



Electrodynamics HW Problems

04 – Displacement Current, etc...

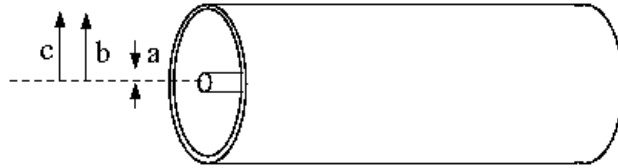
1. Displacement current in a coaxial cable
2. Loop antenna
3. Rod antenna
4. Dimensional analysis
5. Electromagnetic mass

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4.01. Displacement current in a coaxial cable

[Pollock FA11, Kinney SP11, Rogers SP09]

Consider (again!) our standard coax cable as an “infinite” length wire of radius a surrounded by a thin conducting cylinder, coaxial with the wire, with inner radius b and outer radius c .



Again, assume $a \ll b$ and $c - b \ll b$ (thin shell and wire), as show in the figure.

a) We have previously found the self-inductance per length of the coax. Now let's investigate the induced \mathbf{E} field for a particular time dependent current $I(t) = I_0 \cos(\omega t)$, which flows along the wire, and a corresponding current $I(t)$ flows in the opposite direction on the outer cylinder. Assume that we have a quasi-static situation in which the currents are identical in magnitude at each moment in time, and the changes in current are sufficiently slow. Find $\mathbf{E}(s, t)$ in the “coax region” ($a < s < b$) where s is the usual radial coordinate, and the current on the wire is I_0 in the $+z$ direction at $t = 0$. (Assume that the magnitude of $\mathbf{E} \rightarrow 0$ as $s \rightarrow \infty$.)

(b) Find the displacement current density \mathbf{J}_d in the “coax region” ($a < s < b$) for this electric field \mathbf{E} , and integrate it to get the total displacement current I_d . Using physically reasonable numbers for a real coax (say $a = 1$ mm and $b = 1$ cm), determine the frequency ω for which I_d finally equals 1% of I_0 . Briefly, comment. (E.g, What sort of frequency is this, and do we need to worry about the displacement current story for *lower* or *higher* frequencies than this? What does this tell you about our usual “quasi-static” approximation?)

4.02. Loop antenna [Munsat FA10]

Consider a plane wave in free space with electric field given by

$$E(z, t) = E_0 \exp[i(kz - \omega t)] \hat{y} \quad (E_0 \text{ is real})$$

An N -turn wire loop of radius a and resistance R is used to try to detect the changing B -field of the wave, by connecting the ends of the loop to a fast oscilloscope. Assume $a \ll \lambda$ (the loop radius is much smaller than the wavelength).

(a) Which direction should the loop be pointed to maximize the signal on the oscilloscope?

(b) In terms of the quantities given, what will the voltage reading on the oscilloscope be?

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4.03. Rod antenna [Munsat FA10]

A radio station broadcasts at $f = \omega/2\pi = 1190$ kHz with a 7800 watt transmitter. For the purposes of this problem, let's assume that the power is evenly distributed over a hemisphere surrounding the radio tower, and that we set up our receiving antenna 10 km from the tower. (We can also approximate these waves as plane waves).

(a) I create a rudimentary antenna by simply measuring the voltage difference between two points separated by 1 m. What will the voltage reading on the oscilloscope be in this case?

(b) What will the voltage reading be if I use a loop antenna instead? Assume a 10-turn loop with radius of 10 cm.

4.04. Dimensional analysis [Rogers SP09]

(a) Given that m_0 has units of henries/meter and ϵ_0 has units of farads/ meter, use your experience with LC circuits to explain the units of the combination:

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Be sure to reduce your answer to only combinations of the basic units of meters, or kilograms, or seconds of the SI system. What do you suppose is the physical significance of this combination of physical constants?

(b) Using your knowledge of electrical circuits, what are the units of any impedance in an electrical system? Give your answer as a single type of unit that you already know, not as some crazy combination of meters, volts, amps, etc.

(c) With what you have just learned in part (b) above, what are the units of this combination of free space permeability and permittivity:

$$\sqrt{\frac{\mu_0}{\epsilon_0}} \quad ?$$

What do you suppose is the physical significance of this combination of physical constants?

(d) In some region, you find an electric field of strength E and a magnetic field of strength B . What are the units of their ratio E/B , and the units of:

$$\frac{E}{(B/\mu_0)} \quad ?$$

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4.05. Electromagnet mass [Dubson SP12]

According to Dr. Einstein, energy of any kind is equivalent to mass according to the equation $E = mc^2$. So, if you have an empty box and you fill it with energy U of any kind, then the mass of the box will increase by U/c^2 . Electric and magnetic fields contain energy, so they have an effective mass. It was first suggested by J. J. Thomson that the mass of the electron is due to the energy of the field in the space surrounding the electron. (This turns out to be not quite true.)

(a) Assume the electron consists of a uniformly charged spherical shell of radius r_0 , with total charge $-e$. (Assume a thin hollow shell, not a solid sphere.) Compute the total energy in the electric field of the electron by integrating the field energy density over all space. What is the total energy in the field if the electron is a point particle ($r_0 = 0$)?

(b) If we assume that the mass m of the electron is due entirely to the field energy, what is the formula for r_0 ? (Write r_0 in terms of m , e , and other constants) This value of r_0 is called the Compton radius. What is the numerical value of r_0 for an electron? How big is this compared to the size of an atom? Of a nucleus?

(c) A proton is a composite object (meaning it has internal parts) consisting of 3 quarks, plus gluons that hold the quarks together. The radius of a proton is about 0.9 fm (femto-meters). Assume that the proton is uniform sphere of charge (a solid sphere of charge, not a shell of charge). Compute expressions for the field energy both inside the proton sphere and outside the proton sphere. What is the ratio of the field energy inside the proton to the field energy outside the proton? Now put numbers into your equations, and determine what fraction of the proton's mass is due to its field energy.