Electrodynamics Tutorials/In-Class Activities
Instructor's Guide

The guiding principle in creating these activities was that students would gain more from being active (instead of passive) participants in the classroom. The tasks are generally focused on either promoting an understanding of important topics from second-semester E&M, or guiding students through derivations that would typically be done during lecture. They were often inspired by in-class observations of student difficulties, and have been tested in focus-group student interviews (designed to mimic a tutorial setting) and in the classroom. We provide here information about how they were implemented, and a summary list of the activities and the topics they cover, including an estimate of the amount of class time they require. Please feel free to share with us any changes you make, or your observations regarding their use. We are currently working at refining these materials, so please contact us if you want to be sure to have the latest versions!

We ask for your cooperation in not making solutions to these tutorials/activities available on the open web under any circumstances, out of respect for instructors at other institutions, and for maintaining the integrity of our research. Other reasons for this are addressed in the implementation notes (Section II, below).

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I. Creative Commons License

II. Notes on Implementation

All of the tutorials/activities contained in this package were developed for use **during** class, to augment or replace standard lectures on the topics they address. This style of implementation is in contrast to the "usual" tutorial setting, being separate from the lecture portion of the class, but they could certainly be adapted for use in such environments, and we encourage instructors to do so. Depending on the topic, they require anywhere from 5 to 50 minutes of class time, and versions of them were recently used intermittently during a course having 50-minute class periods (three times per week over a 15-week semester). ~40% of the CU SP12 lectures were partly or fully replaced with tutorial activities. We typically oriented students to the activities before they began with a concept test and/or discussion. Otherwise, they were implemented at appropriate times during lecture, most often during the middle, sometimes at the very end of class. Incorporating these student-centered tasks into the classroom was sometimes challenging, and we describe below some lessons we have learned about getting the most out of the time spent.

(1) Sell students on group work. Students will have their own ideas about what a junior-level physics classroom should be like, and some may at first be reluctant to engage in activities that differ from the standard lecture format (even when they are familiar with them from introductory courses, and particularly if they associate them only with "lower-level" work). Aside from the belief that they will gain more through active participation, instructors may also remind students that scientific argumentation (oral or written) is a skill is developed with practice, and that scientists work almost exclusively in group settings. Stronger students benefit from working with weaker students (and not just the other way around) since, as we should know from our own teaching experiences, they will never understand something so well as when they can explain it to someone else!

(2) Hand out just before activity begins. We've found that handing out the printed activities at the beginning of class (or a significant amount of time before starting them) is not ideal. There will inevitably be some students who immediately start reading through the pages or working the problems, and mostly tune out the instructor from that point on, so instructors should be aware they might not have the undivided attention of the class once the activity sheets are in front of students. This can also discourage students from collaborating with others at their table, since they'll be ahead of everyone else and may be reluctant to go back.

(3) Keep it closed note. We have tried to provide students with sufficient information to complete these activities without having to refer to their notes. Some of the tasks *do* require them to recall facts from memory, but this is only in cases where we feel they should have this knowledge at their fingertips, and instructors can certainly write out necessary equations on the board if they wish. If there are instances where students feel they *must* refer to outside sources, this should be an indication to them that they may need to devote a little more time to studying that particular topic. We actively discouraged them from copying equations or following examples from the textbook, since this does not involve the kind of understanding we are trying to promote.

(4) Introduce the activities. Students may require some kind of orientation to the topic at hand, or need an important piece of information to get started; they may also need you to be explicit in connecting the tasks as a whole to your overall learning goals. We have tried to be as clear as possible in the problem statements, and their wording has (in most cases) already been tested with students, but what seems "obvious" to instructors may not be so for students. We have also noticed that, even at the junior-level, some students don't always read each problem statement completely, often only skimming the words and trying to glean as much information as possible from the diagrams.

(5) Activities may take longer than anticipated. All of the activities (except where explicitly noted) have been validated through student interviews (in their preliminary forms) and field-tested in a classroom setting. The summary that accompanies each tutorial has an estimate of the amount of time it should take for *most* students to complete the entire activity, but an actual implementation may take more (or less) time than anticipated. We notice there is a tendency for instructors to *underestimate* the amount of time it will take students to complete these activities; a general rule is that students usually take around 10 minutes per page.

