12B – Snell's Law

Topics: Snell's law from boundary conditions, with complex exponentials.

Summary: The sole task of this short activity is to use BC's to derive Snell's law (connect the angle of transmission to index of refraction for the material.)

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Comments: We gave students about 15 minutes, and many were not done! Lots felt stuck, didn't know how to start. I have added a bold hint (not tested in class yet) to see if that helps students not get bogged by the distractors.

Interestingly I had DONE this derivation in the previous lecture for them, in full detail! This Tutorial does have a lot of "distraction" (the complex equations, the full blown diagram), with the big "clue" or trick in the little separated central paragraph. When groups were stuck, I pointed them there and said "write out those dot products, when r = xhat, and see what you get." I also pointed out that, on the board, we had written:
omega = v k
v= c/n

Note that the vectors in the diagrams all have the correct proportions, so it is important that students can justify their answers in terms of the reduced wave speed, and are not simply judging from the diagram. But the diagram is "correct", for the case n2 > n1.

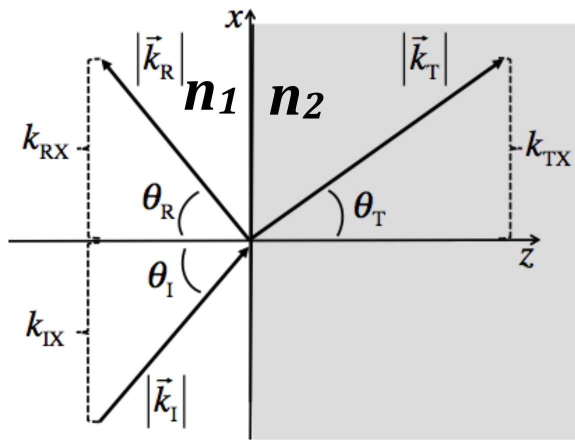
Followup clicker question: See powerpoint, we told them to assume n2 > n1, and decide if kT,z is >, < , or = kI,z I the figure. (I used this to figure out when people were finishing up) It was 65% correct (kT,x = kI,x, that's what the continuity equation gave us, but |kT| > |kI| since k is proportional to n…

An EM plane wave traveling in the x-z plane (i.e., $\vec{k}_I = k_{IX}\hat{x} + k_{IZ}\hat{z}$) is incident on a material at an angle θ_I relative to the z-axis:

$$\vec{\mathbf{E}}_I(\vec{r},t) = \vec{E}_I \exp\left[i\left(\vec{k}_I \cdot \vec{r} - \omega t\right)\right]$$

$$\vec{\mathbf{E}}_R(\vec{r},t) = \tilde{\vec{E}}_R \exp\left[i\left(\vec{k}_R \cdot \vec{r} - \omega t\right)\right]$$

$$\vec{\mathbf{E}}_T(\vec{r},t) = \vec{E}_T \exp\left[i\left(\vec{k}_T \cdot \vec{r} - \omega t\right)\right]$$



We argued from continuity of $E_{//}$ that $\vec{k}_I \cdot \vec{r} = \vec{k}_R \cdot \vec{r} = \vec{k}_T \cdot \vec{r}$ for any **r** in the z=0 plane. (E.g. $\vec{r} = \hat{x}$ **)**
THIS IS THE STARTING POINT!

- Derive the following ratio for incident and transmitted wave angles. *(Express it first in terms of magnitudes of **k** vectors, but then write it simply in terms of the index of refraction in the two regions, n_1 and n_2)*

$$\frac{\sin\theta_T}{\sin\theta_I} =$$