

Phys 3310, HW #6, Due in class Wed Feb 27.

(I suggest you get started on this one early, it's only 5 problems but it's pretty long)

Q1. UNIQUENESS THEOREM – Ch3 – laplace and uniqueness

Griffiths section 3.1.6 is a discussion and proof of the "Second uniqueness theorem".

Read through that theorem (and his proof), make sense of it! Then for this homework problem, prove it yourself, using a slightly *different* method than what Griffiths does (though you may find some common "pieces" are involved!) Do it like this:

Go back to Green's Identity (stated in Problem 1.60c, p. 56, you may recognize this from our HW#2) This identity is true for ANY choice of T and U, so let the functions T and U in that identity both be the SAME function: specifically, you should set them both equal to $V_3 = V_1 - V_2$. Then, Green's Identity (along with some arguments about what happens at the boundaries, rather like Griffith's uses in *his* proof) should let you quickly show that E_3 (which is defined to be the negative gradient of V_3 , as usual) must vanish everywhere throughout the volume. QED.

Understand the game. We are checking if there are two different potential functions, V_1 and V_2 , each of which satisfies Laplace's equation throughout the region we're considering. You construct (define) V_3 to be the difference of these, and you prove that V_3 (or in this case, $E_3 = -\nabla V_3$) must vanish everywhere in the region. Which means there really is only one unique E-field throughout the region after all! This is another one of those "formal manipulation" problems, giving you a chance to practice with the divergence theorem and think about boundary conditions...

Some student confusion about why you can pull V_3 out of the integrals. There was some confusion in following the proof in the book. Why $\nabla \cdot E_3$ was zero was confusing until we noticed the words "in the space between the conductors." Most students seemed to get the proofs OK, though they were intimidated by them. Overall, good practice.

Q2. METHOD OF IMAGES – spherical – Ch3 – method of images

Take a look at Griffiths' Fig 3.12, which shows a grounded metal sphere with a charge q outside it. He argues (leading up to Eq. 3.17) that there is a simple "method of images" trick available here - you just have to put the right charge (q') at the right spot (b , inside the radius of the sphere). Your task:

i) Solve Griffiths' problem 3.7a (p. 126)

(which shows WHY this particular "image trick" works for a spherical conductor)

ii) Solve Griffiths' problem 3.7b

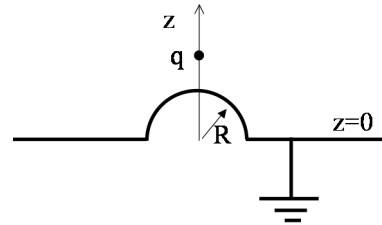
iii) Now let's apply this result to a novel situation:

Imagine a grounded infinite conducting plane in the x - y plane, that has a (conducting) hemispherical bump (radius R) in it, centered at the origin, as shown.

A charge q sits a distance " a " above the plane, i.e. at the point $(0,0,a)$

I claim that you can find the potential V anywhere in the plane above the conductor using the method of images, with *three* image charges.

Where should they be? (Explain your reasoning- you need to ensure the boundary condition $V=0$ on the entire conductor.)



Several students struggled with the math involved in integrating sigma, although they managed to work it out. There was almost universal confusion about what the answer SHOULD be, nobody was comfortable with a Gauss' law explanation, they sort of wanted for the integral of sigma to come out to be zero (maybe since the sphere is grounded).

Q3. SEPARATION OF VARIABLES - CARTESIAN 2-D ch3 – separatin of variables

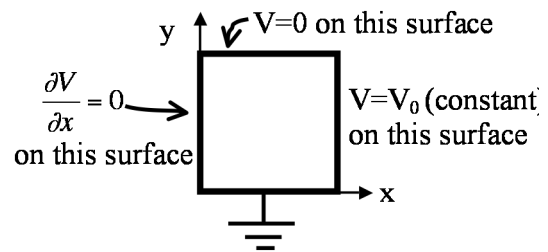
A square rectangular pipe (sides of length a) runs parallel to the z -axis (from $-\infty$ to $+\infty$)

The 4 sides are maintained with boundary conditions given in the figure. (Each of the 4 sides is insulated from the others at the corners)

i) Find the potential $V(x,y,z)$ at all points in this pipe.

ii) Sketch the E-field lines and equipotential contours inside the pipe. (Also, state in words what the boundary condition on the left wall means - what does it tell you? Is the left wall a conductor?)

iii) Find the charge density $\sigma(x,y=0,z)$ everywhere on the bottom conducting wall ($y=0$).



Students easily managed to get to the point where they had $X(x) = C \cdot \cosh(n \pi x/a)$ and $Y(y) = A \cdot \sin(n \pi y/a)$. But there is a mixture of student ability to proceed from there, how to apply the last boundary condition, whether Fourier's trick is familiar or not. Among those who are familiar with Fourier's Trick, however, some still do not quite understand orthogonality of the sin functions. The sketch is challenging for students. I saw several who had gotten the E field sketch right just by thinking about where a charge would go when placed on one face, but then struggled with the equipotentials, finally figuring it had to be perpendicular to the field lines but still having a bit of trouble from there.

Q4. SEPARATION OF VARIABLES - CARTESIAN 3-D – Ch3 = sep of variables

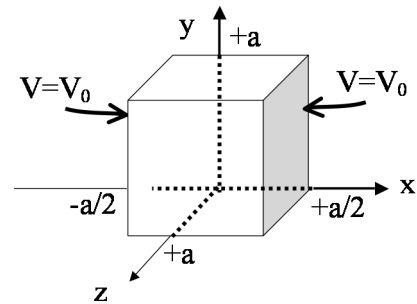
You have a cubical box (sides all of length a) made of 6 metal plates which are insulated from each other.

The left wall is located at $x=-a/2$, the right wall is at $x=+a/2$.

Both left and right walls are held at constant potential $V=V_0$.

All four other walls are grounded.

(Note that I've set up the geometry so the cube runs from $y=0$ to $y=a$, and from $z=0$ to $z=a$, but from $x=-a/2$ to $x=+a/2$. This should actually make the math work out a little easier!)



Find the potential $V(x,y,z)$ everywhere inside the box.

(Also, is $V=0$ at the center of this cube? Is $E=0$ there? Why, or why not?)

Most students have the procedure right, but don't know they should use different summation index to distinguish $\sin(\pi n_y y/a)$ and $\sin(\pi n_x z/a)$, so there is only one summation in their results instead of two.

Q5. SEPARATION OF VARIABLES - SPHERICAL

The potential on the surface of a sphere (radius R) is given by $V = V_0 \cos(2\theta)$.

(Assume $V(r=\infty)=0$, as usual. Also, assume there is no charge inside or outside, it's ALL on the surface!)

i) Find the potential inside and outside this sphere.

(Hint: Can you express $\cos(2\theta)$ as a simple linear combination of some Legendre polynomials?)

ii) Find the charge density $\sigma(\theta)$ on the sphere.

Overall students seemed to do well with this. Some studentes seemed totally unfamiliar with the Gauss' Law trick to determine the discontinuity in E at a surface charge, which is a trick that could use emphasis.

Extra Credit: CHARGED METAL SPHERE

You have a conducting metal sphere (radius R), with a net charge $+Q$ on it.

It is placed into a pre-existing uniform external field E_0 which points in the z direction.

(So, this is *exactly like* Griffiths Example 3.8, except the sphere is not neutral to start with.) Find the potential everywhere inside and outside this sphere. Please explain clearly *where* you are setting the zero of your potential. Do you have any freedom in this matter? Briefly, explain.