

**TOTAL HW STATS (Mean 70.97%, Median 79.21% , Standard Deviation 21.93)**

**Q1. BOUND CHARGES (avg 87.94%, median 100, stddev 23.53) – ch4 – field of polarized object**

Consider a long teflon rod, (a dielectric cylinder), radius  $a$ . Imagine that we could somehow set up a permanent polarization  $\mathbf{P}(s,\theta,z) = ks$  ( $= ks\hat{s}$ ), where  $s$  is the usual cylindrical radial vector from the  $z$ -axis, and  $k$  is a constant). Neglect end effects, the cylinder is long.

- (Max: 5; avg 4.67, median 5, stddev 0.73) Calculate the bound charges  $\sigma_b$  and  $\rho_b$  (on the surface, and interior of the rod respectively). What are the units of " $k$ "?
- (Max 5; avg 4.43, median 5, stddev 1.21) Next, use these bound charges (along with Gauss' law) to find the electric field inside and outside the cylinder. (Direction and magnitude)
- (Max 5; avg 4.09, median 5, stddev 1.73) Finally, find the electric displacement field ( $\mathbf{D}$ ) inside and outside the cylinder, and verify that Griffiths' Eq 4.23 (p. 176) is correct. Explain briefly in words why your answer might be what it is.

**This problem appears to be useful practice, though most students in the transformed class did not have conceptual problems with bound charge. Some confusion about whether  $\mathbf{D}$  should vanish in those two problems (and why); we spent some time in several groups talking about "when can you get  $\mathbf{D}$  easily" and when can't you?**

**Q2. BOUND CHARGES II (avg 89.84%, median 100, stddev 0) – ch4 – electric displacement**

Now let's hollow out that teflon rod, so it has inner radius  $b$ , and outer radius is (still)  $a$ . Just to make things a little different here, suppose I now set up a different polarization within the teflon material, namely  $\mathbf{P}(s,\theta,z) = \frac{k}{s^2}\hat{s}$  for  $b < s < a$ .

- (max 10; avg 8.9, median 10, stddev 2.28) I'm assuming we have vacuum for  $s < b$  and  $s > a$ . What does that tell you about  $\mathbf{P}$  in those regions? Find the bound charges  $\sigma_b$  and  $\rho_b$  ( $\sigma_b$  on the inner AND outer surfaces of the hollow rod, and  $\rho_b$  everywhere else. Use these bound charges, along with Gauss' law, to find the electric field everywhere in space. (Direction and magnitude)
- (max 5; avg 4.57, median 5, stddev 1.25) Use Griffiths' Eq 4.23 (p. 176) to find  $\mathbf{D}$  everywhere in space. (This should be quick - are there any *free* charges in this problem?) Use this result for  $\mathbf{D}$  (along with Griffiths basic definition/relation of  $\mathbf{E}$  to  $\mathbf{D}$ , Eq 4.21) to find  $\mathbf{E}$  everywhere in space. (*This should serve as a check for part a, and shows why sometimes thinking about  $\mathbf{D}$  fields is easier and faster!*)

**Students need this practice with reasoning the direction of  $\mathbf{E}$ ,  $\mathbf{D}$ , and  $\mathbf{P}$ .**

**Q3. FIELD PATTERNS FROM BOUND CHARGES** (avg 62%, median 70, stddev 32.34 – Ch4 – Electric displacement

Let's go back to the "solid" teflon rod (radius  $a$ ), but let's change the polarization once again - now assume it is uniform and parallel to the  $z$ -axis, i.e.  $\mathbf{P} = k \hat{z}$ . This time, let's also keep it finite, and consider realistic "end effects"

a) (max 5; avg 4.24, median 5, stddev 1.55) Sketch the electric field lines, inside and outside, in two different limits: First, when the cylinder is very LONG compared to its diameter, and second, when it is very SHORT compared to its diameter. Explain in words and formulas your physical reasoning behind the sketches.

b) (max 10; avg 7.09, median 8, stddev 3.08) In both of the above two limiting cases, sketch (and explain in words)  $\mathbf{P}$  everywhere in space, and also  $\mathbf{D}$  everywhere (inside and outside the rods)

c) (max 10; avg 4.90, median 6, stddev 4.17) In both of the above limits, explain how Griffiths' "boundary condition" (page 178) Eq 4.26 and 4.28 work out near the top end of the rod.

Also (just briefly, *very* qualitatively, e.g. simply in terms of signs or directions of these fields) explain how boundary conditions 4.27 and 4.29 makes sense/agree with your sketches in part b. (Here, you only need to really consider the outer edge of the middle of the cylinder at  $z=0$  i.e. at  $(s=a, z=0)$  if the rod runs from  $z=-L$  to  $z=+L$ )

*Such an object is referred to as a "bar electret", it is an electrical analogue of a permanent bar magnet. Do you see why, from your sketches? Griffiths points out that this is quite unusual, most materials cannot maintain such a permanent electric polarization in the absence of an external field, although he claims they do exist. But if you did have one, it would attract charged ions from the air which would negate the interesting fields you found above.*

**This is a hard problem for students and many leave it blank. Some students struggle with how to find the  $\mathbf{E}$  field or  $\mathbf{D}$  field. This was especially troublesome for the long thin rod, where they hadn't realized they could estimate  $\mathbf{E}$  in various "limiting regions" and thence get  $\mathbf{D}$ . Many think that  $\mathbf{E}$  field is uniform inside the rod in the limiting case of a long rod.**

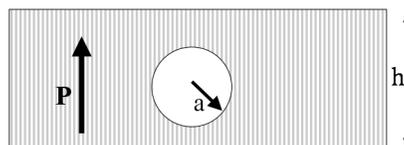
**Q4. MATH OF BOUND CHARGES** (avg 91.43%, median 100, stddev 13.52) – ch4 – bound charges

Using Griffiths' formal definition of bound charge distributions, eqns 4.11 and 4.12 (p. 167-8) prove (mathematically) that if you have a dielectric which starts off neutral, and then it gets polarized, that the total bound charge is still exactly zero. Explain how this formal mathematical result makes (simple) physical sense. Look back at all previous questions, does the total charge in those problems satisfy what you found here?

