Electricity and Magnetism II

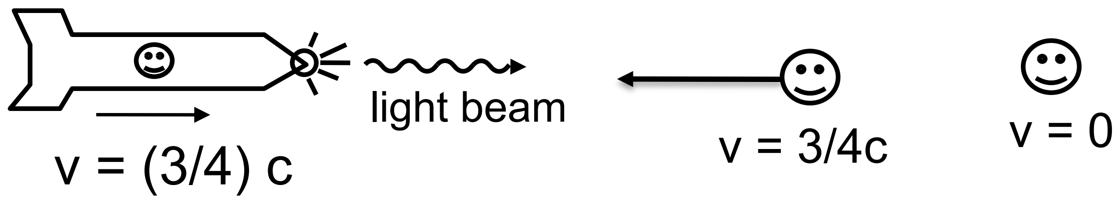
Griffiths Chapter 12 Relativity

Clicker Questions

Two major results of special relativity are Time Dilation and Lorentz Contraction. Please pick one of the choices below which best describes how well you feel you understand them.

A. No idea what these effects are
B. I remember having heard about these, but couldn't define them precisely right now.
C. I know what these effects are, (but I've forgotten how to derive them)
D. I know what these effects are, and I even sort of remember the derivation, but it would take me a while to sort it out
E. I'm confident I could derive these results right now

A rocket is moving to the right at speed v = (3/4)c, relative to Earth.
On the front of the rocket is a headlight which emits a flash of light.



In the reference frame of a passenger on the rocket, the speed of the light flash is

A) c B) 7/4 c C) 1/4 c D) None of these

According to a person at rest on the earth, the speed of the light flash is

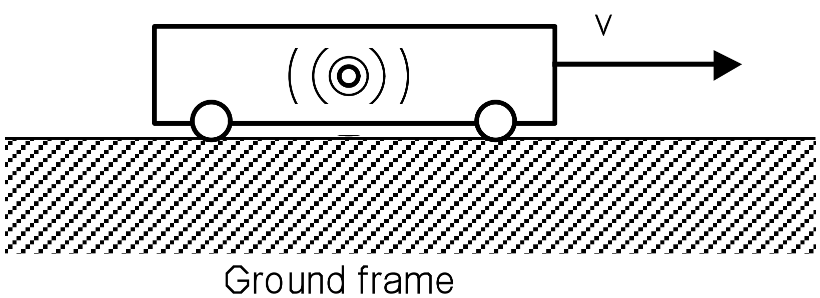
A) c B) 7/4 c C) 1/4 c D) None of these

According to a person moving toward the rocket at speed (3/4)c, relative to earth, the speed of the light flash is

A) c B) 7/4 c C) 1/4 c D) None of these

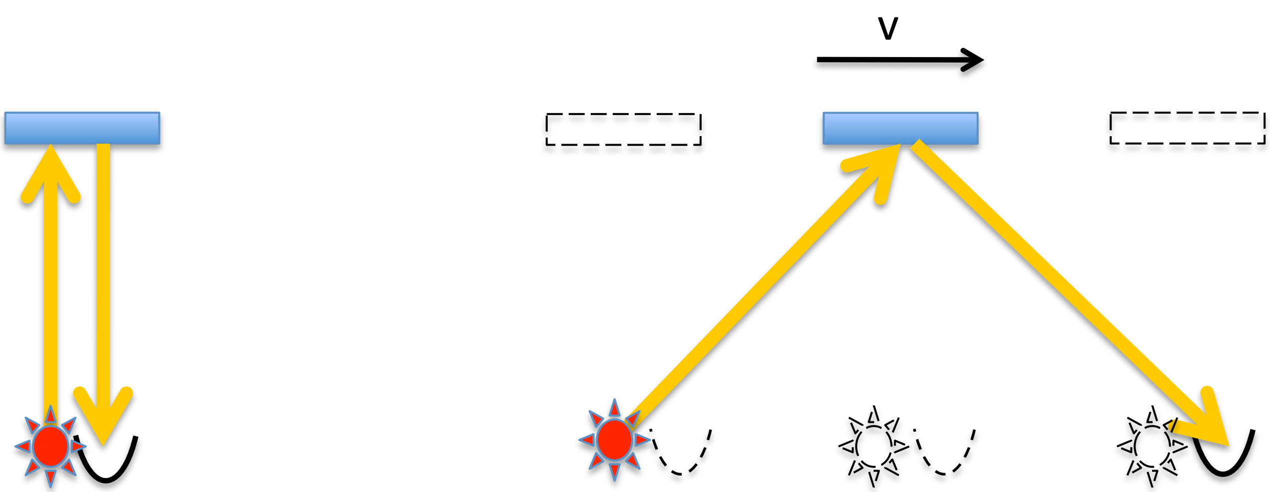
A light bulb flashes in the center of a train car that is moving at speed v with respect to the ground. In the frame of reference of the train car, light wave from the flash strikes the front and back of the train simultaneously.
In the frame of reference of the ground, the light strikes the back of the train ____________ (fill in the blank) the light strikes the front of the train.