(6) Use challenge problems, or create new ones. When students are working at their own pace, there will always be some who are much quicker than others to complete the tasks. To keep these students using their time productively, many of the activities have one or two *challenge questions* at the end, which usually involve taking their conclusions one or two steps further. If there isn't a challenge question, instructors should be prepared with a question or task for students that builds in some way off of the tasks they've just completed

(7) Don't provide written solutions. There have been studies that suggest students will learn and retain more when they are not given *written* solutions to tutorials, though it is *essential* that instructors ensure that students are arriving at correct answers as they progress through the tasks.[1] Some students may feel frustrated by this policy, but we suspect that referring to an answer key while studying may short circuit an important aspect of the learning process, namely arriving at a correct answer through their own reasoning, and being able to justify the correctness of that answer. For our class, activities were posted on a secure site for students who were unable to attend, and we encouraged them to speak with us (and each other) outside of class about any questions they might have. Most importantly, they should ask questions *during* class time when they recognize that they're confused. **We do ask that instructors not post solutions to these activities on the open web under any circumstances, out of respect for instructors at other institutions, and for maintaining the integrity of our research.**

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[1] See Slezak, et al. *PRST-PER* **7**, 020116 (2011); Koenig, et al. *PRST-PER* **3**, 010104 (2007).

III. Summary List of Activities

The ordering of topics for these activities follows the presentation in Griffiths (except for *AC* circuits, which is sparsely covered in his book). The activities are *usually* sufficiently self-contained that they can be used independent of each other, but they sometimes come in two parts, or use language we expect students to be familiar with from earlier tasks. We have tried to make note of this in the summaries when applicable.

The general topics covered in each of the tutorials/activities are listed below, along with the estimated time it will take for *most* students to complete them, followed by a brief summary of the tasks involved, and abbreviated comments on their implementation (including some common student difficulties). The pdf files in this package contain the complete notes at the beginning of each activity.

00 – Review Material	
A – Divergence and Stokes' Theorems	(~15 min)
B – Gauss' Law	(~15 min)
C – Ampere's Law	(~15 min)
01 – Current Density & Charge Conservation	(~15 min)
02 – Ohm's Law	(< 50 min)
03 – Faraday's Law	(< 30 min)
04 – Complex Exponentials	(< 50 min)
05 – Complex Impedance	(< 50 min)
6A – Maxwell-Ampere Law, Part 1	(~25 min)
6B – Maxwell-Ampere Law, Part 2	(~25 min)
07 – Boundary Conditions	(< 50 min)
08 – Energy Flow in a Simple Circuit	(< 40 min)
09 – Linear Operators	(< 15 min)
10 – EM Wave Equation	(~10 min)
11 – Complex Plane Waves	(< 40 min)
12 – Reflection & Transmission	
A – Normal Incidence	(< 50 min)
B – Oblique Incidence	(~30 min)
13 – Gauge Invariance	(< 50 min)
14 – Time-Retarded Potentials	(~30 min)
15 – Special Relativity	
A – Length Contraction	(< 15 min)
B – Inelastic Collision	(~10 min)
C – Velocity Transformation	(~10 min)

0 – Review Material

These are meant to be short review activities, so the time-estimates are based on students already having a reasonable familiarity with these topics from the first semester of the course.

0A – Divergence and Stokes' Theorems (~15 minutes)

Topics: Divergence theorem, Stokes' theorem, Gauss' law, Ampere's law.

Summary: Students are asked to state the divergence theorem and Stokes' theorem, and then work backwards from the integral forms of Gauss' law and Ampere's law to derive these expressions in differential form.

Comments: Many students will have difficulty recalling these two mathematical theorems from memory, but we encourage them to do this because perpetually copying out of a book does not demonstrate understanding, and we also believe that writing them down should be straightforward if they genuinely understand what they mean. Students are typically asked to derive the integral forms from the differential forms, and these tasks have them do it in the other direction. The greatest difficulty for them was in justifying dropping the integration symbols in the final step of their derivations; students may recognize that two integrals being equal doesn't necessarily mean the integrands are equal, yet still make the mistake of implicitly assuming this.

0B – Gauss' Law (~15 minutes)

Topics: Gauss' law, symmetries, electric field from a line charge distribution.

Summary: Students consider the symmetry of a line charge distribution to argue for why the electric field is entirely in the radial direction, and why a Gaussian cylinder is needed to solve for the electric field (instead of a sphere or a cube). Students are then asked to recall Gauss' law in integral form, find the charge contained in a section of wire, and solve for the electric field.

