Phys 3310, HW #13, Due in class Wed Apr 30

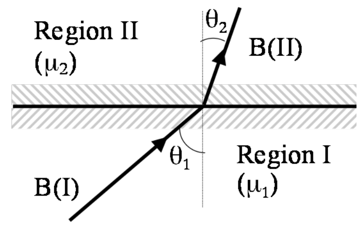
Q1. B AND H IN A CAVITY (Average: 80%, Median: 85%, Stddev: 28.79%, % Score: 18.18%) – CH6 – linear and nonlinear media

Find **B** and **H** at the center of a hollow spherical cavity carved out of a *large* chunk of uniform, linear, magnetic material (susceptibility χ_m) which has total field $\mathbf{B} = B_0\,\hat{\mathbf{z}}$ through its volume, where B_0 is the magnetic field prior to the carving of the cavity.
(So, this material will be magnetized, and will have a uniform $\mathbf{H}_0 = (1/\mu_0)B_0\,\hat{\mathbf{z}} - M_0\,\hat{\mathbf{z}}$)
(Express your answer in terms of B_0)
Note: In my lecture notes I did two similar examples for a "needle-like" cavity, and a "wafer-like" cavity. However, for this problem, you might prefer to think of the problem as the superposition of a large totally uniform magnetized system with a sphere of uniform but opposite magnetization) This problem could help you to "model" magnetic materials - knowing the B field in the cavity would tell you how a single atom placed there would magnetize...

Lots of people were rather confused about how to proceed on this problem and they were making it into a much harder problem than it needed to be. Many were really struggling with the basic idea of superposition, not recognizing that "cavity in material = solid uniform material + reversed sphere". Several people expressed surprise that if you could picture the magnetization as a superposition of physical materials, that the B field which results would be the same simple superposition of those materials.

Q2. BOUNDARY CONDITIONS FOR B AND H FIELDS (Average: 78.89%, Median: 85%, Stddev: 26.32%, % Score: 18.18%) – CH6 – boundary value problems

In class awhile ago, we considered the situation of a static E field spanning a boundary between two different materials (with different dielectric constants, ε)
Do the same thing with static B fields: in the configuration shown in the figure, assuming medium one has relative magnetic permeability μ_1, and medium two has permeability μ_2, find the ratio $\tan\theta_2/\tan\theta_1$. Please show/explain your work clearly.
(Find the ratio in terms of μ_1 and μ_2, that should be simplest. Assume there is no free current anywhere in the figure).



• In the figure as shown, if one of the regions is vacuum, and the other one is paramagnetic, which is which? (I.e. which region is vacuum, region I or region II?) How about if one is vacuum, and one is diamagnetic, then which is which? Briefly explain.

We saw no questions on this problem.

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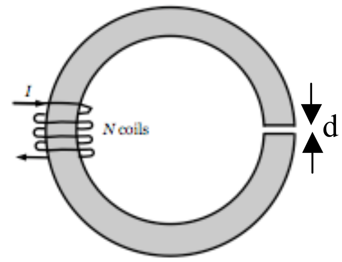
Q3. CONTINUITY B AND H FIELDS (Average: 73.89%, Median: 100%, Stddev: 38.83%, % Score: 18.18%) – Ch6 – boundary value problems



A toroidal piece of "soft iron" (iron that is roughly linear, but has a very large permeability μ characteristic of ferromagnetic materials) has a very thin gap in it, of width "d". A wire carries current I , and is wrapped N times around a section of the toroid. The toroid has a constant cross-sectional area A.
Find the B and H fields in the gap.
You may assume many things: that B (and H and M) inside the soft iron are quite uniform, smooth, and continuous all the way around... except of course in the gap. (Which is continuous through THAT little region, H or B?) Assume further that fringe fields are negligible, and that $\mu/\mu_0 >> 1$. *And lastly, assume that* $\mu/\mu_0 >> (2\pi R/d)$, *in other words that although the gap may be reasonably small,* μ/μ_0*(iron) is really quite huge, typically of order 1000.*

This was a good, tough question which generated a lot of enthusiasm and discussion. People appreciated the puzzle-like nature of it. Some people confused "toroidal piece of iron" with the "toroid" that Griffiths solves for, where the windings go around the whole toroid, and erroneously used this result. People just weren't sure what Amperian loop to draw, what continuity to use/assume. Taking the limit was also a challenge. When pushed to discuss/interpret their final answer, they did a good job overall of seeing what was interesting about this little setup (the "B-magnifying" aspect of it)

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Q4. POWERFUL MATH NOTATION (Average: 88.89%, Median: 100%, Stddev: 25.97%, % Score: 27.27%) – ch6 – boundary value problems.

Do Griffiths Problem 6.22

Comment: This is a derivation of Eq. 6.3 (which I never proved in class or notes!), i.e. proving $\mathbf{F} = \nabla(\mathbf{m} \bullet \mathbf{B})$. *I think you will find this to be a long, challenging problem. Proceed with care, take your time, use lots of paper. Think hard about symbols and notation. Don't give up, the proof "solves itself" if you keep on going! Although this method may look pretty awful to you, the methods developed in this problem are really THE WAY that many proofs (in many physics courses you will take in the future) are done! You may have seen this* method *before - but if not, you introduce numerical subscripts (1,2,3) instead of (x,y,z) to describe Cartesian coordinates, and you introduce the "Kronecker delta",* δ_{ij} *which is a shorthand, DEFINED to be +1 if i=j, and 0 otherwise. It's a convenient notation - you'll see it many times in the future. Same thing for the "Levi-Civita" symbol,* ε_{ijk}, *(which is defined in this Griffiths problem). It takes getting used to, but once you've practiced a little with Levi-Civita, Kronecker delta, and this "ijk" notation, many proofs (like e.g. the identities in the front flyleaf) become mechanical, and thus easy. It becomes mere "careful bookkeeping" to prove many otherwise formidable things!*

This one generated LOTS of questions, just about every single group struggled with it. However, they seemed to be enjoying it, there was a sense of either pride or at least desire to deal with it. I think having a more elementary question leading up to this might have helped - but I still think it was a great problem to end the term with, and worth having them wrestle with.

Many groups struggled right away with the first step, before getting to the notational issue, they didn't see why the (two) B0 terms vanished in Griffiths first step. They were not at all clear about what variables were being integrated over, or how you make sense of "pulling the "cross B0" out of the integral" idea. Then came the Kronecker delta/Levi-Cevita part. This was the particularly difficult portion. One student suggested we make up a simpler problem to get used to the notation. It was a good idea. So we worked on proving one of the triple product rules from the front flyleaf of Griffiths. We had to talk about dummy indices, and the cylic aspec of the Levi-Cevita symbol.