Phys 3310, HW #7, Due in class Wed Mar 5

Overall HW statistic: MEAN 79; MEDIAN 91; STANDARD DEV 24.

Q1. SEPARATION OF VARIABLES - SPHERICAL SIGMA **(percent: Mean 9.29, Median 10, SD 2.03) – ch 2 – separation of variables**
The surface charge density on a sphere (radius R) is a constant, σ_0
(As usual, assume $V(r=\infty)=0$, and there is no charge anywhere inside or outside, it's ALL on the surface!)
i) *Using the methods of section 3.3.2* (i.e. explicitly using separation of variables in spherical coordinates), find the electrical potential inside and outside this sphere.
ii) **(out of 5, Mean 3.29, median 4, SD1.76)** Discuss your answer, explain how you might have just "written it down" without doing all that work! (Be explicit - what about all the specific coefficients you got in i?)
iii) **(out of 10, Mean 7.76, median 9, SD 2.86)** Now, suppose the surface charge density is $+\sigma_0$ on the entire northern hemisphere, but $-\sigma_0$ on the entire southern hemisphere. Again, find voltage inside and outside. (This time, you will in principle need an infinite sum of terms - but for this problem, just work out explicitly what the first *two* nonzero terms are. (In both cases, for $V(r<R)$, and $V(r>R)$)
Note: some terms you might have expected to be present will vanish. Explain physically or mathematically why the first "zero" term really *should* be zero.
Griffiths solves a generic example problem, for which part i above is a simple special case (and for that matter, so is part iii). But, please work through the details on your own - you're welcome to use Griffiths to guide you if/whenever you need it, but in the end, solve the whole problem yourself and show your work!

Many questions on this one.

Students were confused about the simpleness of parts (i) and (ii) and didn't understand why they had to go through the whole procedure when they already knew the answer. And even when they DID start going through the separation of variables game, it wasn't clear to them when they could start taking advantage of the simplifications that occur once you know sigma is just a pure P0 (constant). I told everyone to treat the problem as through sigma was GENERAL/unknown, get as FAR as they could, and only then see what happens when you realize that sigma is pure P0. This seemed to help clarify.

Many students struggled with the + sigma and – sigma part of the problem. They weren't sure how to take that into account. Some broke up the integral at the wrong point (for example, before they solved for their Al's using Fourier's trick, they tried to just solve for the Al's in one region or the other). I tended to probe them by writing sigma as sigma(theta) instead, so that when they set up their integral in Fourier's trick they noticed that sigma was a function of theta and

they had to have two separate integrals. Most saw it at that point. Lot of students were a little stumped in part iii, when sigma can not "by inspection" be written as one (or a couple) legendre terms. They asked surprisingly elementary questions, like whether the integral from zero to pi "could be" split up into two intregrals, one from 0 to pi/2 the other from pi/2 to pi. (I was just surprised they had to ask me if that was legit.). Several expressed confusion as to why every other term was vanishing, and a couple were frustrated by the fact that they had to actually look up and integrate P3!

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Q2. SEPARATION OF VARIABLES - CONCENTRIC SPHERES **(out of 10, Mean 7.7, Median 9, SD 2.92) – ch3 – separation fo variables**

Two concentric spherical surfaces have radii of a and b. If the potential on the inner surface, at r=a, is just a nonzero *constant* (call it V_{in}) and the potential on the outer surface is given by $V(b,\theta) = V_{out}P_1(\cos\theta) \quad (i.e. = V_{out}\cos\theta)$, find the potential in the region *between* the two surfaces (a < r <b). Can you think of a fairly simple physical/experimental setup that might yield a situation something like this?

There was a lot of difficulty in setting up the BC's for this. They got stuck thinking that we knew the potential AT the boundary and didn't see how to use this to match BC's. They didn't see that the potential had a functional form on either SIDE of the boundary, and that was what they needed to use and then match for the coefficients. Many people (on this and on the previous) didn't quite seem to recognize that if we had, for example, a term that went as 1/r on one side of the equation, then the coefficient of that term HAS to match the term that goes as 1/r on the other side of the equation.

There can be a tendency to jump the gun, writing down the boundary condition for JUST the l=0 term at the inner boundary, and the l=1 term at the outer boundary, not realizing that there is information about ALL terms at EACH boundary. I also thought at first I didn't have enough information/boundary conditions to solve the problem.

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Q3. SEPARATION OF VARIABLES – DISK **- ch3 – sep of variables and multipole expansion**

A disk of radius R has a uniform surface charge density σ_0. Way back on Set #2 you found the E-field along the axis of the disk (and on the midterm, you again solved a very similar (but *harder*) version of this where σ was not uniform). You can check for yourself by direct integration, (but don't have to): I claim that along the z axis, (i.e. θ=0),

$$V(r,\theta=0)=\frac{\sigma_0}{2\varepsilon_0}\left(\sqrt{r^2+R^2}-r\right)$$

i) **(out of 10, Mean 8.57, Median 10, SD 2.82)**Find the potential *away* from the axis (i.e nonzero θ) , for distances r > R, by using the result above and fiddling with the Legendre formula, Griffiths' 3.72 on page 140. You will in principle need an infinite sum of terms here - but for this problem, just work out explicitly what the first *two* *non-zero* terms are.

