Physics 3310: First Homework.

YOUR NAME (neatly!)____

This is a regular hw, due at the start of class Wed Aug 27

Use whatever resources you need, including Griffiths, your old 2210 text, talking to peers, office hours - whatever you need. In the end, though, what you turn in must be your own work, reflecting your own understanding

Note that in general, we grade homeworks for clarity of explanation as much as for correctness of final answer.

Please *show your work* or explain your reasoning whenever possible.
For each question, rate your level of confidence (A, B, C) in the material with this scale:
A) The material is very familiar to me. I was able to answer the question without assistance of a book, human, Internet, etc
B) The material was familiar, but in order to answer the question, I needed some assistance (from a book, human, Internet, etc)
C) The material was unfamiliar to me. I had to learn it in order to answer the question.
Write the letter (A, B, or C) , with a circle around the letter, near your answer .

1.) Given a triangle (NOT necessarily a "right triangle") with sides a, b, and c, and an angle (θ) opposite side c. Suppose I tell you a, b, and θ, and ask "what is c?"
What is the formula for c?
Remember to rate this as A, B, or C, with a circle around the letter.

2.) $\int \frac{4x}{\left(x^2+a^2\right)^{3/2}}\, dx = ?$ (a is a known constant. Note that it is an indefinite integral)

3.) $\frac{d}{dx}\int_1^x f(y)dy = ?$ [where f(y) is some given, known (well behaved) function of y]

4.) $\frac{d}{dx}\int_0^1 (y+x)\ dy = ?$

Make a quick sketch, in the x-y plane, of the following vector functions.
Plot enough different vectors to get a feeling for what this field looks like in the x-y plane.

5.)(a) $y\,\hat{\mathbf{x}}$

(b) $r\,\hat{\mathbf{r}}$ (The symbol **r** here refers to the usual **r** in spherical coordinates)

(c) $\dfrac{x}{\sqrt{x^2+y^2}}\hat{\mathbf{x}}+\dfrac{y}{\sqrt{x^2+y^2}}\hat{\mathbf{y}}$

Also just for this one (part c) - can you explain in words what this plot is showing?

6.) Given the scalar function T(x,y,z) (e.g. the temperature at any point in the room), which of the three operations (div, grad, or curl) can be sensibly operated on T?

In each case that makes sense:

a) give a formula for the result
b) explain in words how you would interpret the result.
c) state whether the result is a vector or scalar.

7.) Given an arbitrary vector function **V**(x,y,z) (e.g. the velocity of a flowing liquid), which of the three operations (div, grad, or curl) can be sensibly operated on **V**?

In each case that makes sense:

a) give a formula for the result
b) explain in words how you would interpret the result.
c) state whether the result is a vector or scalar.

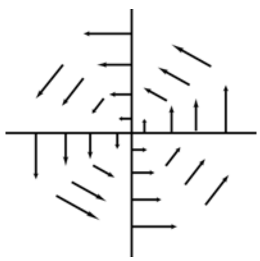
8.) For each of the four vector fields sketched below....

Which of them have nonzero *divergence* somewhere? ______________
(If the divergence is nonzero *only* at isolated points, which point(s) are those?)

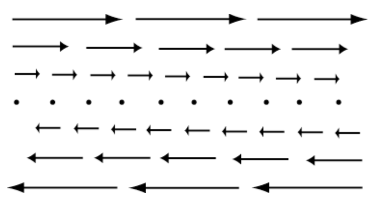
Which of the following fields have nonzero *curl* somewhere? ___________________
(If the curl is nonzero *only* at isolated points, which point(s) are those?)

(A brief explanation of your answers below each figure would be welcome)

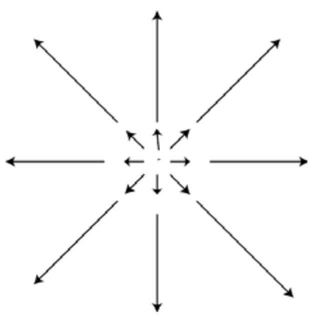
A.



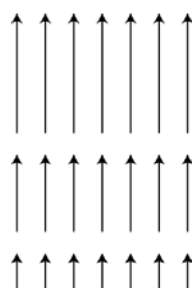
B



C.



D.



9.) Vectors **A** and **B** are known. ("known" means you know the components, or alternatively, the length and angle of the vectors)

Define the dot product mathematically in two different looking ways.
(Hint: one way should involve the components, the other, the length/angles)

Give a brief physical interpretation of what the dot product means or tells you (you can give a concrete example if you like)

10.) Define the vector cross product mathematically in two different looking ways.
(Hint: one way should involve the components, the other, the length/angles)

Give a brief physical interpretation of what the cross product means or tells you (you can give a concrete example if you like)

11.) Compute the gradient of the following two scalar fields:

a) $e^x \cos(y)$

b) $\cos(x^2 + y^2 + z^2)$

12.) Compute the divergence and curl of $\hat{\imath}(x^2 + yz) + \hat{\jmath}(y^2 + zx) + \hat{k}(z^2 + xy)$.

13.) Evaluate the line integral $\int(y^2 dx - 2x^2 dy)$ along the parabola $y = x^2$ from the point (0,0) to the point (2,4).

14.) In Phys 1120, one of the early chapters was on Gauss' law, one of the most fundamental laws of electricity. It looks like this: $\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\ dA = q(\text{enclosed})/\varepsilon_0$,

where **E** is the electric field, S is a closed surface, **n** is a unit vector which points everywhere *outward* from the surface.
Suppose I fill a cube (length L on a side) uniformly with electric charge. I then imagine a larger, closed cubical surface symmetrically surrounding this cube (length 2L on a side)

A) Is Gauss' law TRUE in this situation? (Briefly, why or why not?)

B) Can one *use* Gauss' law to easily compute the value of the electric field at arbitrary points outside the charged cube (Don't try, just tell me if you *could*, and why/why not?)

C) What exactly is **E**, the electric field? (Define it, and explain how you think about it, first mathematically *and* then in words? Please define any new technical words you introduce into your definition.)

15.) Have you ever learned about orthogonality of functions in a math or physics class? If so, tell me briefly what you remember about the orthogonality of sin and/or cos.

16.) Have you ever learned about the delta function (also called the "Dirac delta function", $\delta(x)$, in either a math or a physics class)? If yes, tell me briefly what you remember.

If you can, evaluate $\int_{-\infty}^{\infty}(x-6)\,\delta(x-6)\,dx$.