Comments: Many students had difficulty articulating their reasoning on the symmetry questions, and were more inclined to argue in terms of the curl (or closed line integral) of an electrostatic field being zero. A significant number of students will believe that the electric field can be solved for using Gauss' law and a non-symmetric surface, but that we don't use such surfaces because the integral would be too difficult to calculate. All of this indicates that students may have the rote application of Gauss' law down, without necessarily having a strong grasp of the important role of symmetry when calculating fields.

0C – Ampere's Law (~15 minutes)

Topics: Ampere's law, symmetries, magnetic field of a long wire.

Summary: Students first argue for why the magnetic field is entirely in the tangential direction for a straight current-carrying wire. They are then asked to recall Ampere's law in integral form, and solve for the magnetic field around the wire.

Comments: The previous activity on Gauss' law was more explicit about making symmetry arguments, and many students may still do this after having completed that prior activity. Others were more comfortable thinking in terms of there being no magnetic charges, and the curl (or closed line-integral) of the B-field being zero where there is no current (enclosed). Instructors should be aware that understanding the symmetry arguments in applying Gauss' law doesn't necessarily translate to the context of Ampere's law. A *challenge question* at the end asks if Ampere's law can be used to find the B-field at the center of a circular loop of current, which inspired a great deal of good discussion/questions.

01 – Current Density & Charge Conservation (~15 minutes)

Topics: Current density, conservation of charge (continuity equation).

Summary: Students first consider a cylindrically symmetric conductor having three regions of different cross-sectional area. The task here is to rank order the three regions in terms of several physical quantities in those regions: conductivity, total current, current density and electric field. The remaining tasks connect the flux of the current density through a closed surface to the rate of change of the charge enclosed within the volume.

Comments: These tasks were overall relatively straightforward for students. A common difficulty that arose had to do with whether the outward flow of current corresponding to positive flux, and if $-\partial\rho/\partial t$ is a positive quantity. A challenge question at the end has them convert the integral form of the continuity equation to its differential form.

02 – Ohm's Law (< 50 minutes)

Topics: Ohm's law, continuity equation, boundary conditions on the electric field inside a conductor

Summary: A steady current flowing through a cone-shaped resistor is used as the context for addressing the implications of the microscopic version of Ohm's Law $\mathbf{J} = \sigma\mathbf{E}$. The initial multiple-choice question orients students to the situation by having them consider the current density inside the resistive material. They are then led to make conclusions about the electric field and local charge density inside the resistor by using Ohm's law in conjunction with the continuity equation and

Gauss' law. Students are presented with two possible configurations for the electric field inside the conductor, and are asked to identify which aspects of those configurations are allowed, and which are precluded by boundary conditions or conservation of charge/current. The final activity asks them to interpret a graph of the correct field and equipotential lines inside the resistor in terms of the concepts discussed in the previous sections.

Comments: Instructors should be sure that students reconcile their mathematical conclusions ($\nabla \cdot \mathbf{E} = 0$ inside the resistor) and the fact that the field lines are spreading outwards (which may *look* to them like a "diverging" field). It is not essential that the field lines drawn by students on the second page are completely correct before moving on – we just want them to develop some kind of expectation for what they ought to look like.

03 – Faraday's Law (< 30 minutes)

Topics: Faraday's Law, fields of a solenoid with time-varying current.

Summary: Students first sketch the B-field for a long solenoid, and then consider whether there is a non-zero electric field anywhere in space when the current in the solenoid is changing with time. They then use Faraday's law in integral form to compute the electric field inside and outside the solenoid, and sketch the induced field as a function of distance from the center.

Comments: This is a shortened version of a tutorial on *EMF* from a series created by the University of Colorado for the first semester of this course. The biggest conceptual difficulty for students has been with the idea that there is a non-zero electric field in a region of space where the magnetic field is zero (outside the solenoid). This can lead to good discussions on the difference between the differential and integral forms of Faraday's law. There have been a few students who were concentrating only on the electric field driving the current in the coil, and weren't thinking there could be an electric field anywhere but inside the wire. This can lead to interesting discussions about the relative magnitudes of the induced electric field and the field driving the current, and how this could depend on the dimensions of the solenoid or the rate of change in the current.

04 – Complex Exponentials (< 50 minutes)

Topics: Complex exponentials as oscillatory functions, representations of complex numbers, simple AC circuit with resistor.