(It might help to remember that $P_l(1)$ is always equal to 1, and you will have to think mathematically about how the formula above behaves for r>>R)

ii) **(out of 5, Mean 3.33, Median 4, SD 1.82)** Griffiths Chapter 3.4 talks about the "multipole expansion". Look at your answer to part i, and compare it to what Griffiths says it *should* look like (generically) on page 148. Discuss - does your answer make some physical sense? Note that there is a "missing term" - why is that?

Some students, not surprisingly, struggle with the "theta" dependence in the Legendre formula and want to delete the theta-dependent term, not recognizing that in this case theta=0 and Pl(1)=1. The idea of matching terms with the same (1/r) dependence is a logical step that many don't immediately recognize. Many do not have Taylor series memorized.

In part (ii), many students saw that the dipole was missing, but had no ability to argue WHY that should be the case. They were not making the math/physics connection. Some argue that the absence of negative charge results in the lack of a dipole moment.

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Q4. MULTIPOLES - point charges **(Mean 3.9, Median 5, SD 1.64) – ch3 - multipole**
You have four point charges. Their location and charges in Cartesian coordinates are:
A charge -q located at (a,0,0), another charge -q located at (-a,0,0), a third charge +3q located at (0,0,b), and finally a fourth charge -q located at (0,0,-b)
Find a simple approximate formula for the V(r,θ) (i.e. in spherical coordinates!) valid at points far from the origin. ("Simple" means only the *first non-zero term* is needed!)
ii) **(Mean 3.76, Median 5, SD 1.79)** Find a simple approximate expression for the electric field valid at points far from the origin. (Again, express your answer in spherical coordinates, so we want
$\vec{\mathbf{E}}(r,\theta) = E_r(r,\theta)\hat{\mathbf{r}} + E_\theta(r,\theta)\hat{\theta}$, and you should figure out what E_r and E_θ are.)
Sketch this E field. (Don't worry about what happens near the origin, I just want a sketch of the simple approximation)

Some students add the dipole moments like scalars, not vectorially. Graphing seems to be difficult for students, and there is some evidence of cheating on this problem (graphs that are right despite differences in the calculated E field).

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Q5. MULTIPOLES - spherical shell charge distribution **- ch3 - multipole**
Griffiths derives (on page 142-144) the exact potential $V(r,\theta)$ everywhere outside a spherical shell of radius R which has a surface charge distribution $\sigma(R,\theta)= k\cos\theta$
i) **(Mean 7.9, Median 10, SD 3.67)** Calculate the dipole moment of this object, and also the total charge on this object.
ii) **(Mean 3.62, Median 5, SD 1.94)** Use the methods of "the multipole expansion" (Griffiths p. 148) to find an *approximate* form for the potential far from the sphere. You can stop with the leading nonzero term. Now compare with Griffiths *exact formula* from the earlier example (on p.144). What does this tell you about the quadrupole moment (and higher moments) of this surface charge distribution?

Almost all students fail to argue by symmetry that px=py=0. Most fail to show interest in the fact that this distribution gives a pure dipole (part ii), suggesting they are not deeply considering the physics.

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Q6. REAL DIPOLE. **(Mean 8.43, Median 10, SD 3.34)**
Griffiths 4.1 (p. 163) Only, instead of just using/assuming the Bohr radius (like Griffiths suggests) please *estimate* the radius of hydrogen, wherever you may need it, by using the experimental atomic polarizability of hydrogen given in Table 4.1 (p. 161) of Grifffiths. (Griffiths' Example 4.1 should tell you how this would give you a quick estimate of the hydrogen atom radius. How does it compare with the Bohr radius?)

Q6: Students were not clear on where/how to start. The idea of using Griffiths' model problem (Ex 4.1) was not jumping out at them, I wonder if they had actually read it? Seemed like maybe they had not. S. and R. were struggling just to visualize what the problem was asking in the first place. C. wanted a calculator, and when I told her this was an estimation problem, she seemed suddenly pleased, remembering e.g. that 1/4 pi epsilon0 = k = 9E9, and so on...

Extra Credit:
Go back through all your returned old homeworks (but preferably one of the more recent ones), and/or the midterm, and find a problem (or two) that you got some serious points taken off. i)clearly identify the set # and question # you are correcting, ii) state what was wrong with your original wrong answer (assuming it wasn't just blank) iii) explain where your original reasoning was incorrect (if it wasn't blank) or what you were missing (if it was blank), and then outline the correct *reasoning* for the problem, and how it leads to the right answer. Of course, solutions are posted, so we're not interested in having you just copy my solutions! What we're looking for is more your reflections on where you went wrong, and what *you understand* about the problem now.