Phys 3310, HW #9, Due in class after spring break, i.e. Wed Apr 2

Q1. ITERATING FIELDS

Griffiths solved the problem of a dielectric sphere in a uniform external field in Example 4.7 using Separation of Variables (and applying boundary conditions). There's another approach, which in some ways is perhaps *conceptually* simpler (?)

a) You've put the object into a uniform external field $\mathbf{E_0}$, so it would be logical (but wrong) to assume that the polarization of the dielectric would just be simply $\mathbf{P}=\varepsilon_0\chi_e\mathbf{E_0}$. (Briefly - why is that wrong?) Go ahead and assume it, it's like a "first approximation", let's call it $\mathbf{P_0}$. Now this polarized sphere generates its own additional (induced) E field, call that $\mathbf{E_1}$. What is that field? (You don't have to rederive it from scratch, Griffiths has shown us how to figure out the E field in a sphere, created by a uniform polarization of that sphere. You've also probably used it in the last homework) OK, but now $\mathbf{E_1}$ will modify the polarization by an *additional* amount, call it $\mathbf{P_1}$. What's that? That in turn will add in a new electric field by an additional amount $\mathbf{E_2}$. And so on. The final, total, "real" field will just be $\mathbf{E_0}+\mathbf{E_1}+\mathbf{E_2}+\ldots$

Work it out, and check/compare your answer with Griffiths result at the end of Ex 4.7 (p. 188)

b) If the sphere was made of Silicon (see Table 2 in Griffiths), compare the "first approximation" for $\mathbf{P_0}$ with the true result for polarization - how important was it to go through the summation? (In what limit, large or small ε_r, does this summing procedure make a big difference?)

You will need the handy trick for summing the infinite geometric series: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

If that looks mysterious to you, you should convince yourself that it's correct, by doing our usual Taylor expansion of the function 1/(1-ε).

Q2. DERIVING CLAUSIUS-MOSSOTTI

Griffiths 4.38 (on page 200). Please *discuss* your result, briefly. (In words, what is 3.72 telling us, and how (and when) is it different from the "naive expectation" Griffiths talks about at the start of the problem?)

Q3. APPLYING CLAUSIUS-MOSSOTTI

Use the Clausius-Mossotti equation (which you just derived in the previous problem) to determine the polarizability of atoms in air (which is largely N_2). (Table 2 in the book gives dielectric constants for N_2 and "air" as a separate entries but the numbers are basically identical.) Combine this result with the simple classical calculation we did in class a couple of weeks ago (basically example 1 in chapter 4) to *roughly estimate* a typical radius of an air molecule.

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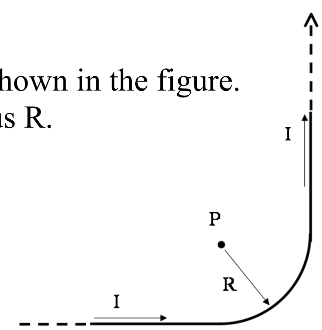
Q4. CURRENT DENSITIES:
a) A solid cylindrical straight wire (radius a) has a current I flowing down it. If that current is uniformly distributed over the outer surface of the wire (none is flowing through the "volume" of the wire, it's all surface charge), what is the surface current density **K**?
b) Suppose that current *does* flow throughout the volume of the wire, in such a way that the volume current density J grows quadratically with distance from the central axis, what then is the formula for **J** everywhere in the wire?
c) A CD has been rubbed so that it has a fixed, constant, uniform surface electric charge density σ everywhere on its top surface. It is spinning at angular velocity ω about its center (which is at the origin). What is the surface current density **K** at a distance r from the center?
d) A sphere (radius R, total charge Q uniformly distributed throughout the volume) is spinning at angular velocity ω about its center (which is at the origin) What is the volume current density **J** at any point (r, θ, ϕ) in the sphere?
e) A very thin plastic ring has a constant linear charge density, and total charge Q. The ring has radius R and it is spinning at angular velocity ω about its center (which is at the origin). What is the current I, in terms of given quantities? Also, how would you write down the volume current density **J** in cylindrical coordinates? (This may be a little tricky, since the ring is "very thin", there will be some delta functions involved. I suggest trying to write down a formula for $\rho(r,\phi,z)$ first. And, remember that **J** should be a vector!)

Q5. MAGNETIC MOTION:
Griffiths Problem 5.2. For each part - is kinetic energy constant with time? (Where it isn't, very briefly explain) Also, for part a: the result is in many respects quite special - give some physical example(s) where this result could be useful for some practical purpose.

Q6. MAGNETIC FIELD:
An infinitely long wire has been bent into a right angle turn, as shown in the figure.
The "curvey part" where it bends is a perfect quarter circle, radius R.
Point P is exactly at the center of that quarter circle.
A steady current I flows through this wire.

Find the magnetic field at point P (magnitude and direction)



EXTRA CREDIT: A conducting ball bearing (radius R) is coated with a dielectric paint (dielectric constant ε_r), which extends out to a radius R'. (The coating is very thick, so R' is measurably larger than R) This object sits in a uniform external electric field, $\mathbf{E_0}$.
Find the electric field *in* the "coat" region.
This problem lets you work out for yourself the various methods of Ex. 4.7 of Griffiths, with slightly different boundary conditions. One hint/warning: you might be temped to apply the equation for "ε dV/dr" at the boundary between conductor and paint. This is not a good idea, because the conductor is polarized, so there ARE "free" charges at this boundary (which you don't know yet!) But it's ok - you simply don't NEED that one particular boundary condition. (If you invoke it, you will learn *the charge distribution on the conductor: perhaps interesting, but not what we're asking for) You can (and will) invoke the ε dV/dr boundary condition at the OUTER boundary (dielectric to air), which is fine, since there are no free charges there!*