A) before B) after C) at the same time as



Events that are simultaneous in one frame are not simultaneous in other frames. There is no such a thing as “now”. The time “now” in the Andromeda galaxy, depends on whether we are in the Earth rest frame or the Andromeda rest frame, or…

Light clock



Rest frame of clock Δt_0

Moving frame Δt

In which frame of reference is the time between tics of the clock **longer**?

A) Rest frame of clock B) moving frame C) no difference

"Moving clocks run slower": $$\Delta t = \gamma \Delta t_0$$

I have a stick of length L sitting in front of me.
In the reference frame of a passing train,(moving parallel to the stick) what is the measured length of the stick?

A) L B) γL C) L / γ

D) I'm sure it's either B or C, but I'm NOT sure which one!
E) I'm pretty sure it's "none of the above" or "it depends"

I flash a lightbulb (event 1).
The light reaches a mirror, and
returns to me (event 2)

I measure the time $\Delta t = t_2 - t_1$ for the complete trip of the light.

A long train was passing (speed v) during this experiment.
In the reference frame of the train, what is the interval $\Delta t'$ between those two events?

(As usual, $\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$)

A) $\Delta t' = \Delta t$ B) $\Delta t' = \gamma \Delta t$ C) $\Delta t' = \Delta t / \gamma$

D) I'm sure it's either B or C, but I'm NOT sure which one!
E) I'm pretty sure it's "none of the above"

Lorentz Transformations

We now have the tools to compare positions and times in different inertial reference frames.

In Galilean relativity

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

In Special relativity

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - \frac{v}{c^2}x) \qquad t = \gamma(t' + \frac{v}{c^2}x')$$

Newton worked with these…

but needs reworking of momentum and energy to work with these!

I have a stick of length L sitting in front of me.

In the reference frame of a passing train,(moving parallel to the stick) what is the measured length of the stick?

A) L B) γL C) L / γ

D) I'm sure it's either B or C, but I'm NOT sure which one!

E) I'm pretty sure it's "none of the above" or "it depends"

Can one change the order of events in time
by measuring them in a different inertial reference frame?

A. Always
B. Sometimes
C. Never

The interval between two particular events is positive:

$$I = (\Delta d)^2 - (c\Delta t)^2 > 0$$

Could these events be causally connected? That is, could one of these events have caused the other?

A) Yes B) No

C) Answer depends on the frame of reference

A clock flies over a town at high speed (constant velocity). In the rest frame of the town, the clock reads 0 when it is over the church steeple and it reads 2 when it is over the Old Watch Tower. So according the townsfolk, the flying clock face advanced 2 units between these two events.

Do observers in all other references frames agree that the flying clock face advanced 2 units between these two events?

A) Yes B) No

Is the time interval Δt between two events Lorentz invariant?

A) Yes B) No

Is the proper time interval $\Delta\tau = \dfrac{\Delta t}{\gamma}$ between two events Lorentz Invariant?

A) Yes B) No

The displacement between two events Δx^m is a 4-vector.

Is $5\Delta x^m$ also a 4-vector? A) Yes B) No

Is $\Delta x^m / \Delta t$ a 4-vector? A) Yes B) No

Is $\Delta x^m / \Delta \tau$ a 4-vector? A) Yes B) No

I'm in frame S, and Charlie is in Frame S'
(which moves with speed V in the +x direction.)

