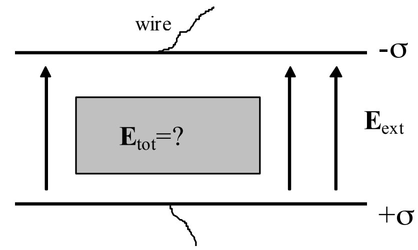
HW 10 HINTS FOR Q6



Q6. MODELING A SOLID DIELECTRIC
The "given constants" here are n, q, k (the spring constant), and $\mathbf{E}_{ext}$ is given, too. However, *note that Δz is not really a "given constant". It will obviously depend on how big an external field you apply! But at first, we'll just give it the name Δz , and later we will eliminate it!*

We want to figure out the dielectric constant of this material.

Below are some fairly detailed steps/suggestions. See if you can use as few of these steps/hints as possible- but if you get stuck, perhaps looking at the next step will help move you along.

Before you begin, review the in-class tutorial, and e.g. figure out the bound surface charge density on the surface of the dielectric, and the polarization in the dielectric, in terms of the givens and Δz.

i) FIND THE INDUCED FIELD (IN TERMS OF GIVENS AND UNKNOWN Δz)
We have a large (assume infinite) slab, with some bound charge on it. I might start by figuring out the induced electric field, $\mathbf{E}_{ind}$, inside the plastic slab. (Express it in terms of Δz!)

ii) FIND THE TOTAL FIELD (IN TERMS OF GIVENS AND UNKNOWN Δz)
So now we know the induced E field. What is then the total electric field inside the plastic slab, $\mathbf{E}_{tot}$? *(First express it very generically in terms of the magnitude of the induced field $\mathbf{E}_{ind}$ and the external field $\mathbf{E}_{ext,.}$ Then, given part i, write it purely in terms of givens, including Δz) Watch out for minus signs!*

iii) USE THE "MODEL" TO FIGURE OUT Δz IN TERMS OF THE GIVENS.
So now you have an expression for the total E field in there. Good – it's $\mathbf{E}_{tot}$ which is "real", that's the actual E field that the little atoms are experiencing! Think physically of this simple model (it's merely charges on springs, with a spring constant k, sitting in a total field $\mathbf{E}_{tot}$)
Can you write a simple expression for Δz in terms of $\mathbf{E}_{tot.}$? Nothing fancy here, what's the force of the spring? What's the electric force? How do they relate in equilibrium?!

iv) ELIMINATE Δz, FIND A RELATION BETWEEN TOTAL AND EXTERNAL FIELD.
Now hook it all together: you have Δz in terms of $\mathbf{E}_{tot}$ (from part iii) But in part ii, you also had a rather different expression for $\mathbf{E}_{tot}$ written in terms of Δz ! Ah! Combine these two equations to relate $\mathbf{E}_{tot}$ directly to $\mathbf{E}_{ext}$. This is progress! Δz is now gone from the story, and you have a direction connection between the physical E field in the medium ($\mathbf{E}_{tot}$) to the applied field ($\mathbf{E}_{ext}$)

v) FIGURE OUT THE D FIELD.
Inside the dielectric slab, it should be fairly quick and easy to write a simple formula for the D field in terms of $\mathbf{E}_{ext}$. (Why? Because D arises from FREE charges only, not bound charges! It's a simple Gauss' law situation) But given D in terms of $\mathbf{E}_{ext}$, now we're pretty close to done:
The fundamental definition of the dielectric constant is from the relation $\mathbf{D} = \varepsilon_0\, \varepsilon_r\, \boldsymbol{E}_{tot}$,
where ε_r is the dielectric constant we're after.
You already found (part iv) the relation between $\mathbf{E}_{tot}$ and $\mathbf{E}_{ext.}$ So I think at this point, you should be able to "read off" the dielectric constant.