15-2 – Relativistic Velocity Transformation

Topics: Special relativity, Lorentz transformations, relativistic addition of velocities.

Summary: Students derive the velocity addition formula using the Lorentz transformations and the definition for the velocity in two different inertial frames.

Written by: Michael Dubson, Charles Baily and Steven Pollock.

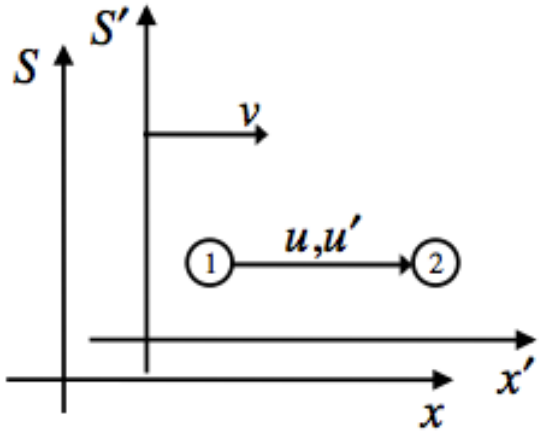
Contact: Steven.Pollock@Colorado.EDU

Comments: Students should be able to complete these tasks in 10 minutes. The tasks are fairly straightforward, and the biggest difficulty for students may be the algebra and simplification choices. A common problem is for students to be confused about the velocity of the frame *v*, and the velocity of the particle *u* in that frame of reference. We have also noticed some conceptual difficulty for students regarding an event taking place at a single point in spacetime, and the different coordinate representations of that point in different inertial frames. The Lorentz transformations are written in a form that makes the symmetry between the two equations more obvious. The final result is in the "velocity subtraction" form, which is slightly more confusing to interpret the final result.

15-Velocity addition

$$\Delta x = \gamma\left(\Delta x' + \beta c \Delta t'\right)$$

$$c\Delta t = \gamma\left(c\Delta t' + \beta \Delta x'\right)$$



Frame S moves with a constant velocity v relative to frame S'. An object moves from Event 1 to Event 2 with constant speed along the x/x' direction.

In frame S, its velocity is $u = \dfrac{\Delta x}{\Delta t}$.

In frame S', its velocity is $u' = \dfrac{\Delta x'}{\Delta t'}$.

Rewrite u in terms of the primed variables $\Delta x'$ and $\Delta t'$ using the Lorentz transformations at the top of the page.

Simplify this result to find a relationship between u and u'.
(Notice that this is not a simple Lorentz transformation. Why not?)

Challenge questions: What is u if v<c, but u'=c?