An object moves in the S' frame in the +y direction
with speed v'_y.

Do I measure its y component of velocity to be $v_y = v_y'$?

A) Yes
B) No
C) ???

Displacement is a defined quantity:

$$\Delta x^{\mu} \equiv \left(x_A^{\mu} - x_B^{\mu} \right)$$

Is displacement a 4-vector?

A) Yes
B) No
C) Ummm… don't know how to tell
D) None of these.

Be ready to explain your answer.

Which of the following equations is the correct way to write out the Lorentz product?

A) $a \bullet b = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3$

B) $a \bullet b = +a_0 b^0 + a_1 b^1 + a_2 b^2 + a_3 b^3$

C) $a \bullet b = a_\mu b^\mu$

D) More than one (but not all three)

E) All three are equally correct

Velocity is a defined quantity: $$\vec{u} \equiv \frac{\Delta\vec{r}}{\Delta t} = \begin{pmatrix} \frac{\Delta x}{\Delta t} & \frac{\Delta y}{\Delta t} & \frac{\Delta z}{\Delta t} \end{pmatrix}$$

In another inertial frame, seen to be moving to the right, parallel to x, observers see:

$$\vec{u}' \equiv \frac{\Delta\vec{r}'}{\Delta t'} = \begin{pmatrix} \frac{\Delta x'}{\Delta t'} & \frac{\Delta y'}{\Delta t'} & \frac{\Delta z'}{\Delta t'} \end{pmatrix}$$

Is velocity a 4-vector?

A) Yes
B) No
C) Sometimes yes, sometimes no
D) None of these.

4-velocity?

Imagine this quantity: $$u^\mu \equiv \begin{pmatrix} c \\ \dfrac{\Delta x}{\Delta t} \\ \dfrac{\Delta y}{\Delta t} \\ \dfrac{\Delta z}{\Delta t} \end{pmatrix}$$

Is this quantity a 4-vector?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

4-velocity?

Imagine this quantity: $$u^{\cancel{\mu}} \equiv \begin{pmatrix} c \\ \dfrac{\Delta x}{\Delta t} \\ \dfrac{\Delta y}{\Delta t} \\ \dfrac{\Delta z}{\Delta t} \end{pmatrix}$$

Is this quantity a 4-vector?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

This object does not Lorentz Transform. NOT a a 4-vector.

4-velocity?

Imagine this quantity: $$\eta^{\mu} \equiv \frac{1}{\Delta\tau}\begin{pmatrix} c\Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

Proper time

Is this quantity a 4-vector?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

Can we define?

$$c\Delta\tau \equiv \sqrt{-(\Delta x)_{\mu}(\Delta x)^{\mu}}$$

Proper time

$$= \sqrt{c^2(\Delta t)^2 - \left[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2\right]}$$

Is this a reasonable definition?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

Displacement is a defined quantity: $\Delta x^{\mu} \equiv \left(x_A^{\mu} - x_B^{\mu}\right)$

Interval is a defined quantity: $I \equiv \Delta x_{\mu} \Delta x^{\mu}$

Written in terms of the spatial distance, d, between events and the times separation between events, t, the interval is:

A) Working on it.
B) Think I've got it.
C) None of the above.

In my frame (S) I measure two events which occur at the same place, but different times t_1 and t_2
(they are NOT simultaneous)

Might Charlie (in frame S') measure those SAME two events to occur simultaneously in his frame?

A) Possibly, if he's in the right frame!
B) Not a chance
C) Definitely need more info!
D) ???

The 4-velocity η^μ is defined as

$$\eta^\mu = \frac{1}{\sqrt{1-u^2/c^2}} \begin{pmatrix} c \\ \mathbf{u} \end{pmatrix}$$

What is the invariant length squared of the 4-velocity, $\eta_\mu \eta^\mu$?

