Phys 3310, HW #3, Due in class Wed Jan 30. In general, we will try to grade homeworks for clarity of explanation as well as for mere "correctness of final answer".

Q1. DELTA FUNCTIONS!

a) Calculate the following integrals, assuming c=2 in all parts!
In part iv, assume the volume V is a sphere of radius 1 centered on the origin, and the constant vector $\mathbf{r_o} = (0,3,4)$

i) $\int_{-1}^{1} c\,\delta(x-c)\,dx$ (Hint: watch out for limits of integration. Remember, c=2 throughout)

ii) $\int_{-10}^{10} 2cx\,\delta(x-c)\,dx$

iii) $\int_{-\infty}^{\infty} |x-c|^2\,\delta(2x)\,dx$ (Hint: see Griffiths Example 1.15)

iv) $\iiint_V |\vec{\mathbf{r}}-\vec{\mathbf{r}}_0|^2\,\delta^3(\vec{\mathbf{r}})\,d^3\vec{\mathbf{r}}$

v) $\iiint_V |\vec{\mathbf{r}}-\vec{\mathbf{r}}_0|^2\,\delta^3(2\vec{\mathbf{r}})\,d^3\vec{\mathbf{r}}$ (Hint: $\delta^3(\vec{\mathbf{r}}) = \delta(x)\delta(y)\delta(z)$)

b) Evaluate the integral $\int_V (1+e^{-r})\,(\nabla\cdot\frac{\hat{r}}{r^2})\,d\tau$ where volume V is a sphere of radius R centered at the origin, by two different methods, just as Griffiths does in his Ex 1.16.
(Hints: one method, surely the easier one, is to use your knowledge of what $\nabla\cdot\frac{\hat{r}}{r^2}$ is (Griffiths section 1.5.3 covers this) The other is to use "integration by parts in 3-D", described in Griffiths section 1.3.6. *Don't forget that the inside flyleaf of Griffiths tells you how to take div, grad, and curl in, e.g. spherical coordinates!*

c) For the volume charge density $\rho(x,y,z) = b\cdot\delta(x-3)$:
What is the physical situation represented by this volume charge density? (Describe it in words) What are the *units* of all symbols in this equation (including, specifically the symbols ρ, x, b, the delta function itself, the number written as "3" inside the δ function) - Given all this, what is your physical interpretation of "b" in this equation?

d-i) On the previous homework we had two point charges: +q located at x=-D, and -q at x=+D. Write an expression for the *volume* charge density ρ(**r**) of this system of charges.

ii) On the previous homework we had another problem with a spherical surface of radius R (Fig 2.11 in Griffiths) which carried a uniform surface charge density σ. Write an expression for the *volume* charge density ρ(**r**) of this charge distribution.
Verify explicitly that the units of your final expression are correct.
(Hint: use spherical coordinates. Be sure your total integrated charge comes out right.)

The delta function, both 1-D and 3-D appears throughout this course. Physically, it represents a highly localized source (like a point charge), but as in part b, it will often appear "mathematically" when integrals arise involving Gauss' law. It can be tricky to translate between the math and the physics in problems like this, especially since the idea of an infinite charge density is not intuitive. Practice doing this translation is important (and not just for delta functions!)

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Q2. Gauss' law and Divergence

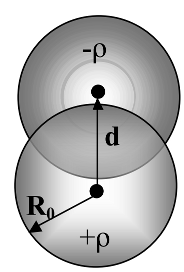
The electric field outside an infinite line that runs along the z-axis is $\vec{\mathbf{E}} = \frac{2\lambda}{4\pi\varepsilon_0}\frac{\hat{\mathbf{s}}}{s}$ in cylindrical coordinates. (This is derived in Griffiths Example 2.1)

a) Find the divergence of the E field for s>0.
b) Calculate the electric flux out of an imaginary Gaussian cylinder of length "L", and radius "a", centered around the z axis. Please do this 2 different ways to check yourself: by direct integration, *and* using Gauss' law.
c) Given parts a and b, what is the divergence of this E field? (We want a math formula) [Hint: use cylindrical coordinates. Your answer can't be zero everywhere! Why not?]
Notation: Griffiths uses s *for "distance from the z axis" in cylindrical coordinates. I sometimes use the symbol r for that quantity (if it's clear I won't confuse it with spherical coordinate), or sometimes* $\boldsymbol{\rho}$ *(if I don't think I'll confuse it with charge density!)*

Q3. E FIELD IN HYDROGEN.

