12C – EM waves in conductors

Topics: Continuity equation in conductors (dynamics).

Summary: Just a quick, heavily guided activity to bring them to the equation from Griffiths for the time dependence of charge buildup in a conductor. (This is the key argument that allows us to "toss" the rho term in Gauss' law when solving for EM waves in a conductor. This approximation holds only for relatively low frequencies, and this activity helps us set at least ONE of the relevant time-scales for this physics)

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Comments: Our students got through this activity in about 5-10 minutes. More than the 2 minutes I would have taken at the board doing it for them, but I felt this one went well, it was time well-spent.

Note: we are glossing over the "free" vs "total" story here (so the students will likely not quite get the technically correct formula, they will probably have ε_0 rather than ε…)

Lots were stumped on line 1, they could not remember the continuity equation! Everyone came up with it, many by "buzz" in the room, some looking it up. A few remembered that you can take div(Maxwell-Ampere equation) (I had written all the Maxwell equations up on the board pre-class!) and some students upon hearing that decided on their own to to "rederive it", taking a few extra minutes. Nice!
Some students got a little hung up about how exactly to use Ohm's law.

Some students got stuck on "Apply Gauss' law", reading that to mean that somehow there were supposed to convert to integral form or DO a surface/volume integral or some such thing. One student thought sigma = 1/rho cancels the rho from Gauss' law (oops, one symbol, two meanings!)
Most could "solve" the simple PDE by inspection, but interesting some felt stuck. Of those, some had not "eliminated" all other terms, so they really weren't looking at the PDE I intended them to be at by stage 5, d(rho)/dt = - sigma rho/epsilon
Some had come to reasonable conclusions – we followed this up with a discussion about "where does the charge go" and what is the timescale for copper, etc...

Clicker question to wrap up with:
What are the SI units of epsilon0/sigma? (s, s^-1, it's complicated,…)

Charge conservation in metals: dynamics

Suppose you dump some charge into the bulk of a metal at t=0.
What happens next?

1) Start with the continuity equation:

$$\nabla \cdot \vec{J} =$$

2) Use Ohm's law for the current density, to get

$$\nabla \cdot \underline{\hspace{3cm}} =$$

3) Assume the material is homogeneous (pull constants out of the derivative) so…

$$\underline{\hspace{1.5cm}}\ \nabla \cdot \underline{\hspace{3cm}} =$$

4) Apply Gauss' law in that last equation to simplify it
(We can't automatically assume $\rho=0$ inside a metal if we just DUMPED charge into it!)

5) You should have a first order differential equation. Can you solve it by inspection?

What do you conclude about the charge density inside the metal as time goes by?
(What is happening physically?)