Phys 3310, HW #9, Due in class after spring break, i.e. Wed Apr 2

## TOTAL HW STATS (Avg: 81.35%, median: 87.69%, stddev: 10%)

**Q1.** ITERATING FIELDS (avg: 85.88%, median: 100%, stddev: 23.79% - ch4 – linear dielectrics

Griffiths solved the problem of a dielectric sphere in a uniform external field in Example 4.7 using Separation of Variables (and applying boundary conditions). There's another approach, which in some ways is perhaps *conceptually* simpler (?)

a) (partial points: 10, avg: 9.12, median: 10, stddev: 2.00) You've put the object into a uniform external field $\mathbf{E_0}$, so it would be logical (but wrong) to assume that the polarization of the dielectric would just be simply $\mathbf{P}=\varepsilon_0\chi_e\mathbf{E_0}$. (Briefly - why is that wrong?) Go ahead and assume it, it's like a "first approximation", let's call it $\mathbf{P_0}$. Now this polarized sphere generates its own additional (induced) E field, call that $\mathbf{E_1}$. What is that field? (You don't have to rederive it from scratch, Griffiths has shown us how to figure out the E field in a sphere, created by a uniform polarization of that sphere. You've also used in the last homework) OK, but now $\mathbf{E_1}$ will modify the polarization by an *additional* amount, call it $\mathbf{P_1}$. What's that? That in turn will add in a new electric field by an additional amount $\mathbf{E_2}$. And so on. The final, total, "real" field will just be $\mathbf{E_0}+\mathbf{E_1}+\mathbf{E_2}+\ldots$

Work it out, and check/compare your answer with Griffiths result at the end of Ex 4.7 (p. 188)

b) (partial points: 5, avg: 3.76, median: 5, stddev: 1.95) If the sphere was made of Silicon (see Table 2 in Griffiths), compare the "first approximation" for $\mathbf{P_0}$ with the true result for polarization - how important was it to go through the summation? (In what limit, large or small $\varepsilon_r$, does this summing procedure make a difference?)

*You will need the handy trick for summing the infinite geometric series:* $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

*If that looks mysterious to you, you should convince yourself that it's correct, by doing our usual Taylor expansion of* $1/(1-\varepsilon)$.

**This is a very hard problem for students. There were a lot of question on what P0, P1, etc represented. Many wanted P1 to be the "new" P due to the effects of E0 + E1, and were confused that E1 is an "additional" field instead of the *total* E field to 2$^{nd}$ order. This question could be crafted better. There was also a lot of question about what the iterative approach meant… only perhaps half of the students could verbalize the game that was being played here. Several asked if this is what actually happens, in time, and thought about the corrections as being corrections that happen in time, that first it polarizes due to E0 and then it reacts to that polarization, and so on. Many only calculate to 2$^{nd}$ order and then somehow come up with the correct formula, whereas one should really calculate to 3$^{rd}$ order to be more certain.**

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**Q2.** DERIVING CLAUSIUS-MOSSOTTI (avg: 83.53%, median: 80%, stddev: 13.67%) – Ch4 – linear dielectrics. (Do keep this one in, Nina, even thoughit's from Griffiths)
Griffiths 4.38 (on page 200). Please discuss the result briefly. In words, what is 3.72 telling us, and how (and when) is it different from the "naive expectation" Griffiths talks about at the start of the problem.

**This is a very hard question for students. Many can solve it but not understand what they had done. The question of "Eself" vs "Eelse" is subtle and a little confusing. Many realized that Example 4.2 and/or 4.3 should help, but weren't quite sure how/why it was relevant or what it meant. I think it's a good problem, and important, I just suspect that many students, without the guidance of an instructor, will just do the simple algebra and "mumble" without really thinking about what it all means.**

**Q3.** APPLYING CLAUSIUS-MOSSOTTI (avg: 89.41%, median: 100%, stddev: 21.35%) – ch4 – linear dielectrics
Use the Clausius-Mossotti equation (which you just derived in the previous problem) to determine the polarizability of atoms in air (which is largely $N_2$). (Table 2 in the book gives dielectric constants for $N_2$ and "air" as a separate entries but the numbers are basically identical.) Combine this result with the simple classical calculation we did in class a couple of weeks ago (basically example 1 in chapter 4) to *roughly estimate* a typical radius of an air molecule.

**Not one of about half a dozen students were able to tell me the volume of one mole of ideal gas (although when I said 22.4 l, they all kind of went "oh yeah, I remember that...") There were several questions about whether N (# of atoms/unit volume) should just be Avogadro's number or not. Many students don't do the discussion on their final writeup.**

**Q4.** CURRENT DENSITIES: (avg: 79.41%, median: 85.7%, stddev: 44%) – Ch5 - currents
a) (partial points: 2, avg: 1.94, median: 2, stddev: 0.24) A solid cylindrical straight wire (radius a) has a current I flowing down it. If that current is uniformly distributed over the outer surface of the wire (none is flowing through the "volume" of the wire, it's all surface charge), what is the surface current density **K**?
b) (partial points: 2, avg: 1.53, median: 2, stddev: 0.80) Suppose that current *does* flow throughout the volume of the wire, in such a way that the volume current density J grows quadratically with distance from the central axis, what then is the formula for **J** everywhere in the wire?
c) (partial points: 2, avg: 1.71, median: 2, stddev: 0.47) A CD has been rubbed so that it has a fixed, constant, uniform surface electric charge density $\sigma$ everywhere on its top surface. It is spinning at angular velocity $\omega$ about its center (which is at the origin). What is the surface current density **K** at a distance r from the center?

