

Phys 3310, HW #3, Due in class Wed Jan 30. In general, we will try to grade homeworks for clarity of explanation as well as for mere "correctness of final answer".

### Q1. DELTA FUNCTIONS

a) Calculate  $\int_{-1}^1 |x - c|^2 \delta(2x) dx$ , with the constant  $c=3$ .

b) Calculate  $\iiint_V |\vec{r} - \vec{c}|^2 \delta(2\vec{r}) d\tau$ , where the volume  $V$  is a sphere of radius 1, and the constant vector  $\vec{c}=(3,4,0)$

c) Evaluate the integral  $\int_V (1 + e^{-r}) (\nabla \cdot \frac{\hat{r}}{r}) d\tau$  where volume  $V$  is a sphere of radius  $R$

centered at the origin, by two different methods, as Griffiths does in Ex 1.16.

(one method, surely the easier one, is to use your knowledge of what  $\nabla \cdot \frac{\hat{r}}{r}$  is.

The other is to use "integration by parts in 3-D", described in Griffiths section 1.3.6. Don't forget that your purple crib sheet (inside flyleaf of Griffiths) tells you how to take the divergence of any arbitrary function in, e.g. spherical coordinates!)

*The delta function, both 1-D and 3-D appears throughout this course. Physically, it represents a highly localized source (like a point charge), but as in part c, it will often appear "mathematically" when integrals arise involving Gauss' law.*

### Q2. $\delta$ FUNCTIONS AND CHARGE DISTRIBUTIONS

a) On the previous homework we had two point charges:  $+3q$  located at  $x=-D$ , and  $-q$  located at  $x=+D$ .

Write an expression for the *volume* charge density  $\rho(\mathbf{r})$  of this system of charges.

b) On the previous homework we had another problem with a spherical surface of radius  $R$  (Fig 2.11 in Griffiths) which carried a uniform surface charge density  $\sigma$ . Write an expression for the *volume* charge density  $\rho(\mathbf{r})$  of this charge distribution. (Hint: use spherical coordinates, and be sure that your total integrated charge comes out right.) (Verify explicitly that the units of your final expression are correct)

c) Suppose a linear charge density is given to you as  $\lambda(x) = q_0 \delta(x) + 4q_0 \delta(x - 1)$

Describe in words what this charge distribution looks like.

How much total charge do we have?

Assuming that  $q_0$  is given in Colombs, what are the *units* of all other symbols in this equation (including, specifically the symbols  $\lambda$ ,  $x$ , the delta function itself, the number written as "4" in front of the second term, and the number written as "1" inside the last  $\delta$  function.

*It can also be tricky to translate between the math and the physics in problems like this, especially since the idea of an infinite charge density is not intuitive. Practice doing this translation is important (and not just for delta functions!)*

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### Q3. $\delta$ FUNCTIONS AND GAUSS

The electric field outside an infinite line that runs along the z-axis is equal to  $\vec{E} = \frac{2\lambda}{4\pi\epsilon_0 s} \hat{s}$

in cylindrical coordinates. (This is derived in Griffiths Example 2.1)

- Find the divergence of the E field for  $s > 0$ .
- Calculate the electric flux out of an imaginary "Gaussian" cylinder of length "L", and radius "a", centered around the z axis. Do this 2 different ways to check yourself: by direct integration, and using Gauss' law
- Given parts a and b, what is the divergence of this E field?

[Hint 1: use cylindrical coordinates. Hint 2: your answer can't be zero everywhere! Why not?]

*Notation note: s is the symbol Griffiths uses for "distance from the z axis" in cylindrical coordinates. I may sometimes use the symbol r for that quantity (if it's clear I won't confuse it with spherical coordinate), or sometimes even  $\rho$  (if I don't think I'll confuse it with a charge density!)*

### Q4. E FIELD IN HYDROGEN.

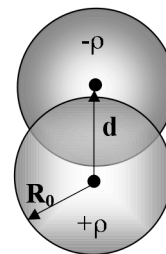
- Find the E-field in a hydrogen atom. Quantum mechanics tells us that the electron is effectively "smeared out", so the electron's resulting contribution to the charge density is  $\rho_0(r) = \rho_0 \exp(-\frac{2r}{a_0})$  where  $a_0$  is the Bohr radius. (Hint: Don't forget to normalize  $\rho$  so the electrons charge is  $-e$ . This charge density is not uniform, so you must integrate to get enclosed charge. Also, don't forget there is also a pointlike proton in the middle of this atom!) Sketch and briefly discuss your result (compare the E field you get to what you'd have from the proton alone)
- Having done the above, briefly discuss the advantages and disadvantages of using Gauss' Law to find the electric field instead of using Coulomb's Law (Griffiths Eq 2.8)? What role does symmetry play?

