CHAPTER 1

Perspectives in Quantum Physics

"Why do some textbooks not mention *complementarity*? Because it will not help in quantum mechanical calculations or in setting up experiments. Bohr's considerations are extremely relevant, however, to the scientist who occasionally likes to reflect on the meaning of what she or he is doing." – Abraham Pais [1]

I. Introduction

I.A. Notions of Classical and Quantum Reality

Albert Einstein considered the aim of physics to be "the complete description of any (individual) real situation (as it supposedly exists irrespective of any act of observation or substantiation)." [2] His statement on the *purpose* of science speaks also of his predisposition toward thinking of the universe itself in terms of *realist* expectations: there *is* an objective reality that exists independent of any human observation. In other words, a *complete* description (or theory) of that objective reality is minimally comprised of elements in one-to-one correspondence with physical quantities (such as position or momentum) that are assumed to have definite, objectively real values at all times. [3]

Such assumptions about the nature of reality are built into the equations of classical mechanics – in describing the position of a free electron with a given momentum at some later time, it is already assumed the electron was initially located at some definite, single point in space (x_0), and that its specific momentum (p) *predetermines* its definite location (x) at all later times (t):

$$(1.1) \qquad x(t) = x_0 + \frac{p}{m_e} \cdot t$$

Just as with the assumption of *universal time* for all observers in Galilean relativity, these classical assumptions are based on intuitive notions grounded in everyday experience, and it may not occur to classical thinking that these are even assumptions to begin with, or anything other than axiomatic.

Contrast this with an expression from quantum mechanics for the wave function (Ψ) describing a free electron with definite momentum (and therefore definite energy, E) as a function of position and time:

$$(1.2) \qquad \Psi(x,t) = A \exp\left[\frac{i}{\hbar}(p \cdot x - E \cdot t)\right].$$

Although the electron's momentum is well defined (Einstein would say it has *reality*), its location may only be described in terms of the probability for where it might be found when observed, which (according to Born's probabilistic interpretation of the wave function) is given by the modulus squared of this complex exponential:

$$(1.3)\quad \rho[x]=|\Psi(x)|^2=\left(A^*\exp\left[-\frac{i}{\hbar}p\cdot x\right]\right)\cdot\left(A\exp\left[\frac{i}{\hbar}p\cdot x\right]\right)=|A|^2=\text{constant}$$

(where the energy/time term has been suppressed). When its momentum is certain, the probability density for its location is constant, and the electron has an equal likelihood of being found anywhere in space – in the mathematics of quantum physics, the location of this free electron is not well defined, and the outcome of a position measurement cannot be predicted with any certainty. If, as Einstein assumed, the electron always exists as a localized particle and is indeed located at a specific point in space at all times (its position also has reality), then a probabilistic (statistical) description of the true state of that electron must be considered *incomplete*. [3] A physical quantity that has some definite value, but is not described by a theory, is known as a *hidden variable*.

According to quantum mechanics, the observables **p** and **x** are *incompatible* (their mathematical operator representations do not commute):

$$(1.4)\quad [\hat{p},\hat{x}]=\hat{p}\cdot\hat{x}-\hat{x}\cdot\hat{p}=\frac{\hbar}{i}\qquad\leftrightarrow\qquad\Delta x\cdot\Delta p\geq\frac{\hbar}{2}$$

and so the position and momentum of a particle cannot be simultaneously described with arbitrary precision. If, in this scenario, the electron does not actually exist at some single, definite location until observed, then the theory of quantum mechanics is *not necessarily* an incomplete description of that reality.

In 1935, Einstein (along with Boris Podolsky and Nathan Rosen, collectively known as EPR) posited a second assumption about the nature of reality (which they considered to be "reasonable"): "If, at the time of measurement, two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done in the first system." [3] This intuitive *assumption of locality* says that the outcome of a measurement performed on some System A can have no influence (or dependence) on any measurement performed on some other System B that is sufficiently isolated from the first. With their *condition of completeness* and the *assumption of locality* in hand, EPR argued that the position and momentum of a particle can be logically demonstrated to have simultaneous reality, and that the quantum mechanical description is therefore incomplete.

