.Pollock@Colorado.EDU

Comments: Students should be able to complete these tasks in less than 10 minutes. This activity was straightforward for most students, as long as they were clear on the following: definition of relativistic 4-momentum; the total momentum of a system is the linear sum of the momenta of the particles; and that this quantity is conserved before and after the collision. Some students may momentarily forget the velocity dependence of γ when first working out the total momentum; the spatial velocities of the two particles cancel, but the γ-factor that appears in the total momentum is not also zero.

$$p^{\mu} = \left(\frac{E}{c}, p_x, p_y, p_z \right) = \gamma m \left(c, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right)$$

Consider the following 1-D collision: Two objects of mass m, both with speeds u, collide head-on and merge into a single object of mass M.



(A) Write down an expression for the total 4-momentum of this system *before* the collision.

(B) Write down an expression for the total 4-momentum of this system *after* the collision.

(C) Solve for M in terms of m and u.

(D) Notice that the kinetic energy *after* the collision is less than the kinetic energy *before* the collision. Where did this energy go?

(E) Check that your answer for M makes sense in the non-relativistic limit $u \ll c$.