*Note: sketch means sketch - just a rough plot which shows key features (e.g. what's it do near the origin? Near infinity?) We don't want to just calculate E fields, we want to be able to imagine what they look like, so sketching fields is important. (You can always use a program (like Mathematica) to check your sketch, but try on your own first...)*

**Q5.** You have two spheres. The first is centered at the origin, has uniform positive charge density  $\rho$ , and radius  $R_0$ . The second has uniform negative charge density  $-\rho$ , same radius  $R_0$ . It is shifted up by a distance  $d$ .

- Show that the field in the region of overlap of the two spheres is constant, and find its value. (Please check that the units are correct)

[Hint: You will first want to figure out what the E field is in a *single* sphere with uniform charge density  $\rho$ . It is definitely not zero, nor is it uniform!]

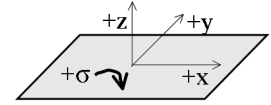


- In the limit that  $d$  becomes small compared to  $R_0$ , discuss in words what the resulting (total, physical) charge distribution in space really *looks* like (so that later in the course when we encounter such a charge distribution, we will know where it came from and what the E field looks like inside!)

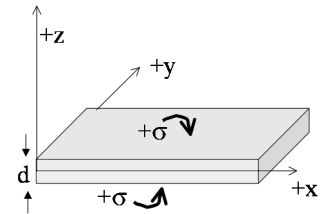
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### Q6. FLAT SLABS AND CONDUCTORS

a) Griffiths (Ex 2.4) works out the E-field everywhere in space due to an infinite thin plane of charge, with surface charge density  $\sigma$ . Let's assume our sheet lies in the x-y plane. Sketch the z-component of the E field as a function of z (include both + and - z)

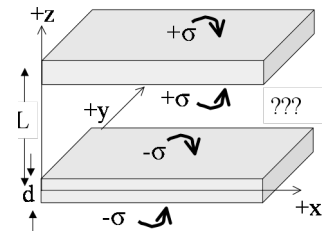


b) Now we have an infinite *conducting slab* of metal which has a charge density  $+\sigma$  on each surface (both top *and* bottom) Write a mathematical function for  $E(z)$  (and again sketch the result) for the E-field everywhere in space. (This slab is oriented like the sheet in part a, parallel to the xy plane, and is still infinite in the x-y directions, but *this* slab has a finite thickness d, in the z-direction).



Briefly state what would *change* if the charge density had been negative, i.e.  $-\sigma$  on *each* surface, top and bottom.

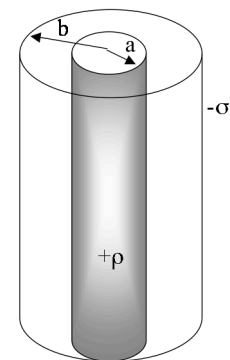
c) Now imagine *two* such conductors, well separated by a distance  $L \gg d$ . Suppose, for the sake of this problem, that the upper one could still be exactly as described in part b (i.e. it has  $+\sigma$  on each surface, top and bottom) and the lower one is like part b only with  $-\sigma$  on each surface, top and bottom) Use the principle of superposition to find the E field everywhere in space. (That means between the slabs, outside either slab, but also *INSIDE* each slab).



As you may recall from Phys 1120, the E field inside of a slab of metal is supposed to be zero. So... what do you suppose might have gone wrong? Have you applied superposition right? Is Gauss' law invalid here? How does nature resolve this inconsistency?

### Q7. COAXIAL CABLE.

A long coaxial cable carries a uniform volume charge density  $\rho$  throughout an inner cylinder (radius  $a$ ) and a uniform surface charge density  $\sigma$  on the outer cylinder (radius  $b$ ). The cable is overall electrically neutral. Find  $E$  everywhere in space, and sketch it.



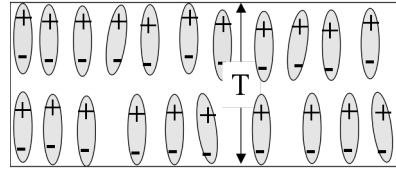
b) Last week you considered how much charge a child's balloon might hold before sparking. Now let's consider the same question for a real life coax cable. Making any reasonable guess as to physical dimensions for a cable like one that might attach your stereo to your TVs - what would be the maximum static charge you could put onto that cable?

(This is *estimation* - I don't care if you're off by a factor of 2, or even 10, but would like to know the rough order of magnitude of the answer!) From where to where do you expect a spark to go, if it *does* break down?

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**Extra Credit:** (worth half of any of the above, but *won't* count off if you don't do it)

Smectic-C liquid crystals are made of long rodlike molecules with a positive head and a negative tail which pack together to form a long, thin sheet as shown. The volume charge density is obviously complicated, but can be quite successfully modeled with the rather simple form  $\rho(x,y,z) = \rho_0 \sinh(z/z_0)$ ,



i.e. uniform in  $x$  and  $y$ , but varying in  $z$ , with  $z = 0$  defined to be the middle of the sheet. (That's the "hyperbolic sinh" function) Find the electric field everywhere in space, in terms of the constants  $\rho_0$ ,  $z_0$  and the sheet thickness  $T$ , and sketch it. Where is it biggest?