

0A – Divergence and Stokes' Theorems

Topics: Divergence theorem, Stokes' theorem, Gauss' law, Ampere's law.

Summary: Students are asked to state the divergence theorem and Stokes' theorem, and then work backwards from the integral forms of Gauss' law and Ampere's law to derive their expressions in differential form.

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Comments: Students should be able to complete these tasks within 15 minutes. This is meant to be a short review activity, so the time-estimate is based on students already having a reasonable familiarity with using vector calculus in the context of E&M. Many students will have difficulty recalling the two mathematical theorems from memory, but we encourage them to do this because perpetually copying out of a book does not demonstrate understanding, and we also believe that writing them down should be simple enough if one genuinely understands what they mean. Some had difficulty making a formal definition of the current density in terms of the current enclosed, particularly with respect to the area being perpendicular. Students are typically asked to derive the integral forms from the differential forms, and these tasks have them do it in the other direction. The greatest difficulty for them was in justifying dropping the integration symbols – the correct argument is that the integrals are equal for an arbitrary line or surface. Students may recognize that two integrals being equal doesn't necessarily mean the integrands are equal, yet still make the mistake of implicitly assuming this in their derivations



Review of Divergence and Stoke's Theorems

A. Begin by writing down the divergence theorem. Try to do this without referring to your notes!

The integral form of Gauss' law is: $\oiint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$

Write down the relationship between Q_{encl} and the charge density ρ .

Use the divergence theorem to *derive* the differential form of Gauss' law from the integral form. Be sure to briefly explain each of your steps.



Review of Divergence and Stoke's Theorems

B. Now, write down Stokes' theorem (the "curl theorem"). Again, try to do this without referring to your notes.

The integral form of Ampere's law is: $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{encl}$

Write down the relationship between I_{encl} and the current density $\vec{\mathbf{J}}$.

Use Stokes' theorem to *derive* the differential form of Ampere's law from the integral form. Be sure to briefly explain each of your steps.