a) Find the E-field in a hydrogen atom. Quantum mechanics tells us that the electron is effectively "smeared out", so the electron's resulting contribution to the charge density is $\rho_0(r) = \rho_0 \exp\left(-\frac{2r}{a_0}\right)$ where a_0 is the Bohr radius. *(Hint: Don't forget to normalize ρ so the electrons charge is –e. This charge density is not uniform, so you must integrate to get enclosed charge. Also, don't forget there is also a pointlike proton in the middle of this atom!)*
Sketch and briefly discuss your result (compare the E field you get to what you'd have from the proton alone)
b) Having done the above, briefly discuss the advantages and disadvantages of using Gauss' Law to find the electric field instead of using Coulomb's Law (Griffiths Eq 2.8)? What role does symmetry play?
Note: sketch means sketch - just a rough plot which shows key features (e.g. what's it do near the origin? Near infinity?) We don't want to just *calculate E fields, we want to be able to imagine what they look like, so sketching fields is important. (You can always use a program (like Mathematica) to* check *your sketch, but try on your own first...)*

Q4. OVERLAPPING SPHERES OF CHARGE



You have two spheres. The first is centered at the origin, has uniform positive charge density $\boldsymbol{\rho}$, and radius R_0. The second has uniform negative charge density $-\boldsymbol{\rho}$, same radius R_0. It is shifted up by a distance **d**.
a) Show that the E field in the region of overlap of the two spheres is constant, and find its value. (Please check that the units are correct)
[Hint: You will first want to figure out what the E field is in a single sphere with uniform charge density $\boldsymbol{\rho}$*. It is definitely not zero, nor is it uniform!]*

b) In the limit that **d** becomes small compared to R_0, discuss in words what the resulting (total, physical) charge distribution in space really *looks* like (so that later in the course when we encounter such a charge distribution, we will know where it came from and what the E field looks like inside!) Don't go to the complete limit of d=0, we want a description when d is SMALL but still finite.

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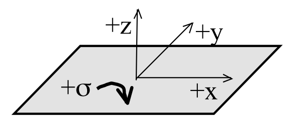
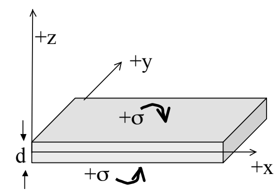
Q5. FLAT SLABS AND CONDUCTORS

a) Griffiths (Ex 2.4) works out the E-field everywhere in space due to an infinite thin plane of charge, with surface charge density σ. Let's assume our sheet lies in the x-y plane. Just sketch the z-component of the E field as a function of z (include both + and - z)

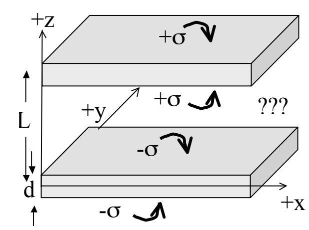


b) Now we have an infinite *conducting slab* of metal with charge density +|σ| on each surface (top *and* bottom) Write a mathematical function for **E**(z) and <u>sketch the result</u>.
(Note: this slab is oriented like the sheet in part a, parallel to the xy plane, infinite in the x-y directions, but this slab has a finite thickness d, in the z-direction).

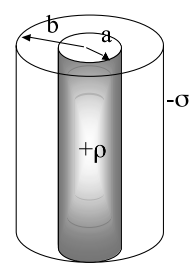


Briefly state what would *change* if the charge density had been negative, i.e. -|σ| on *each* surface, top and bottom.

c) Now imagine *two* such conductors, separated by a distance L>>d. Suppose, for the sake of this problem, that the upper one could still be exactly as described in part b (i.e. it has +|σ| on each surface, top and bottom) and the lower one is like part b only with -|σ| on each surface, top and bottom) Use the principle of superposition to find the E field everywhere in space. (That means between the slabs, outside either slab, but also INSIDE each slab).



As you may recall from Phys 1120, the E field inside of a slab of metal is supposed to be zero. So... what went wrong? Have you applied superposition right? Is Gauss' law invalid here? How does nature resolve this inconsistency?



Q6. COAXIAL CABLE (a static version!)

A long coaxial cable has a uniform volume charge density ρ throughout an inner cylinder (radius *a*) and a uniform surface charge density σ on the outer cylinder (radius *b*). The cable is overall electrically neutral. Find **E** everywhere in space, and sketch it.

Extra Credit: (worth any of the above, but *won't* count off if you don't do it!)

Smectic-C liquid crystals are made of long rodlike molecules with a positive head and a negative tail which pack together to form a long, thin sheet as shown. The volume charge density is obviously complicated, but can be quite successfully modeled with the rather simple form $\rho(x,y,z) = \rho_0 \sinh(z/z_0)$, i.e. uniform in x and y, but varying in z, with $z = 0$ defined to be the middle of the sheet. (That's the "hyperbolic sinh" function)

Find the electric field everywhere in space, in terms of the constants ρ_0, z_0 and the sheet thickness *T*, and sketch it. Where is it biggest?