Summary: The first tasks are meant to help students gain some familiarity with complex exponentials as oscillatory solutions to differential equations. They first consider similarities and differences between exponential and trigonometric functions as solutions to a first-order equation, then similarly for the behavior of their second-derivatives. Students then perform a few basic tasks involving different representations of numbers in the complex plane, and draw conclusions about the direction of rotation over time for an arrow representing a complex exponential function. The final task applies to a simple *AC* circuit, where students must find the magnitude of the current through the resistor at a specific time.

Comments: The first task may seem "simple", but we were surprised by the amount of time that some students took to find the first-derivatives of the functions given; a sign error here and there was common. Students didn't necessarily have problems completing the exercises on complex numbers, but seem to require more practice in order to be comfortable with them; some will be rusty on the rules for multiplying exponentials functions. Questions about a vector rotating in the complex plane are given in anticipation of their use in future tutorials (#5-Complex Impedance, #7-Boundary Conditions, and #12-Reflection and Transmission). We've found this to be a very powerful visualization tool for students when working with oscillatory functions. Some students have shown a tendency to confuse their use of complex exponentials in quantum mechanics (multiplying by the complex conjugate to find a physical quantity) when finding the physical voltage or current represented by a complex exponential (instead of looking at just one component).

05 – Complex Impedance (< 50 minutes)

Topics: Complex impedance, phasor diagrams, RLC circuits, *leading* vs. *lagging*

Summary: These activities are meant to help students gain facility with different representations of complex functions, and with relating them to the behavior of an *RLC* circuit. They begin by plotting voltage (and current) relative to a given current (or voltage), using a specific value for the complex impedance. They can compare their answers with trigonometric representations, and resolve difficulties in deciding whether one function *leads* or *lags* the other in time. Students then determine the total impedance in an *RLC* circuit in terms of the impedance for each circuit element, and plot various vectors in a phasor diagram to see how they are related.

Comments: Students are assumed to have either completed the previous activities on complex exponentials, or had some kind of introduction to writing complex numbers in various forms, and the multiplication of complex exponentials (frequent errors come from not being familiar enough with the rules of exponents). There has been a great deal of confusion among students concerning whether a voltage is leading or lagging the current, depending on which representation being used. It seems to be fairly intuitive for them when looking at the phasor diagrams (as long as they're clear on the direction in which the vectors are rotating with time); but the trigonometric representations can be challenging because, in the graph of a function that is *leading* in time, it peaks at a point that is to the physical left of the peak for the function it leads, and therefore "looks" like it's actually lagging.

6A – Maxwell-Ampere Law, Part 1 (~25 minutes)

Topics: Maxwell-Ampere law, conservation of charge, E- and B-fields for a charging capacitor.

Summary: After first converting the Maxwell-Ampere equation from differential to integral form, students draw conclusions about $\partial\rho/\partial t$ and $\nabla \cdot \mathbf{J}$ for a circuit with a charging capacitor, and compare them with what's predicted by the static form of Ampere's law. They are then asked to compare these incorrect predictions with those for the full Maxwell-Ampere equation, and consider how this is related to the continuity of field lines for a divergenceless field (the vector field $\nabla \times \mathbf{B}$).

Comments: The other activity on this topic (Maxwell-Ampere Part 2, #6B) can also be done in approximately 25 minutes, so the two parts could potentially be used in the same class period, or just split between two classes. When deriving the Maxwell-Ampere law in integral form, 40% of our students incorrectly substituted $Q_{enclosed} / \varepsilon_0$ for the open-surface flux integral of $\mathbf{E}$ (an incorrect application of Gauss' law, where the flux integral is over a closed surface). Many students were confused about the sign of the net flux of the current density in a region where a capacitor plate is charging – usually because they were not considering the different directions the area vector points in around the Gaussian surface; many were incorrectly thinking that a net charge flowing into the volume would correspond to positive flux. About 1/4 of our students were confused by the questions regarding charge conservation, thinking they were instead asking about whether there was an equal but opposite amount of charge on the two capacitor plates. In a handful of cases, students initially believed that charge was actually flowing through the space between the capacitor plates, so that the charge flow was continuous through the circuit. Students may need to be reminded that the divergence of the curl of a vector field is always zero.