d) (partial points: 4, avg: 3.18, median: 3, stddev: 1.07) A sphere (radius R, total charge Q) is spinning at angular velocity $\omega$ about its center (which is at the origin) What is the volume current density **J** at any point (r, $\theta$, $\phi$) in the sphere?

e) (partial points: 4, avg: 2.76, median: 3, stddev: 1.15) A very thin plastic ring has constant charge density, and total charge Q. The ring has radius R and it is spinning at angular velocity $\omega$ about its center (which is at the origin). What is the current I, in terms of given quantities? Also, how would you write down the volume current density **J** in cylindrical coordinates? (This may be a little tricky, since the ring is "very thin", there will be some delta functions involved! I suggest trying to write down a formula for $\rho(r,\phi,z)$ first)

**Many students don't know how to think about the vector direction of J or K. Some thought that v was in the direction of omega; few had thought of v = *w* X r, and NONE spontaneously knew even what the direction of omega for a spinning disk or sphere or ring was, and thought that omega pointed in the phi direction. Several were confused about how to deal with the last part, the one with delta functions in r and z.**

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**Q5.** MAGNETIC MOTION: (avg: 73.16%, median: 81.25%, stddev: 19.74) – NINA, DON'T INCLUDE THIS ONE
Griffiths Problem 5.2.
Part a (partial points: 6, avg: 4.59, median: 5, stddev: 1.18)
Part b (partial points: 5, avg: 3.47, median: 3, stddev: 1.23)
Part c (partial points: 5, avg: 3.64, median: 4, stddev: 1.32)
For each part - is kinetic energy constant with time? (Where it isn't, very briefly explain)
Also, for part a - the result is in many respects quite special - give some physical example(s) where this result could be useful for some practical purpose.

**Steve: I saw C and R discuss this together for a long time, but they never asked me anything about it. Preston was confused about how Griffiths got the "cycloid" from his equations. (But he hadn't really read it/thought about it in advance either). No other questions/observations about this one.**

**EDWIN: All KE discussions are very mathematical; none of the students try to argue it physically (i.e. using net force on the ptcl.) On part a), some students use force argument to come up with constant velocity and straight line trajectory. Most students didn't supply a practical application. Sadly, I don't see any use on most of the suggested application (did they misunderstand the question?) R. and C. suggested to use this for a magic trick (I think they forgot gravity, so it does not work), Steve suggested to use this as a means to evaluate momentum, but I think he oversimplified a lot of things.**

**On part b, many students drew sinusoidal trajectory. Some even play around with the parametric equation and get an equation of a circle, then argued (wrongly) that it's a sinusoidal trajectory. I think they just forgot that their circle is shifted up in the z-direction, thus creating a cycloid instead of a sinusoid.**

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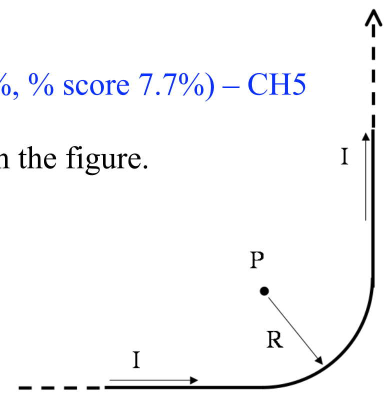
**Q6.** MAGNETIC FIELD: (avg: 87.06%, median: 100%, stddev: 29.95%, % score 7.7%) – CH5 – BIOT SAVART

An infinitely long wire has been bent into a right angle turn, as shown in the figure.
The "curvey part" where it bends is a perfect quarter circle, radius R.
Point P is exactly at the center of that quarter circle.
A steady current I flows through this wire.

Find the magnetic field at point P (magnitude and direction)




**Students who started to go astray (by doing Biot-Savart) were generally set straight by other students.**

EXTRA CREDIT:
A conducting ball bearing (radius R) is coated with a dielectric paint (dielectric constant $\varepsilon_r$), which extends out to a radius R'. (The paint is thick, at least on an atomic scale, so R' is measurably larger than R) This object is sitting in a uniform external electric field, $\mathbf{E_0}$. Find the electric field *in* the paint region.
*This is a slightly nasty little problem, which lets you work out for yourself the various methods invoked in Example 4.7 of Griffith, but with different boundary conditions. I recommend you do NOT try to apply the boundary condition on dV/dr at the* inner *boundary - you will get a paradox (it may look like E=0 everywhere in the paint region, but that's not right). This is another one of those cases where you have to be careful making assumptions about D: even though Qfree(enclosed) is 0, you can't assume D is purely radial in the paint region! But that's ok, you simply don't need this boundary condition anyway!)*