Phys 3310, HW #11, Due in class Wed Apr 16

Q1. VECTOR POTENTIAL I

A) A long (infinite) wire (cylindrical conductor, radius R, whose axis coincides with the z axis) carries a uniformly distributed current I_0 in the +z direction. Assuming $\nabla \bullet \vec{\mathbf{A}} = 0$ (the "Coulomb gauge"), and choosing A=0 at the edge of the wire, show that the vector potential *inside the wire* could be given by $A = c\, I_0(1-s^2/R^2)$. Find the constant c (including units.)
Things to explicitly find/discuss: What is the vector direction of **A**? (Does it "make sense" in any way to you?) Is your answer unique, or is there any remaining "ambiguity" in **A**?
*(Note that I'm not asking you to derive **A** from scratch, just to see that this choice of **A** "works")*

B) What is the vector potential *outside* that wire? (Make sure that it still satisfies $\nabla \bullet \vec{\mathbf{A}} = 0$, and make sure that **A** is continuous at the edge of the wire, consistent with part a)
Here again, is your answer unique, or is there any remaining "ambiguity" in **A(**outside)?

Q2. VECTOR POTENTIAL II

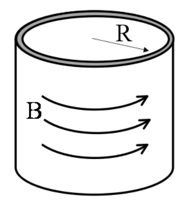
A) Griffiths Fig 5.48 (p. 240) is a nice, and handy, "triangle" summarizing the mathematical connections between **J**, **A**, and **B** (like Fig. 2.35 on p. 87) But there's a missing link, he has nothing for the left arrow from **B** to **A**. Notice that the *equations* defining **A** are really very analogous to the basic Maxwell's equations for **B**:

$$\nabla \bullet \vec{\mathbf{B}} = 0 \quad \Leftrightarrow \quad \nabla \bullet \vec{\mathbf{A}} = 0$$

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \quad \Leftrightarrow \quad \nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$$

So **A** depends on **B** in the same way (mathematically) the **B** depends on **J**. (Think, Biot-Savart!) Use this idea to just write down a formula for **A** in terms of **B** to finish off that triangle.

B) In lecture notes (and/or Griffiths Example 5.9) we found the B field everywhere inside (and outside) an infinite solenoid (which you can think of as a cylinder with uniform surface current flowing around it in an azimuthal fashion. See Griffiths Fig 5.35, p. 227, to see what I mean) Use the basic idea from the previous part of *this* question to, therefore, quickly and easily just write down the vector potential **A** in a situation where **B** looks analogous to that, i.e. $\vec{\mathbf{B}} = C\delta(s-R)\,\hat{\varphi}$, with C constant. (Sketch this **A** for us, please) *(You should be able to just "see" the answer, no nasty integral needed!) It's kind of cool - think about what's going on here. You have a previously solved problem, where a given **J** led us to some **B**. Now we immediately know what the vector potential is in a* very different *physical situation, one where **B** happens to look like **J** did in that previous problem.*



To discuss: What physical situation creates such a B field? (This is tricky - think about it!) Also, if I gave you some B field and asked for **A**, can you now think of an "analogue method", i.e. an experiment where you could let nature do this for you, instead of computing it?

Q3. MULTIPOLES - DIPOLE MOMENT

Do Griffiths Problem 5.33, which amounts to finding a general (coordinate free) formula for **B**, given the dipole vector potential $\vec{\mathbf{A}} = \dfrac{\mu_0}{4\pi}\dfrac{\vec{\mathbf{m}} \times \hat{\mathbf{r}}}{r^2}$.

Note: I think the easiest way to do this one is by assuming 5.87, and then showing *that you get 5.86 if you define your z axis to lie along **m**, rather than trying to go the other way around. Coordinate free formulas are nice, because now you can find B for more general situations!*

Q4. SQUARE LOOP - FAR AWAY

A) Last week we considered a square current loop (current I running around a wire bent in the shape of a square of side a) sitting flat in the x-y plane, centered at the origin. (You found B at the center). Now redo that problem but find **B**(0,0,z) (mag and direction), i.e. a distance z above the center. (Check yourself by setting z=0 and making sure you get the result from last week!)

B) Take your result from part A and now take the limit z>>a, finding an approximate simple formula for B_z (0,0, large z). Do the same for a circular current loop of radius a, and compare. (The exact expression is derived in Griffiths, Eq. 5.38, you don't have to rederive that, just consider the large-z limit) Check both answers by comparing with Griffiths dipole approximation (Eq. 5.86), looking only along the +z-axis of course.
Notice how simple everything gets far away, and how your two expressions for very different wire shapes differ only by the constant out front, which should go like $m = I \times$ (area of loop)
This is the magnetic dipole moment!

Q5. FIELDS AND STRENGTHS

A) Find the density ρ of mobile charges in a piece of gold-wire speaker wire, assume each atom contributes one free electron. (Look up any necessary physical constants!) Then, think about the definition of current, and *estimate* the average electron speed in a gold speaker wire of ordinary household size carrying an ordinary household current. *Your answer will come out quite slow - it might surprise you.* If you flip on the stereo, and the speakers are, say, 2 meters away, would there be a noticeable "time lag" before you hear the speaker come on? Why/why not?

B) If you cut open this wire, you'll see it is really two wires, each insulated, and wrapped close together in a single plastic cylinder (since you need a complete circuit, current has to flow TO and FROM the speaker, right?). Making reasonable guesses for the dimensions involved in a real wire, estimate the TOTAL magnetic force between the "outgoing" and "return" wires. Is it attractive or repulsive? Now - if you could somehow remove the stationary positive ions in the metallic conductor (which play no role in the *flow of current,* right?), make a rough estimate for the total *electrical* force of repulsion between the two wires. How does it compare with the magnetic force you just found. Does this seem at all plausible?

Q6. MORE MAGNETIC DIPOLES

Do Griffiths Problem 5.56. *(For a token point of extra credit, find the current best-value for this electron magnetic dipole moment. Let me know where you got the results, and how recent they are. If you compare theory and measurement, I think you will be extremely impressed at the agreement, it may make you "believe" in quantum physics in a way you might not have before! Although that's not how it works in practice - people usually use this measurement to extract a particular fundamental constant of nature, and then use that value to predict OTHER experiments, since this is one of the most precise measurements you can make in subatomic physics)*

A more usual extra credit problem is on the next page!

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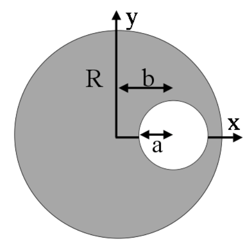
Extra Credit: AMPERE and SUPERPOSITION

A) A long (infinite) wire (cylindrical conductor, radius R, whose axis coincides with the z axis) carries a uniformly distributed current I_0 in the +z direction. (Basically, like question #1). But now, a long (infinite) cylindrical hole is drilled out of the conductor, parallel to the z axis, (see figure for geometry). The center of the hole is at x = b, and the radius is a. Determine the magnetic field *inside the hole region*.



B) If this is an ordinary wire carrying ordinary household currents, and the drilled hole has dimensions roughly shown to scale in the figure above, make an order of magnitude estimate for the strength of the B field in that region. How does it compare to the earth's field?
You should find that the B field in the hole is uniform - that was just a little surprising to me!
The "comparing with B(earth)" question is always interesting to me from a health perspective, I'd say that sets the scale for fields that surely can't be worth worrying much about!