A. c^2
B. $-c^2$
C. $-c^2 + u^2$
D. $c^2 - u^2$
E. None of the above

Two events have a timelike separation. In a “1+1”-dimensional spacetime (Minkowski) diagram
(x horizontal, ct vertical),
the magnitude of the slope of a line connecting the two events is

A. Greater than 1
B. Equal to 1
C. Less than 1

For isolated systems, the total 4-momentum is CONSERVED (this is an experimental fact).

Is 4-momentum invariant ?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

The rest mass m of an object is the mass measured in the rest frame of the object.

Is $p^\mu = m\eta^\mu$ a 4-vector ?

A) Yes B) No C) Sometimes

$$p^\mu = [\gamma mc,\ \ \gamma m\vec{u}]$$

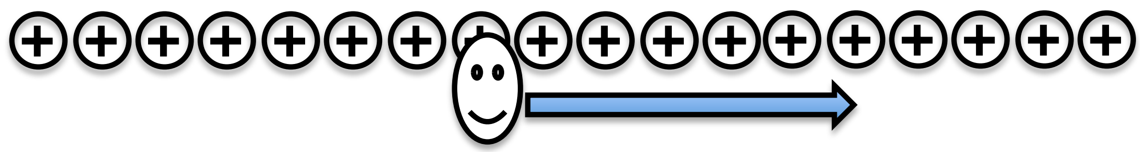
$$p_\mu p^\mu = -\left(\gamma mc\right)^2 + \left(\gamma mu\right)^2$$

What is the invariant length-squared of the 4-momentum, $p_\mu p^\mu$?

A. $+(mc)^2$
B. $-(mc)^2$
C. $-(mc)^2 + (mu)^2$
D. $+(mc)^2 - (mu)^2$
E. None of the above

A row of positive charges is stationary on the ground. A person with a gauss-meter is running to the right along the row of charges, at the same height as the charges and in front of them. What is the direction of the B-field which the observer measures?

A) Right B) left C) up D) down E) B = 0



Are energy and rest mass Lorentz invariants?

A. Both energy and mass are invariants
B. Only energy is an invariant
C. Only rest mass is an invariant
D. Neither energy or mass are invariants

Are energy and rest mass conserved quantities?

A. Both energy and mass are conserved
B. Only energy is conserved
C. Only rest mass is conserved
D. Neither energy or mass are conserved

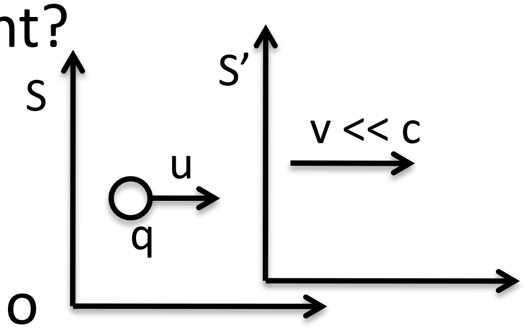
Components of magnetic and electric fields parallel to the relative velocity of two reference frames are transformed between frames by being

A. Multiplied by γ
B. Multiplied by β
C. Multiplied by $\gamma\beta$
D. Multiplied by 1
E. None of the above

A charge q is moving with velocity **u** in a uniform magnetic field **B**. $\vec{F} = q\vec{u}\times\vec{B} = m\vec{a}$

If we switch to a different Galilean frame (a low speed Lorentz transform), is the acceleration **a** different?

A) yes B) No



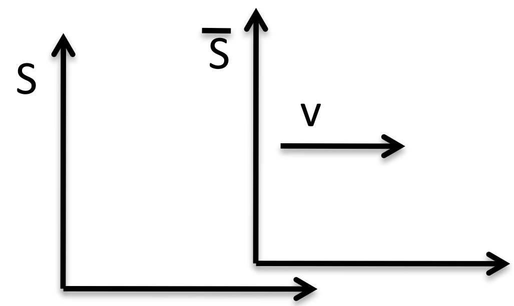
Is the particle velocity **u** different? A) yes B) No

