Phys 3310, HW #5, Due in class Wed Feb 13.

Total HW Stats: Ave 68% w/o EC; median 83.5.

Q1. ENERGY OF POINT CHARGE DISTRIBUTION (Ave 8.6; median 10) – Ch2 - energy
Imagine a small square (side "a") with four point charges +q, one on each corner. Calculate the total stored energy of this system (i.e. the amount of work required to assemble it).
Then, calculate how much work it takes to "neutralize" these charges by bringing in *one* more point charge (-4q) from far away and placing it right at the *center* of this square.
(When studying crystal structures, it is sometimes convenient to model them as rectangular grids of charged ions, this problem forms the starting point for such a model)

This seemed to be an easy question for students.

Q2. ENERGY OF A CONTINUOUS CHARGE DISTRIBUTION – ch2 - energy
a) Find the total electrostatic energy stored in a uniformly charge sphere of radius R and charge q. (Note: this is uniform throughout the whole volume, it's not a shell) Express your answer in terms of q, R, and constants of nature. There are many different ways to do this, I want you to use two *different methods* so you can check yourself. The most obvious are:
i) figure out E(r) and then use Griffiths Eq. 2.45 (being careful to integrate over *all space*, not just where the charge is! Think about that for yourself, it's quite important)
AVERAGE 8.4; MEDIAN 10
ii) figure out V(r), use Griff. 2.43 (what region do you need to integrate this over?)
AVERAGE 7.2, MEDIAN 8
There are still other ways if you prefer - e.g. Griffiths Eq 2.44. Or you could build up the charge "shell by shell" and explicitly integrate the work to build each thin shell, to find the total work done building the complete sphere. But two methods is enough for me!

b-i) According to Einstein, a static electron has a rest mass energy $E = mc^2$. It is tempting (though wrong!) to imagine that the electron is NOT really a point charge, but instead is a tiny sphere, with uniform charge density, whose total electrostatic energy (found above) EQUALS this mass energy of the electron. (In other words, the idea would be that the rest energy of the electron is purely electromagnetic).
What radius would an electron have to have, for this to work out?
You will need to look up the charge and mass of the electron. Don't take your result too literally, we are not taking quantum mechanics into account at all here!

ii) Here's another application of part a: compute the electrostatic energy of an atomic nucleus with Z protons and a total of A nucleons, using an approximation for the nuclear radius as $R = (1.2 \times 10{-}15\text{ m})\, A^{1/3}$. Convert your answer (which is likely in SI units of Joules) to the energy unit of MeV, which is Mega eV. (Give your result in units of MeV times $Z^2 / A^{1/3}$) Now use this result to estimate the *change* of electrostatic energy when a uranium nucleus undergoes fission into two roughly equal-sized nuclei. Go online and check what the experimental answer is - what does this tell you about the source of the energy released in a nuclear explosion - is it mostly electrostatic? (You might be surprised by your answer!)

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(a) There was a lot of confusion on this question. Such as, why can't we use V(r=0)=0? Why do you stop integrating rho *V at the edge, when there is clearly more energy "outside". Why do we integrate over all space of (a) but just the charge density for (b)? We had to move charge through all space in order to create the charge distribution, so why didn't we have to integrate over the space outside the sphere? Which V do we use – V(r) or V(R)? Some wanted to use V(R) even outside the sphere. Students are having a lot of difficulty conceptualizing the formulas for energy physically.

Also, most students calculated V(r) by setting V(0) = 0, and just integrating from r=0 to r=r. Firstly, they didn't recognize that this was *different* from what we usually do (V=0 at infinity) or that this was what they were implicitly assuming by doing that integral. The reason that didn't work was because in the formula for energy, the voltage is implicitly set to zero at infinity, but that formula is not transparently understood.

(b) Quite a few questions about the *language* in ii What is a "nucleon"? What is "A"? Why A^1/3? The problem as stated was just a little confusing, they weren't quite sure how to get started or what exactly I was asking. This problem could be reworded. Also, some students started to use Coulomb's Law (instead of Gauss) to calculate E inside, and were still unsure how to set up Gauss' Law.

Q3. GAUSS' LAW AND CAVITIES: Griffiths 2.36 (p. 101) – Ch2 – conductors (Nina, keep this one)
AVERAGE 9.5; MEDIAN 10
Please be sure to explain your reasoning on all parts, and add to this the problem the following:
In part a, be sure to *sketch* the charge distribution.
In part c, *sketch* the E fields (everywhere in the problem, i.e. in the cavities and also outside the big sphere)
f) Lastly - if someone moved q_a a little off to one side, so it was no longer at the *center* of its little cavity, which of your answers would change? Please explain.
I really like this problem, lots of good physics in it! Try to think it through carefully, make physical sense of all your answers, don't just take someone else's word for it! ‘’

Many people can get the right answer but not explain why. Especially problematic was the "force of q1 on q2" question.

Q4. AVERAGE 8.25; MEDIAN 10 – ch2 - conductor

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In HW 3 you found the E field everywhere in and around a coaxial cable. We can slightly modify that problem, making it more realistic by letting the inner cylinder be a conductor (a wire!) So you have an inner conducting cylinder (radius "a") and an outer conducting cylindrical shell (inner radius "b"). It is physically easy to then set up any fixed potential difference ΔV between the inner and outer conductors that you like.

a) Assuming these are infinite cylinders, find the energy stored per unit length inside this capacitor. Once again, let's do it two ways so we can check:

i) Integrate the energy density stored in the E field (like method i, in Q2 above)

ii) Find the capacitance C of this system, and then use stored energy = $\frac{1}{2} C(\Delta V)^2$

Based on your answers above, *where* in space would you say this energy is physically stored?

b) This is also an excellent model for "axons", which are long cylindrical cells (basically coax cables!) carrying nerve impulses in your body and brain. Using your result from part ii above, estimate the *capacitance* (in SI metric units, which are Farads) of your sciatic nerve.

Assumptions - the sciatic nerve is the longest in your body, it has a diameter of roughly 1 micron, and a length of perhaps 1 m. Note that axons generally have a value of b which is very close to a (i.e. the gap is extremely tiny, b-a is about 1 nanometer.) so you can simplify your expression using $\ln(1+\varepsilon)\approx\varepsilon$

Some students didn't notice that the question was "energy/length" and were concerned about the infinite amount of energy in an infinitely long cylinder.

Extra credit (worth half of any of the above, but *won't* count off if you don't do it) CHILD-LANGMUIR LAW AND SPACE CHARGE: Solve Griffiths problem 2.48.

This problem pull together a lot of physics (including from Phys 1110 and 1120). It was also once a very physically important result, at least in the days of "tube" diodes. It's a little challenging - including a differential equation you have to solve that might not be immediately familiar to you. Work on it, you'll get it! Here's a clue/trick to solving that little diff eq: you can always TRY a solution of the form $V(x) = a x^n$. *Plug it in, see what happens - it won't work, unless n has a very special value, (and a has to be just right also), and there you go! The other part which you might find tricky/unfamiliar is part c, but that's an old 1120 formula which is quite useful and important. You can just use units, current depends on density and velocity of charge carriers in a simple way...*