12A_Modified – Reflection & Transmission (Normal Incidence)

Topics: Reflection and transmission, boundary conditions, complex exponentials.

Summary: Students begin by expressing in exponential notation the boundary condition on the parallel components of an EM plane wave for normal incidence at the interface between two media. They will practice writing the exponential notation for the reflected and transmitted E field, using the correct wave vector. They will also practice how to construct a second boundary equation by considering the electric and magnetic fields of the reflected wave. The remaining tasks connect the amplitude and phase shift of the reflected wave with the refractive indices of the two materials, where students practice reflecting on the results and making sense of the results at certain limits.

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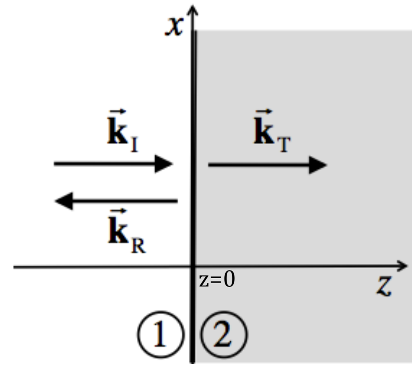
Note: SJP did not try the original 12A in Fa13. There was a concern about the confusing aspect of mixing up "spatial direction" of E with the "complex plane" representation. In this modified version, we delete the complex plane part.

Comments: This version is untested – I changed page 2. The comments below are for the previous version, used in 2014.

In 2014, we tried this new version. It was spread out over two classes, about 8 minutes at the end of one lecture, then the next day (after ~25 minutes of lecture about boundary conditions) they returned to it for another 15 minutes. It's important that students find the correct final equation on page 1, and I found that students were stumped on page 2, so we are rewriting it (see new version) Students had (surprising to me, but the time was short) some difficulties on the last page simply solving 2 equations in 2 unknowns. There is an ambiguity in the notation – we are using the symbol E_R for the the amplitude of the reflected wave, assuming it is positive, but of course if n2>n1, then what we are calling ER will come out **negative** (so it is not truly the amplitude) This is part of what we want them to think about, and it shows up on page two where a minus sign now explicitly arises in the formula from the "flip" of direction of B. In this version, page 2 was a disaster, people didn't know where to start, they were lost. I am rewriting a new version to try to improve on this.



A. An electromagnetic plane wave is traveling through region **1** (a material with index of refraction n_1), and encounters a new material in region **2** (index of refraction n_2), with the boundary located at $z = 0$. The incident wave (I) gives rise to a reflected wave (R) and a transmitted wave (T).

With the incident wave linearly polarized in the <u>*x*-direction</u>, the electric fields of the incident wave can each be represented by the following complex exponentials:

$$\vec{\mathbf{E}}_{\mathrm{I}}(\vec{r},t) = \tilde{E}_{0I} \exp\left[i\left(k_{\mathrm{I}} \bullet r - \omega_{\mathrm{I}} t\right)\right] \hat{x} = \tilde{E}_{0I} \exp\left[i\left(k_{\mathrm{I}} z - \omega_{\mathrm{I}} t\right)\right] \hat{x}$$

Write the complex exponentials for the reflected and transmitted waves. Take the direction of propagation for the reflected and transmitted waves and the magnitude of the wave vectors into consideration and in the final expression, introduce as few new symbols as possible. *Don't try to solve anything yet,* you're merely writing expressions, exactly following the "template" above!

$$\vec{\mathbf{E}}_{\mathrm{R}}(\vec{r},t) = \underline{\hspace{6cm}} = \underline{\hspace{6cm}}$$

$$\vec{\mathbf{E}}_{\mathrm{T}}(\vec{r},t) = \underline{\hspace{6cm}} = \underline{\hspace{6cm}}$$

According to Faraday's Law, the parallel component of the total electric field on either side of the boundary must be the same at all times:

$$\vec{\mathbf{E}}_1^{\parallel}(z = 0,t) = \vec{\mathbf{E}}_2^{\parallel}(z = 0,t) \quad \rightarrow \quad \vec{\mathbf{E}}_{\mathrm{I}}^{\parallel}(z = 0,t) + \vec{\mathbf{E}}_{\mathrm{R}}^{\parallel}(z = 0,t) = \vec{\mathbf{E}}_{\mathrm{T}}^{\parallel}(z = 0,t)$$

Use the information given to re-write this boundary condition using the complex exponential notation from above. (This is a vector relation: which component is physically interesting or useful here?)

Now, write out the "physically interesting" component of this boundary condition at t=0. (Your result should be a very simple relation between the complex numbers $\tilde{E}_{0I}, \tilde{E}_{0R}$ *and* $\tilde{E}_{0T}$)

B. Assume that both materials have a negligible magnetic permeability ($\mu_1 = \mu_2 = \mu_0$)
The following four boundary conditions for the electric and magnetic fields must be true at the interface of materials 1 & 2 :

$$i)\ \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \qquad ii)\ B_1^{\perp} = B_2^{\perp}$$

$$iii)\ \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel} \qquad iv)\ \mathbf{B}_1^{\parallel} = \mathbf{B}_2^{\parallel}$$

(We already used relation iii) in part A)
Explain why *in this simple situation* equations i and ii do not tell us much.

Now use equation iv derive this equation:

$$E_{0I} - E_{0R} = \frac{v_1}{v_2} E_{0T}$$

[Hint: Recall that for EM plane waves $\vec{k} \times \vec{\mathbf{E}} = \omega \vec{\mathbf{B}}$.]
Be sure to discuss with your partners the following:
- Eq. iv is a vector relation: which component is physically useful here?
- WHY does the "magnetic" boundary equation (iv) yield a relation among the *electric* amplitudes?
- WHY is there is a MINUS sign on the second term (do not fudge it! There is a rigorous, unambiguous reason for it)
-WHY does the ratio of speeds appear on the right side like that?

C. For simplicity, let $v_1/v_2 = n_2/n_1 \equiv \beta$ in these two boundary equations, where E_R could be positive or negative, depending on the phase shift.

(Eq. I) $E_{0I} + E_{0R} = E_{0T}$ **(Eq. II)** $E_{0I} - E_{0R} = \beta E_{0T}$

Use equations **I** & **II** to solve for E_{0T} in terms of E_{0I}. Explain in words whether your answer makes sense in the limit $n_2 \to n_1$.

Use this result to solve for E_{0R} in terms of E_{0I}. Explain in words whether your answer makes sense in the limit $n_2 \to n_1$.

For what values of $\beta = n_2/n_1$ will the *reflected* wave be 180^0 out of phase with the incident wave?

If the *reflected* wave is 180^0 out of phase with the incident wave, does this mean that light in medium 2 travels *faster* or *slower* than light in medium 1?