Phys 3310, HW #4, Due in class Wed Feb 6.

Q1. NONUNIFORM SURFACE CHARGE
We typically consider cases in class where charge is distributed completely uniformly, but it's not always that way - let's consider an insulating sphere (radius R) with surface charge density $\sigma=\sigma_0 \sin^2(\theta) \cos^2(\phi)$.
- Find the total charge on this sphere.
- Describe briefly in words and pictures what this charge distribution "looks like".
- Then, briefly but clearly, describe a *procedure* to find **E**(x,0,0) (for x>R)?

Note: you do not need to come up with a final closed-form answer! We just want a discussion, with formulas, of how you would proceed. Get as far as you reasonably can, but stop when the going gets too nasty, and discuss what you would do next if you really *needed* to know this E field in, say, a laboratory/experimental situation.

Q2. DIVERGENCE AND CURL
Consider an electric field $\mathbf{E} = c\dfrac{\vec{\mathbf{r}}}{r^2}$ (Please note the numerator is not $\hat{\mathbf{r}}$: this is NOT the usual E field from a point charge at the origin, which would give $c'\dfrac{\vec{\mathbf{r}}}{r^3}$, right?!)

a) - Calculate the divergence *and* the curl of this **E** field.
- Explicitly test your answer for the divergence by using the divergence theorem. (Is there a delta function at the origin like there was for a point charge field, or not?)
- Explicitly test your answer for the curl by using the formula given in Griffiths problem 1.60b, page 56.

b) What are the units of c? What charge distribution would you need to produce an E field like this? Describe it in words as well as formulas. (Is it physically realizable?)

Q3. ALLOWED E FIELDS
Which of the following two static E-fields is physically *impossible*. Why?

i) $\mathbf{E} = c(2x\hat{\mathbf{i}} - x\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

ii) $\mathbf{E} = c(2x\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

where c is a constant (with appropriate units)
For the one which IS possible, find the potential V(r), using the *origin* as your reference point (i.e. setting V(0)=0)
- Check your answer by explicitly computing the gradient of V.

Note: you must select a specific path to integrate along. It doesn't matter which path you choose, since the answer is path-independent, but you can't compute a line integral without having a particular path in mind, so be explicit about that in your solution.

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Q4. FINDING VOLTAGE FROM CHARGE DISTRIBUTION

a) Find a formula for the electrostatic potential V(z) everywhere along the symmetry-axis of a charged ring (radius a, centered on the z-axis, with uniform linear charge density λ around the ring) Please use the method of direct integration (Griffiths 2.30, on p. 85) to do this, and set your reference point to be $V(\infty)=0$.

- Sketch V(z).
- How does V(z) behave as $z \to \infty$? (Don't just say it goes to 0. HOW does it go to zero? Does your answer make physical sense to you? Briefly, explain.)

b) Use your result from part a for V(0,0,z) to find the z-component of the E field anywhere along the z-axis.

- What is the Voltage at the origin?
- What is the E field right at the origin?
- Do both of these results (for V, and E, at the origin) make physical sense to you, and are they consistent with each other? Explain briefly!

Q5. CALCULATING VOLTAGE FROM E FIELD

Last week, we investigated the electric field outside an infinite line that runs along the z-axis, $\vec{\mathbf{E}} = \frac{2\lambda}{4\pi\varepsilon_0}\frac{\vec{\mathbf{s}}}{s^2}$.

a) This field may look similar to Q2 above, but it is *not* the same - how is it different?

- Find the potential V(s) for points a distance "s" away from the z-axis.

(Note: you will have to have to be very careful to compute a difference of potentials between two points, or something similar, to avoid integrals which are infinite! You'll discuss this in part b)

- Check your answer by explicitly taking the gradient of V to make sure it gives you **E**.

b) Briefly discuss the question of "reference point": where did you set V=0? Can you use $s=\infty$, or s=0, as the reference point, V(s)=0, here?

- How would your answer change if I told you that I wanted you to set V=0 at a distance s=3 meters away from the z-axis?
- Why is there trouble with setting $V(\infty)=0$? (our usual choice), or V(0)=0 (often our second choice).

c) A typical Colorado lightning bolt might transfer a few Coulombs of charge in a stroke. Although lightning is clearly not remotely "electrostatic", let's pretend it is - consider a brief period during the stroke, and assume all the charges are fairly uniformly distributed in a long thin line. If you see the lightning stroke, and then a few seconds later hear the thunder, make a *very rough* estimate of the resulting voltage difference across a distance the size of your heart. (For you to think about - why is this not worrisome?)

What's the model? I am thinking of a lightning strike as looking rather like a long uniform line of charge... You've done the "physics" of this in the previous parts! (But e.g., you need a numerical estimate for λ. How long might that lightning bolt be? For estimation problems, don't worry about the small details, you can be off by 3, or even 10, I just don't want you off by factors of 1000's!)

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Q6. SCREENED POTENTIAL

Consider the "screened Coulomb potential" of a point charge of charge q that arises, e.g. in plasma physics: $V(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0}\frac{e^{-r/\lambda}}{r}$, where λ is a constant (called the screening length).

a) Determine the E-field **E(r)** associated with this potential.

b) Find the charge distribution ρ(**r**) that produces this potential. (Think carefully about what happens at the origin!)

- Sketch this function ρ(**r**) in a manner the clearly describes its characteristics (i.e. what's the best way of representing this three-dimensional charge distribution? Use it, and explain what you're plotting).

c) Show by explicit calculation over ρ(**r**) that the net charge represented by this distribution is zero (!) (If you don't get zero, think again about what happens at r=0)

- Verify this result using the integral form of Gauss' law (i.e. integrate your electric flux over a *very large* spherical surface. By Gauss, that should tell you Q(enclosed))

Extra credit (worth half of any of the above, but *won't* count off if you don't do it)

It is possible to separate normal seeds from discolored ones (or foreign objects) by means of a device that operates as follows. The seeds drop one by one between a pair of photocells. If the color is wrong, a needle deposits a small charge on the seed. The seeds then fall between a pair of electrically charged plates that deflect the undesired ones into a separate bin. (One such machine can sort about 2 metric tons per 24 hour day!)

Setup: If 100 seeds fall per sec, over what distance must they fall if they are to be spaced vertically by 20 mm when they pass between the photocells? (Neglect air resistance.)
Then: Assume the seeds are about 1 gram, and acquire a charge of 1nC (that's nano, 1E-9), the deflecting plates are parallel and 5cm apart, and the potential difference between them is 10,000 volts. How far should the plates extend below the charging needle if the charged seeds must deflect by 45 mm after leaving the plates? (Assume the charging needle and the top of the deflecting plates are close to the photocell.)