Phys 3310, HW #11, Due in class Wed Apr 16

General Comment:

Edwin: **I think students are doing great! People who tried every problem tend to get above 70%. Also, it seems that most of them did the discussion this time, probably because you said it explicitly in the problem set? I find it interesting there's a tendency that students did the discussions from #1A, #1B, #5A&B; but some of them didn't discuss "showing that A works" in part #1A, #1B, #4B (basically those with the question word "Check" or "Show" instead of "To discuss" or "why".**

**Q1.** VECTOR POTENTIAL I (Avg: 71.75%, Median: 70%, Stddev: 18.30%, % Score: 20%)
A) (Partial Pts: 10, Avg: 6.85, Median: 6.5, Stddev: 1.76) - CH5 – VECTOR POTENTIAL
A long straight wire of radius R carries uniform current $I_0$ in the +z direction (basically like the previous question, but without any hole drilled in it). Assuming $\nabla \bullet \vec{\mathbf{A}} = 0$ (the "Coulomb gauge"), and choosing A=0 at the edge of the wire, show that the vector potential *inside the wire* could be given by $A= c\, I_0(1-s^2/R^2)$. Find the constant c (including units.)
Things to explicitly find/discuss: What is the vector direction of **A**?
Is your answer unique, or is there any remaining "ambiguity" in **A**?
*(Note that I'm not asking you to derive **A** from scratch, just to see that this choice of **A** "works")*
B) (Partial Pts: 10, Avg: 7.5, Median: 9, Stddev: 2.59) What is the vector potential *outside* that wire? (Make sure that it still satisfies $\nabla \bullet \vec{\mathbf{A}} = 0$, and make sure that **A** is continuous at the edge of the wire, consistent with part a)
Here again, is your answer unique, or is there any remaining "ambiguity" in **A(**outside)?

Many **students did not try to convince themselves that this form of A does work. A few students did not use the boundary condition that A(s=R)=0, thus missing the "R" in the Ln.**

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**Q2.** VECTOR POTENTIAL II (Avg: 73.5%, Median: 80%, Stddev: 25.65%, % Score: 20%) – CH5 – VECTOR POTENTIAL

Griffiths Fig 5.48 (p. 240) is a nice, and handy, "triangle" summarizing the mathematical connections between **J**, **A**, and **B** (like Fig. 2.35 on p. 87) But there's a missing link, he has nothing for the left arrow from **B** to **A**. Notice that the *equations* defining **A** are really very analogous to the basic Maxwell's equations for **B**:

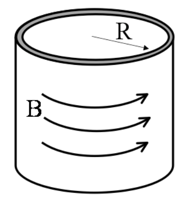
$\nabla \bullet \vec{\mathbf{B}} = 0 \quad \Leftrightarrow \quad \nabla \bullet \vec{\mathbf{A}} = 0$

$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \quad \Leftrightarrow \quad \nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$

So **A** depends on **B** in the same way (mathematically) the **B** depends on **J**. (Think, Biot-Savart!) Use this idea to just write down a formula for **A** in terms of **B** to finish off that triangle.

A) (Partial Pts: 10, Avg: 8.6, Median: 10, Stddev: 3.17)

B) (Partial Pts: 10, Avg: 6.1, Median: 7, Stddev: 2.63) In lecture notes (and/or Griffiths Example 5.12) we found the B field everywhere inside (and outside) an infinite solenoid (which you can think of as a cylinder with uniform surface current flowing around it in an azimuthal fashion. See Griffiths Fig 5.35, p. 227, to see what I mean) Use the basic idea from the previous part of *this* question to, therefore, quickly and easily just write down the vector potential **A** in a situation where **B** looks analogous to that, i.e. $\vec{\mathbf{B}} = C\delta(s-R)\,\hat{\varphi}$, with C constant.




*(You should be able to just "see" the answer, no nasty integral needed!)*

What physical situation creates such a B field? (This is tricky - think about it!)

*It's kind of cool - think about what's going on here. You have a previously solved problem, where a given **J** led us to some **B**. Now we immediately know what the vector potential is in a* very different *physical situation, one where **B** happens to look like **J** did in that previous problem.*

If I gave you a B field and asked for **A**, can you now think of an "analogue method", i.e. an experiment where you could let nature do this for you, instead of computing it?