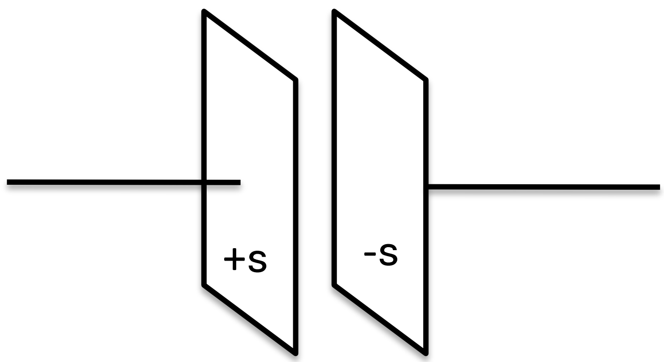
Is the B-field different? A) yes B) No

Suppose we switch to frame with **v** = **u**, so that in the primed frame, **u**' = 0 (the particle is instantaneously at rest). Does the particle feel a force from an E-field in this frame?

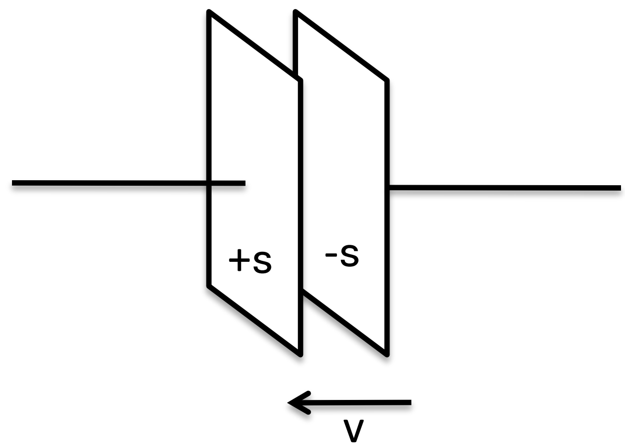
A) Yes B) No C) depends on details

Switch from frame S to frame S-bar:





Frame S (at rest)



Frame $\overline{S}$

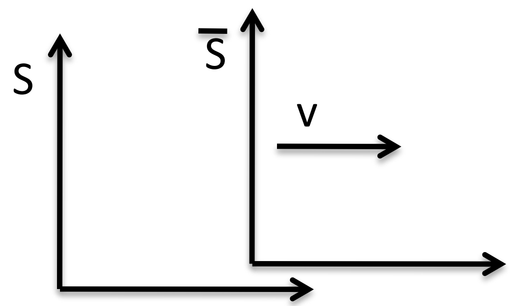
How does E_x compare to $\overline{E}_x$?

A) $\overline{E}_x = E_x$ B) $\overline{E}_x > E_x$ C) $\overline{E}_x < E_x$

Switch from frame S to S-bar. Things change:

$$t \rightarrow \bar{t} \quad x \rightarrow \bar{x} \quad y \rightarrow \bar{y} \quad z \rightarrow \bar{z}$$

$$E_x \rightarrow \bar{E}_{\bar{x}} \quad B_x \rightarrow \bar{B}_{\bar{x}} \quad E_y \rightarrow \bar{E}_{\bar{y}} \quad \text{etc}$$



Do Maxwell’s Equations look the same in S-bar?

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \overset{?????}{\longrightarrow} \quad \bar{\nabla} \cdot \vec{\bar{E}} = \frac{\partial \bar{E}_{\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{E}_{\bar{y}}}{\partial \bar{y}} + \frac{\partial \bar{E}_{\bar{z}}}{\partial \bar{z}} = \frac{\bar{\rho}}{\varepsilon_0}$$

A) Yes   B) No

Can we define a 4-force via the 4-momentum?

$$\frac{dp^\mu}{d\tau} = K^\mu$$

Proper time

Is K, so defined, a 4-vector?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

Minkowski 4-force

To match the behavior of non-relativistic classical mechanics, we might tentatively assign which of the following values to **K**:

A) $K^{1,2,3} = \vec{F}$

B) $K^{1,2,3} = \vec{F}/\gamma$

C) $K^{1,2,3} = \gamma\vec{F}$

D) Something else

Consider the equation

$$\frac{\partial G^{\mu\nu}}{\partial x^{\nu}} = 0$$

How many ordinary equations is that, really?

A) 1
B) 4
C) 6
D) 16
E) ????