

**University of Colorado, Department of Physics**  
**PHYS3220, Spring 11, HW#1**  
**due Wed, Jan 12, 9AM at start of class**

On all homework assignments this term please show your work and explain your reasoning. We will grade for clarity of explanation as much as we do for correctness. You may not get full credit for a correct final answer without explanation or derivation.

*This first assignment is intended as a review of some mathematics and a bit of physics. If there is something in this set which sounds unfamiliar to you, it should give you an idea where you might need to catch up.*

1. Consider the complex number  $z = 4 - 4i$ .
  - a) Evaluate  $z^2$  and  $|z|^2 = zz^*$ ?
  - b) Plot the number  $z$  in the complex plane.
  - c) Rewrite  $z$  in the form  $z = Ae^{i\theta}$ . What are the values of  $A$  and  $\theta$ ?
2.
  - a) Given the matrices  $A = \begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix}$ , what is their product  $AB$ ?
  - b) Consider the following equation and solve for  $\lambda$  ( $||$  stands for the determinant):

$$\begin{vmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{vmatrix} = 0 \quad (1)$$

3. Consider the three vectors

$$\mathbf{x} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}, \quad \mathbf{z} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad (2)$$

- a) Are  $\mathbf{x}, \mathbf{y}$  and  $\mathbf{z}$  (in the three-dimensional Euclidean space) linearly independent or dependent? Show your work.
  - b) Do  $\mathbf{x}, \mathbf{y}$  and  $\mathbf{z}$  form a basis of the three-dimensional Euclidean space? Why or why not?
4. Evaluate the following integrals ( $\delta$  is the Dirac delta function)

$$\int_{-7}^7 (x-7)\delta(x+5)dx \quad \text{and} \quad \int_1^3 (x+2)\delta(x)dx \quad (3)$$

Explain your work (use words!).

**- There are more problems on the back -**

5. Consider the function  $f(x) = 3e^{-(x^2/a^2)}$ .
- a) Sketch a graph of  $f(x)$  vs.  $x$  for  $a = 10$ . Be sure to indicate and scale the axes.
  - b) Does the function show any symmetries? What is the limit  $x \rightarrow \pm\infty$ ?
  - c) What is the significance of the parameter  $a$  in the exponent?
6. What does it mean for two (one-dimensional) functions to be orthogonal? Give also a specific example of two orthogonal functions.
7.   a) What is the relation between the energy  $E$  and the frequency  $f$  of a photon? What is the approximate energy of a photon of yellow light (both in Joules and eV)?
- b) What is the relation between the wavelength  $\lambda$  and the momentum  $p$  of a particle? Roughly what kinetic energy should an electron have in order that its wavelength be of roughly an "atomic" distance scale?
8. Write down an equation describing a sinusoidal traveling wave (in 1D). Tell (using words and/or equations) what in your equation gives the speed and the direction of the wave.