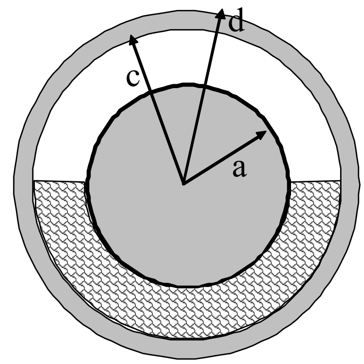
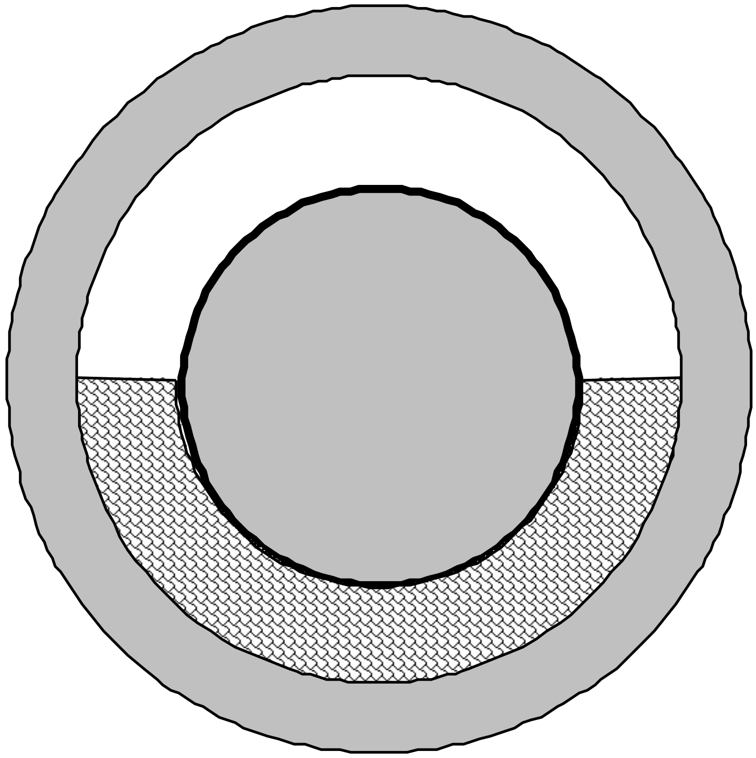
✯ TUTORIAL 8: Is the sphere half full or half empty? ✯
Spherical linear dielectric

Part 1 – Symmetry and boundary conditions

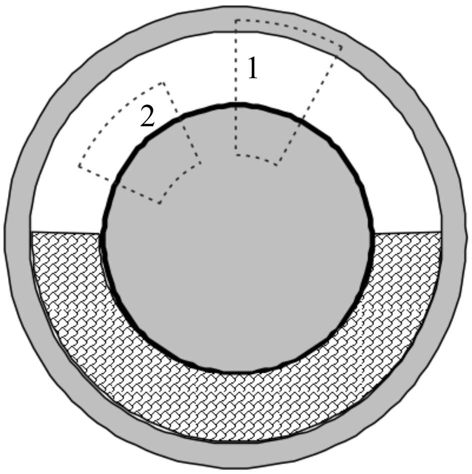


Consider a conducting spherical shell of radii a that is concentric with a conducting sphere of radius c as shown in the figure. The space between them is filled with a liquid having an electric susceptibility χ_e. A total charge of "+Q" is placed in the inner conducting shell and "-Q" in the outer shell.

i. Predict where there would be free and bound charge on all surfaces. Sketch your predictions on the diagram below.
Predict what the total E-field would look like everywhere. Sketch your predictions on the diagram below. Don't worry too much about getting this exactly right, it is just your intuitive guess for now.



ii. In the empty space between the two conductors, what direction does $\vec{E}$ point? What variables (r,θ,ϕ) could $\vec{E}$ depend on in this region? *(You might want to use think about $\oint \vec{E} \cdot \vec{d\ell}$ for the loops drawn below to help you figure this out).*



iii. In the dielectric between the two conductors, what direction does $\vec{E}$ point? What variables (r,θ,ϕ) could $\vec{E}$ depend on in this region?

iv. Is this $\vec{E}$ the same or different as in part ii? *(You might want to think about the boundary condition between the air and the dielectric, or consider drawing more Stokes loops).*

v. Imagine some Q_{eff} distributed on the surface of the inner conductor which includes both the free charges (Q) and some bound charges from the dielectric. How is Q_{eff} distributed on the surface of the sphere?

Is Q_{eff} greater than, less than, or equal to Q? How do you know?

Part 2 – Polarization and the dielectric story

i. Find $\vec{E}$ between the two conductors in terms of Q_{eff}.

ii. Find $\vec{P}$ between the two conductors in terms of Q_{eff}. (*Hint: it's a **linear** dielectric with known* χ_e).

iii. Find $\vec{D}$ between the two conductors in terms of Q_{eff}.

iv. Find σ_b on the dielectric surface at r=a in terms of Q_{eff}.

v. Find σ_b on the dielectric surface at r=c in terms of Q_{eff}.

vi. Find σ_b on the dielectric surfaces at the interface between the dielectric and the air in terms of Q_{eff}.

vii. Find σ_{free} everywhere on the inner conductor in terms of Q_{eff}. Is it safe to assume that σ_{free} is the same on the upper and lower half of the sphere?

viii. Solve for Q_{eff} in terms of Q (and other given quantities, a, c, d, and χ_e).

ix. Does your answer for Q_{eff} make sense? Does it match your prediction from Part 1-v.? Do the limits for large and small χ_e make sense?

Part 3 – Charge distributions

i. How does Q_{eff} (the total bound and free charge) on the outer conductor compare to Q_{eff} on the inner conductor? How did you decide?

ii. Describe (qualitatively) σ_{free} everywhere on the outer conductor. Is it safe to assume that σ_{free} is the same on the upper and lower half of the shell?

iii. Is your dielectric oil overall neutral (support your answer with a calculation)? Should it be?

iv. Do you need to revise your initial predictions from Part 1?

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v. What would change if instead of –Q on the outer conductor, this conductor was neutral?

vi. Is the sphere half full or half empty?