Phys 3310, HW 2, Due in class, 1:00pm, Wed Sept 3. In general, we will try to grade homeworks for clarity of explanation as well as for mere "correctness of final answer".

Q1. SOME VECTOR CALCULUS

a) Make a diagram representing the vector equation $\vec{C} = \vec{A} - \vec{B}$. Take the dot product of each side of this equation with itself and proof the Law of cosines:
$C^2 = A^2 + B^2 - 2AB\cos\theta$ (show the angle θ in your diagram).

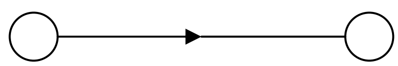
b) Sketch the vector function $\vec{v} = \frac{\hat{r}}{r^2}$ and compute its divergence. The answer may surprise you. Explain the answer qualitatively.

Q2. ANGLE BETWEEN SUSPENDED CHARGES

Two charges of identical mass *m*, one with charge *q*, the other with charge 2*q*, hang from strings of length *L* from a common point. Assume *q* is sufficiently small so that any angle you're looking for is very small, and find an approximate expression for the angle θ each charge makes with respect to the vertical. Check (show us!) that the units work out, and that the limiting behavior for large mass, large length, and/or small q are sensible.

Q3. TWO CHARGES

a) Sketch the field lines for the configuration of charges shown below. I drew one to get you started. Make sure that you are consistent with the number of field lines that you attach to each charge. Think about what the field should look like (how many field lines there should be) far away from the charges.



+3q -q

b) In the previous question, assume the +3q charge is located at x = −D and the -q is located at x = +D. What is a *fourth order* approximation to the electric field **E**(0,y,0) for points far away on the y axis, i.e. |y| >> D? Write your exact answer in terms of the small quantity ε = D/y and Taylor Series expand, keeping terms out to ε^4.
Check (show us!) that the units of your answer are correct, that the direction of your E field makes sense, and that the limiting behavior (y -> infinity) makes sense.
Note: Approximating (in a problem when we have the exact answer!) may seem a little odd, but we'll find as we move on that this is often the only *way to practically solve more complex problems!*

Q4. DISK OF CHARGE:
a) Find the electric field on the z-axis due to a disc of radius R_0 and uniform charge density σ. The disk is in the xy plane and is centered on the origin. [Hint: a disk can be thought of as a series of concentric rings. Can you first find the E field due to a thin ring of charge dq?]
b) Explicitly calculate the limiting forms of your solution at very small and at very large z (compared to R_0) and discuss.
Note: The disk of charge is an idealization of many physical devices: a capacitor plate, a small patch of any surface... Once you have solved this ideal *problem, you will be able to apply it to more realistic situations.*

Q5. SPHERE OF CHARGE:
a) Find the electric field a distance z from the center of a spherical surface of radius R (Fig 2.11 in Griffiths) which carries a uniform surface charge density σ. Do this by explicit integration (i.e. starting from Griffiths Eq. 2.7), please. (Do not use Gauss's Law.)
You only need to treat the case $z>R$ (*outside* the sphere). Express your answer in terms of the total charge q on the sphere.
[Hint: Use the law of cosines to write r in terms of R and θ. I got an integral which I needed to look up using the online Mathematica Integrator. In the end, be careful if you get a square root to take the *positive* root: $\sqrt{R^2+z^2-2Rz}=(R-z)$ if $z<R$, but it's (z-R) if $z>R$].
b) Check your answer using knowledge from 1120 and intuition (briefly discuss - what *should* it be?) What would you expect the answer should be *inside* the sphere? Why?
c) Estimate the maximum charge you can put onto a metal sphere a few inches across.
Hint: If the sphere sparks, it can bear no more charge. Look up the breakdown electric voltage of air. Your answer can be very rough, we only want an order of magnitude.

Historical note: Newton solved part a using geometry (not calculus!) This geometric proof is tricky and still excites debate: see R. Weinstock Am.J.Phys., 52, (1984), p. 883; H. Erlichson, Am.J.Phys.58, (1990) p. 882. Reputedly, Newton thought calculus should be kept secret, and held up publication of Principia until he could work out these non-calculus proofs. He published calculus much later, about the same time as Leibniz.

Q6. E&M Diagnostic.
Take the online Survey of E&M Knowledge here
http://www.colorado.edu/physics/EducationIssues/assessment/
This is a timed "exam" with 35 multiple-choice questions. You have 45 minutes to complete the exam. (After 45 min, your answers are collected automatically.) This exam is participation credit only. You will receive full credit if you take the exam, regardless of your score. The purpose of this exam is to allow us to gauge how much you learned and remember from your previous E&M courses, particularly PHYS1120. So please take the exam without referring to any external aids (no looking in texts, please). Remember, you will receive full credit, regardless of your score.