6B – Maxwell-Ampere Law, Part 2 (~25 minutes)

Topics: Maxwell-Ampere law, Gauss' law, E- and B-fields for a charging capacitor, B-field of a current-carrying wire.

Summary: After converting the Maxwell-Ampere equation from differential to integral form, students find the magnetic field outside a current-carrying wire. They then consider the electric field between the capacitor plates in terms of the current in the wires and the charge density on the plates, and derive a formula for the magnetic field between the plates induced by the changing electric field. They can then compare the magnitude of the field outside the wire with the field between the plates, specifically for the case where the radius of the Amperian loop is such that it encloses all of the electric flux (they are then the same).

Comments: The task of converting Maxwell-Ampere from differential form to integral form is repeated because students have shown they have real difficulty in doing this without a textbook in front of them. Instructors may not want to skip this if Parts 1 & 2 are used in the same day. Many students showed continuing difficulty with choosing the correct surfaces and loops for applying the integral equations – specifically with seeing how the two types of integrals are related to each other by the same surface. An additional task for students could be to explain the final result in terms of the continuity of field lines for the divergenceless field $\nabla \times \mathbf{B}$, which was also addressed at the end of the previous tutorial.

07 – Boundary Conditions (< 50 minutes)

Topics: Boundary conditions, Maxwell's equations in integral form.

Summary: These activities guide students through a derivation of the boundary conditions on the electric and magnetic fields at the interface between vacuum and a general material. Initial tasks have them consider the charge/current/flux enclosed by imaginary surfaces. They are then guided to apply Maxwell's equations to solve for the conditions on the fields at either side of the boundary.

Comments: Just prior to implementation, we gave our class a brief review of sign conventions regarding unit vectors and integration surfaces/loops. During the tasks, many students were still introducing minus signs into the equations from memory (or intuition), without being able to justify them in terms of the direction of the unit vectors. Some students are confused by the distinction between a *surface* current and the *volume* current "right at the very edge" of a material, and this is addressed by having them consider the charge/flux enclosed by surfaces with dimensions that shrink to zero, in this case just across the surface. Some students got very caught up on whether the charge/current/flux enclosed is actually zero, or just vanishingly small – this can be an opportunity to discuss comparisons between finite quantities and ones that are differentially small. We have purposefully avoided reference to the auxiliary fields $\mathbf{D}$ and $\mathbf{H}$, and there is no distinguishing here between free and bound charges/currents, because of the added complexity this would involve.

08 – Energy Flow in a Simple Circuit (< 40 minutes)

Topics: Poynting vector, boundary conditions, surface charges, Ohm's law

Summary: These activities focus on the location and direction of energy flow for a circuit containing just a battery and a resistor; the initial tasks consider only a resistive element with a current flowing through it. Students should first conclude that energy is flowing radially into the resistor (and not along the direction of current), and that Faraday's Law requires the electric field to be nonzero outside the resistor. With no volume charge density inside the resistor, the next conclusion is that surface charges are responsible for the perpendicular components of the electric field, which must vary along the length of the resistor for the field to be conservative. The final conclusion is that energy flows from battery to resistor through the fields outside the conducting wires, and that energy can (counter-intuitively) flow opposite the direction of current.

Comments: The part concerning the parallel components of the electric field outside the resistor may be more challenging for students who did not complete the tutorial on boundary conditions. There were still a few students who believed the volume charge density inside the resistor is non-zero (even though the current is steady), so this activity gives another opportunity to address this (see #2-Ohm's Law). The final conclusion about the location and direction of energy flow was surprising to most students, and instructors should be sure that students don't automatically assume the direction of energy flow is the same through the entire circuit (from positive to negative terminal, instead of outwards from both). Many students strongly associate the Poynting vector *only* with electromagnetic waves, so this activity provides another context for them.

09 – Linear Operators (< 15 minutes)

Topics: Linear differential operators/equations

Summary: Linear operators are defined, and students must determine which of five operators are linear. The second part addresses how the components of a complex solution are themselves solutions to a linear differential equation.

Comments: Students should be sure to check their answers to the first part, since many will mistakenly believe that option *IV* is linear if they don't think too hard about it. The final task is designed to address potential confusion about how to translate between complex exponential representations and physical solutions.

10 – EM Wave Equation (~5-10 minutes)

Topics: Wave equation, Maxwell's equations.

Summary: This is a mostly mathematical exercise, to have students derive the wave equation for the electric field in a vacuum (where there are no charges or currents).