**This is a fairly easy problem for students.**

**Q5. SUPERPOSITION** (avg 69.29%, median 70, stddev 29.0) – ch4 – field of polarized object

A large slab (infinite in the  $x$ - $y$  directions) of dielectric material has thickness  $h$ , and has uniform polarization  $\mathbf{P} = k \hat{z}$ . The polarization is perpendicular to the surfaces of the slab. At the



midplane of the sheet is a spherical bubble, of radius  $a$ . The inside of the bubble contains vacuum: it has zero polarization.

*Hint: Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization!*

a) (max 5; avg 3.62, median 5, stddev 1.77) Assume there are no free charges anywhere (so this polarization  $\mathbf{P}$  is just spontaneous), find the net electric field everywhere *inside* the bubble.

b) (max 10; avg 6.86, median 6, stddev 3.35) Assume there are still no free charges anywhere. Find the bound surface charge on the surface of the bubble. Then, find (or describe) the electric field *outside the slab*.

c) (max 5; avg 3.38, median 4, stddev 1.88) Now assume that there is an external field  $\mathbf{E}_{\text{ext}}$  which *causes* that (given) uniform polarization  $\mathbf{P}$ . What is now the net electric field everywhere *inside* the bubble?

*Hint: I can think of two ways to do part b. Check out Example 3.9 and Example 4.2 of Griffiths, I believe the result of either one could provide a lot of help for this part...*

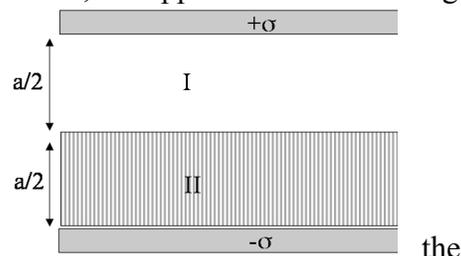
**This is a hard problem. Lots of confusion about the approach, whether they had to rederive the results from Griffiths for a polarized sphere, how/what superposition means... Lots of students try to solve the Laplace equation by using separation of variables, but few of the them got the right answer.**

**Q6. LINEAR DIELECTRIC, RECTANGULAR BOUNDARY** (avg 79.09%, median 95, stddev 95) – ch4 – linear dielectrics

You have a large parallel-plate conductor (two big metal sheets, the upper one has free charge density  $+\sigma$ , the lower one has  $-\sigma$ )

They are a distance  $a$  apart.

The space between the sheets is half filled with a dielectric oil. Region I, (filling the top half of the volume) is vacuum. The lower half, region II, has dielectric constant  $\epsilon_r=3.0$



a) (max 10; avg 8.81, median 10, stddev 2.18) Find

the electric displacement (" $\mathbf{D}$  field") (direction and magnitude) in regions I and II, the electric field  $\mathbf{E}$  in both regions, and the polarization  $\mathbf{P}$  in both regions

b) (max 5; avg 3.86, median 5, stddev 1.82) Find the location and amount of bound charge (surface and volume) everywhere. Given this, go back and compute the  $\mathbf{E}$  field in the two regions to check your answer for  $\mathbf{E}$  in part a.

c) (max 5; avg 3.90, median 5, stddev 1.76) Find the voltage between the plates. If the plates are large but not infinite (area  $A$ ), compare the capacitance before and after you added the dielectric oil.

**The direction of  $\mathbf{E}$ ,  $\mathbf{D}$  and  $\mathbf{P}$  is still difficult for students.**

(There is 1 more question on the next page)

**Q7. LINEAR DIELECTRIC, CYLINDRICAL BOUNDARY** (avg 79.09%, median 95, stddev 30.69) – Ch4 – boundary value problems

Consider two concentric conducting *cylinders* of radii  $a$  and  $b$  as shown in the figure. (They extend infinitely in and out of the plane of the paper. The outer one is really just a conducting shell. This is basically a coaxial cable)

The space between them is *half*\* filled with a dielectric having a dielectric constant  $\epsilon_r$ . (So, this dielectric only extends out to radius  $c = a + (b-a)/2$ . The rest of the gap is vacuum)

A total charge of “+Q” is placed in the inner conducting wire and “-Q” in the outer conducting cylindrical shell.

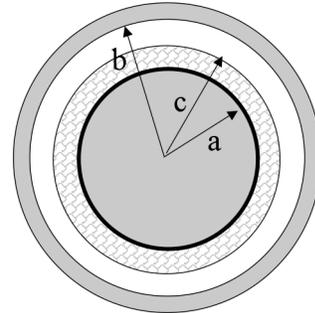
a) Find the magnitude and direction of the fields **E**, **D**, **P** everywhere; namely for  $r < a$ ,  $a < r < c$ ,  $c < r < b$ , and  $r > b$ .

Also determine the bound charge densities everywhere.

b) Determine the potential difference from inner to outer conductor, and then use this to find the capacitance per unit length for this capacitor.

How does it compare with what it would be without the dielectric (i.e. if you just had a vacuum all the way between)

How does it compare with what it would be if the dielectric had filled the entire region?



**Only one student attempted this problem and complained that it was too long and complicated.**