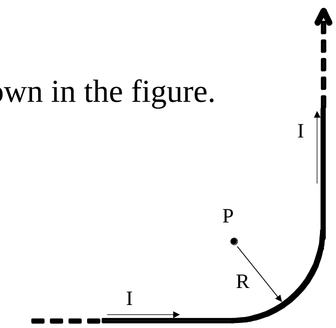
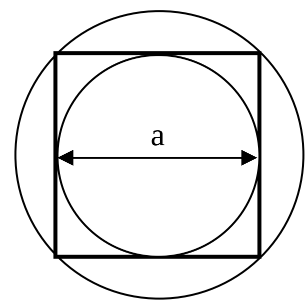
Phys3310 HW10, Due start of class Wed Nov 5

Q1. MAGNETIC FIELD:
An infinitely long wire has been bent into a right angle turn, as shown in the figure.
The curved part is a perfect quarter circle of radius R.
Point P is exactly at the center of that quarter circle.
A steady current I flows through this wire.

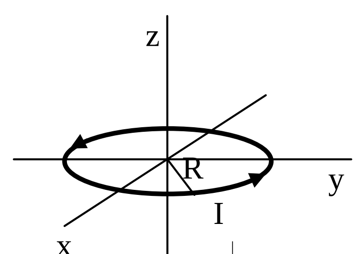
Find the magnetic field at point P (magnitude and direction).

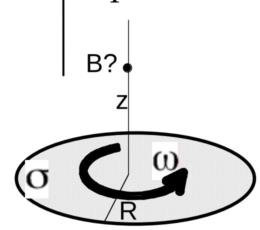


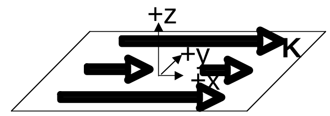
Q2. BIOT-SAVART - SQUARE LOOP
A) Find the magnetic field at the center of a square current loop with current I and edge length *a*.
B) If I had such a loop in my lab and wanted the B field at the center, I might do the above calculation, but if I was planning an experiment and just wanted a *rough estimate* of the B-field, I might "assume a spherical cow": assume the square was really a circle. We've done that problem (**B** at center of a circular loop - it's much simpler than the square.You don't have to rederive it, but do think back to how we got that result, and why it turned out to be a relatively easy application of Biot-Savart.) But what radius circle would you use, to estimate B? Find the B fields for the inscribed and circumscribed circles and then average. How good an approximation does that turn out to be?



Q3. B FIELD FROM ROTATING DISK
A) Compute the magnetic field **B**(0,0,z) on the axis of a circular ring of radius R carrying a current I . The ring is in the xy plane and is centered on the origin.
B) Last week we had a problem with a CD of radius R with a fixed, constant, uniform surface electric charge density σ everywhere on its top surface. It was spinning at angular velocity $\omega\hat{\mathbf{z}}$ about its center (the origin). You found the current density **K** at a distance r from the center. Use that result to find the magnetic field **B**(0,0,z) at any distance z directly above the origin.
Does your answer seem reasonable? Please check its limiting behaviors (e.g. what do you expect if $R\to 0$? $z\to\infty$? $\omega\to 0$? Slightly less obvious, but also worth checking/thinking about, what about $z\to 0$?)







Q4. SHEET OF CURRENT: BIOT-SAVART VS. AMPERE
Consider a thin sheet with uniform surface current density $K_0\ \hat{\mathbf{x}}$ in the xy plane at z = 0.

A) Use the Biot-Savart law to find **B**(x,y,z) both above and below the sheet, by integration.

<u>**Note:**</u> The integral is slightly nasty. Before you start asking Mathematica for help - simplify as much as possible. Set up the integral, be explicit about what curly R is, what da' is, etc, what your integration limits are, etc. Then, make clear mathematical and/or physical arguments based on symmetry to convince yourself of the *direction* of the **B** field (both above and below the sheet), and to argue how **B**(x,y,z) depends (or doesn't) on x and y. (If you know it doesn't depend on x or y, you could e.g. set them to 0... But first you must convince us that's legit!)

B) Now solve the above problem using Ampere's law. (Much easier than part a, isn't it?) Please be explicit about what Amperian loop(s) you are drawing and why. What assumptions (or results from part a) are you making/using?
(Griffiths solves this problem, so don't just copy him, work it out for yourself!)

C) Now let's add a second parallel sheet at z = +a with a current running the other way. $\vec{K} = -K_0\ \hat{\mathbf{x}}$. Use the superposition principle (do NOT start from scratch or use Ampere's law again, this part should be relatively quick) to find **B** *between* the two sheets, and also *outside* (above or below) both sheets.

Q5. UNIFICATION
A) Griffiths 5.12.
B) Along with the necessary addition made by Maxwell, Ampere's law in full glory reads
$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$ (We haven't talked much about the second term yet, since we've been focused on statics this term, but it's there!)
Take the divergence of this equation, and show that electric charge is conserved globally.
I think both *these results are pretty cool, and carry very deep messages about the nature and unification of electricity and magnetism,* and *their connection to special relativity.*