Comments: The initial task of deriving the wave equation should be completed within 5-10 minutes, though some may need help in getting started. The final part asking about static fields has been added since the implementation in our class, but we expect that there will be some students who are confused about whether this statement about fields in a vacuum is general. The biggest confusion we've seen for students is how the wave equation for EM fields is usually written as a compact vector equation, where it can look as though the Laplacian is operating on the entire vector, instead of each of its components separately.

11 – Complex Plane Waves (< 40 minutes)

Topics: Plane waves, complex exponentials

Summary: The initial tasks have students identifying the various quantities that go into a plane wave represented by a complex exponential; the first involves constructing an expression from the quantities given, while the second analyzes the quantities for an expression that's given to them. The remaining tasks involve deriving explicit expressions for the divergence and curl of a plane wave, and relating them to Maxwell's equations in vacuum to determine the orientations of the electric and magnetic fields relative to the direction of propagation.

Comments: We've noticed that students can have trouble parsing out the various vectors and scalar quantities that go into a plane wave expressed in complex exponential notation; the first two tasks give them practice with this. When taking partial derivatives of the complex exponential, many students had difficulty correctly applying the chain rule; in particular, they often didn't see that the dot-product $\vec{k}\cdot\vec{r}$ is compact notation for $k_X x + k_Y y + k_Z z$ - sometimes because they were associating the r-vector only with spherical coordinates. The final page asks students to make a *convincing argument* for how the electric field is related to the magnetic field in terms of a cross-product with the wave vector – several students were initially trying to calculate the actual cross-product using determinants, without recognizing that the divergence and curl operations just replace the spatial derivatives with the corresponding components of $\vec{k}$.

12A – Reflection & Transmission (Normal Incidence) (< 50 minutes)

Topics: Reflection and transmission, boundary conditions, complex exponentials.

Summary: Students begin by expressing in exponential notation the boundary condition on the parallel components of an EM plane wave for normal incidence at the interface between two media. They can then find the phase shift and amplitude for the reflected wave by considering representations of the electric field in the complex plane, first for when the amplitude of the transmitted wave is smaller than for the incident, and then when the opposite is true. Students should conclude that the frequencies of all three equations must match for the boundary condition to hold at all times. A second boundary equation is found by considering the electric and magnetic fields of the reflected wave. The remaining tasks connect the amplitude and phase shift of the reflected wave with the refractive indices of the two materials.

Comments: <u>Warning</u> (!): Portions of this tutorial have not been validated or field-tested, but we expect students to be able to finish the tasks in less than 50 minutes. A somewhat different version was used in our class, and the tasks related to representations in the complex plane are new. We anticipate that this way of representing the electric field will be more intuitive for seeing how the electric fields must match up in order for the boundary condition to be satisfied at all times.

A number of students will have issues with the algebra when solving two equations for two unknowns on the final page. Checking their answers for the case when the refractive indices are equal may help them to see whether they have it right.

12B – Reflection & Transmission (Oblique Incidence) (~30 minutes)

Topics: Reflection and transmission, boundary conditions, complex exponentials.

Summary: Students begin by expressing in exponential notation the boundary condition on the parallel components of the electric field for an EM plane wave at oblique incidence to the interface between vacuum and a material. After finding the phase shift for the reflected wave, students should conclude that the components of k for each of the waves that are parallel to the boundary must match if the equation is true all along the boundary. The remaining tasks connect the angles of reflection and transmission to index of refraction for the material.

Comments: Warning (!): Portions of this tutorial have not been validated or field-tested, but we expect students to be able to finish these tasks in around 30 minutes. A somewhat different version was used in our class, and the tasks related to representations in the complex plane are new. The tasks in this tutorial have been constructed with the assumption that students have completed the tutorial on R&T for normal incidence (#12A); if not, the more abbreviated tasks herein will be more challenging, since they are not scaffolded in the same way as in the prior tutorial. The vectors in the diagrams all have the correct proportions, so it is important that students can justify their answers on the final page in terms of the reduced wave speed, and are not simply judging from the diagram.

13 – Gauge Invariance (< 50 minutes)

Topics: Time-dependent potentials and fields, gauge transformations

Summary: Students are first reminded of why EM fields can be written in terms of a scalar and vector potential. They then show that a gauge transformation in the vector potential results in an identical magnetic field. Students derive an integral relationship between **E** & **A**, and then find the necessary conditions for transforming V. A challenge question asks students to derive Poisson's equation, which would be used to solve for the scalar function λ that transforms the potentials to the Coulomb gauge.