This is a tough question. Many **people had trouble understanding the analogy. For a few, I had to explicitly point out the analogy, drawing the diagram above and then drawing a solenoid so that it popped out at you. Note that this analogy is part of the tutorial, but students were still struggling with it. Many said that since $\nabla^2 \vec{A} = \vec{B}$ then they plugged in formula to find that $\vec{A} = \int \nabla \times \vec{B}$. This works, and you can use a vector identity to arrive at $\vec{A} = \int J \times r$.**

**Lots of questions on the physical situation in part B, and many needed instructor coaching. Many students forgot that A=constant is only INSIDE the solenoid. Also the majority give a solenoid-like picture of A and B without sketching a graph.**

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**Q3.** SQUARE LOOP - FAR AWAY (Avg: 76%, Median: 85%, Stddev: 27.13%, % Score: 20%) – CH6 – MAGNETIC DIPOLES

A) (Partial Pts: 11, Avg: 9.15, Median: 9.5, Stddev: 2.64) Last week we considered a square current loop (current I running around a wire bent in the shape of a square of side a) sitting flat in the x-y plane, centered at the origin. (You found B at the center). Now redo that problem but find **B**(0,0,z) (mag and direction), i.e. a distance z above the center. (Check yourself by setting z=0 and making sure you get the result from last week!)

B) (Partial Pts: 9, Avg: 6.05, Median: 6, Stddev: 3.44) Take your result from part A and now take the limit z>>a, finding an approximate simple formula for $B_z$ (0,0, large z). Do the same for a circular current loop of radius a, and compare. (The exact expression is derived in Griffiths, Eq. 5.38, you don't have to rederive that, just consider the large-z limit) Check both answers by comparing with Griffiths dipole approximation (Eq. 5.86), looking only along the +z-axis of course.

*Notice your two expressions differ only by the constant out front, which should go like* $m = I \times$ (area of loop) *This is the magnetic dipole moment!*

Most student **questions here were about taking the limit of B as z>>a, including some confusion about what parameter is small. Students are not universally comfortable with the binomial expansion (even though one can simply approximate in this problem without doing the whole expansion). Half the students did this problem by integrating from Biot-Savart.**

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**Q4.** FIELDS AND STRENGTHS(Avg: 64%, Median: 77.5%, Stddev: 37.30%, % Score: 20%)
A) (Partial Pts: 10, Avg: 6.8, Median: 8, Stddev: 3.53) – CH6 – MAGNETIZATION
Find the density ρ of mobile charges in a piece of gold-wire speaker wire, assume each atom contributes one free electron. (Look up any necessary physical constants!) Then, think about the definition of current, and *estimate* the average electron speed in a gold speaker wire of ordinary household size carrying an ordinary household current. *Your answer will come out quite slow - it might surprise you.* If you flip on the stereo, and the speakers are, say, 2 meters away, would there be a noticeable "time lag" before you hear the speaker come on? Why/why not?

B) (Partial Pts: 10, Avg: 6, Median: 8, Stddev: 4.22) If you cut open this wire, you'll see it is really two wires, each insulated, and wrapped close together in a single plastic cylinder (since you need a complete circuit, current has to flow TO and FROM the speaker, right?). Making reasonable guesses for the dimensions involved in a real wire, estimate the TOTAL magnetic force between the "outgoing" and "return" wires. Is it attractive or repulsive? Now - if you could somehow remove the stationary positive ions in the metallic conductor (which play no role in the *flow of current,* right?), make a rough estimate for the total *electrical* force of repulsion between the two wires. How does it compare with the magnetic force you just found. Does this seem at all plausible?

As always, students are uncomfortable with estimation, and part (A) generated many questions. At least some students did not know how **physical constants about gold could give them an estimate for how many atoms were in a volume. The error on the estimation can be quite large (1-2 orders of magnitude) with an acceptable approximation scheme. The problem could be amended to state in part B that the length of the cable is assumed to be 2m.**

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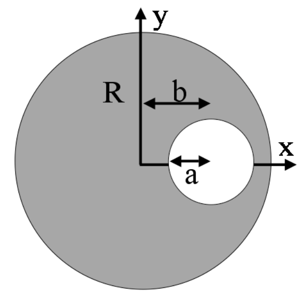


**EXTRA CREDIT** (Avg: 90%, Median: 90%, Stddev: 14%, out of 3 students that did it) – ch5 - ampere

AMPERE and SUPERPOSITION

A) A long (infinite) wire (cylindrical conductor of radius R, whose axis coincides with the z axis) carries a uniformly distributed current $I_0$ in the +z direction.

A cylindrical hole is drilled out of the conductor, parallel to the z axis, (see figure for geometry). The center of the hole is at x = b, and the radius is a. Determine the magnetic field *in the hole region*.

B) If this is an ordinary wire carrying ordinary household currents, and the drilled hole has dimensions roughly shown to scale in the figure above, make an order of magnitude estimate for the strength of the B field in that region. How does it compare to the earth's field?

*You should find that the B field in the hole is uniform - that was just a little surprising to me!*