Comments: The biggest complaint from students has been about not entirely understanding why we would want to transform the potentials in the first place. This is hinted at in the final challenge question, where an equation is found for the function that transforms to the Coulomb gauge, but is not explicitly addressed here. The first task is a review intended to orient students to the remaining tasks, reminding them of why we can write the fields in terms of potentials. Many students have forgotten that the various statements that can be made about divergenceless (or irrotational) fields are all equivalent. [See Sect. 1.6.2 in Griffiths] Some students were unsure about the cross product being a linear operator (whether the curl of the sum of two vectors is equal to the sum of the curls). There has been a modest amount of confusion in simply keeping track of primed and unprimed quantities, with students sometimes mixing them up in their heads.

14 – Time-Retarded Potentials (~30 minutes)

Topics: Retarded time, time-retarded potentials and fields.

Summary: Students explore the concept of *retarded time* for the case of an infinitely long wire with a current that abruptly starts at $t = 0$. They first consider the points in space where an observer would be aware of there being a non-zero current a short time after it starts. Students find that the retarded time has different values at different points in space (relative to an observer at the origin), and decide on the limits of integration for finding the retarded vector potential at the origin. Challenge questions at the end have students calculate the electric field at the origin, and check their answer in the limit of long times.

Comments: Although it is fairly intuitive to students that it takes a finite amount of time for effects to propagate from a source to an observer, the definition of *retarded time* (and how it is used in a calculation) is not. In particular, that the retarded time is a function of two coordinate variables, and has different values at different points in space relative to a fixed observer. There is a "time ruler" on a separate handout that students can use for the questions on the first and second page – they may need a gentle reminder that it's easiest to work with whole numbers for the distance from the origin (some were tempted to estimate the distance for points on the wire that were even with the tick marks on the x and y-axes). There were some students confused about what the primed and unprimed variables are each referring to, which typically shows up in problems involving the separation vector $\vec{r} - \vec{r}'$. The tasks in the *challenge questions* at the end are similar to Example 10.2 in Griffiths, but a simpler method (involving the fundamental theorem of calculus) is used for calculating the electric field from the retarded vector potential.

15 – Special Relativity

These are all relatively short activities, meant to address just some of the basics from special relativity, such as length contraction, 4-momentum, and velocity addition.

15A – Length Contraction (< 15 minutes)

Topics: Special relativity, Lorentz transformations, length contraction, simultaneity.

Summary: Students first establish the relationships between the times and locations that go into the length measurement of a moving body. They then derive a formula for length contraction using the Lorentz transformations, and consider whether the two position measurements occur at the same time in both frames.

Comments: Although some of the questions may seem trivial to instructors, we found that a number of students were confused on even the "simple" tasks, which shows that students may use the Lorentz transformations without understanding exactly what the different primed and unprimed variables correspond to. Our students were told beforehand that length measurements involve a simultaneous determination of the positions of the two ends; still, some were very tentative about simply saying that the two times are equal, or even that the length is simply the difference between the two position measurements.

15B – Inelastic Collision (~10 minutes)

Topics: Special relativity, 4-momentum, relativistic collisions.

Summary: Students use conservation of relativistic 4-momentum to find the final mass of an object resulting from the merging of two colliding particles.

Comments: This activity was straightforward for most students, as long as they were clear on the following: definition of relativistic 4-momentum; the total momentum of a system is the linear sum of the momenta of the particles; and that this quantity is conserved before and after the collision. Some students may momentarily forget the velocity dependence of γ when first working out the total momentum; the spatial velocities of the two particles cancel, but the γ-factor that appears in the total momentum is not also zero.

15C – Relativistic Velocity Transformation (~10 minutes)

Topics: Special relativity, Lorentz transformations, relativistic addition of velocities.

Summary: Students derive the velocity addition formula using the Lorentz transformations and the definition for the velocity in two different inertial frames.

Comments: The biggest difficulty for students may be the algebra involved. A common problem is for students to be confused about the velocity of the frame v, and the velocity of the particle u in that frame of reference. We have also noticed some conceptual difficulty for students regarding an event taking place at a single point in spacetime, and the different coordinate representations of that point in different inertial frames.