Originally formulated in terms of position and momentum measurements, the EPR argument has been reframed [4] in terms of spin measurements performed on systems of *entangled* particles. We imagine a pair of spin-1/2 fermions (Particles A & B) somehow formed in a state of zero total spin angular momentum and traveling in opposite directions.[1] Individual measurements of each particle's spin projection along any given axis will always yield one of two values (*up* or *down*, +1 or −1, however we choose to designate them). Moreover, spin measurements

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[1] The argument does not depend on how this is done, but one method would involve preparing a positronium atom in a singlet state, and then dissociating the electron-positron bound state in such a way that the total linear and angular momentum of the system are conserved. [5]

performed on these entangled fermion pairs will always yield opposite values, so long as the measurements are performed along the same axis. In this way, a spin measurement performed along the z-axis for just one of the particles is sufficient for predicting with 100% certainty the outcome of a spin measurement performed on the second particle along that same axis; the actual measurement on the second particle need not be performed, but can be done so as to confirm the predicted outcome. The same is true for spin measurements performed along the x-axis, or any other axis we choose, so long as the axis of orientation is the same for both analyzers. Quantum mechanics says the operators for the x-component and the z-component of spin angular momentum are non-commuting, and therefore obey a similar uncertainty relation as with position and linear momentum (the components of spin angular momentum for a particle cannot be simultaneously specified along two different axes with arbitrary precision):

(1.5) $\left[\hat{S}_x, \hat{S}_z\right] \neq 0 \quad \leftrightarrow \quad \Delta S_x \cdot \Delta S_z \neq 0$

Now suppose the spin of Particle A is measured along the z-axis: an outcome of +1 for Particle A means that a similar measurement performed on Particle B will *always* yield the result of −1, *before* any such measurement on Particle B is actually made. The *assumption of locality* says that any measurement performed on Particle A can have no causal influence on the outcome of any measurement performed on Particle B.[2] EPR would then argue that the z-component of spin for Particle B must have had a definite (real) value at the time of its separation from Particle A, and that this value can be found without disturbing Particle B in any way. If the measurement on Particle B is instead performed along the x-axis, EPR would conclude that the spin projection for Particle B is now simultaneously specified along two different axes, both x (by the second measurement on Particle B) and z (by the first measurement on Particle A); they therefore have simultaneous reality, which is precluded in the quantum mechanical description. [Eq. 1.5] It follows that quantum mechanics offers an incomplete description of the objectively real state of Particle B.

In defense of the completeness of quantum physics, Niels Bohr took issue mainly with EPR's claim of *counterfactual definiteness* – there can be no definite statements (according to Bohr) regarding the outcomes of quantum measurements that haven't been performed. [6] He further insisted that no definitive line could be drawn between the measurement apparatus and the system being measured: "An independent reality in the ordinary [classical] physical sense can […] neither be ascribed to the phenomena nor to the agencies of observation." [7] Bohr ultimately went so far as to redefine the purpose of science: "It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature." [8]

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[2] Assuming the two measurements are performed at space-like separations (the second measurement lies outside the light cone of the first), then special relativity precludes any cause-and-effect relationship between the two events.

I.B. Philosophy or Science?

It is generally agreed in the physics community that Bohr emerged triumphant in this debate, [8] though many physicists of today might feel hard-pressed to say exactly why. If anything, it has been argued that the positivistic[3] aspects of the *Copenhagen Interpretation* [9] (often referred to as the *orthodox* interpretation of quantum mechanics [10]) have contributed to its popularity over the years by allowing physicists to set aside questions of completeness and locality, and instead just use the wave function to “shut up and calculate!” [11] All the same, it was anyways widely believed that J. von Neumann had successfully ruled out the possibility for hidden quantum variables in 1932. [12]

Such beliefs went largely unchallenged [4] until the appearance in 1964 of a groundbreaking paper by J. S. Bell, who had come to realize that Einstein’s assumptions were not just a matter of philosophical taste, and could be put to experimental test. [13] In his own discussion of the EPR argument, Bell maintained the assumption of locality in his demonstration that a more complete description of an entangled system of particles could never be specified in terms of hidden variables (a set of one or more unknown parameters, λ). If the result for Particle A is a function of the orientation of its Stern-Gerlach analyzer (unit vector ***a***) and the hidden parameters (λ); and if the outcome for Particle B is similarly a function of both the orientation of its Stern-Gerlach analyzer (***b***) and of λ, we may write this as

(1.6) $A(a,\lambda) = \pm 1 \quad \& \quad B(b,\lambda) = \pm 1$,

where A and B represent the measurement outcomes for Particles A & B, respectively. The assumption of locality may expressed as

(1.7) $A \neq A(a,b,\lambda) \quad \& \quad B \neq B(a,b,\lambda)$

which says merely that A cannot depend on how the other analyzer is oriented, and similarly for B. The anticorrelated nature of measurement outcomes along similar axes may be written as

(1.8) $A(a,\lambda) = -B(a,\lambda)$.

We may then find the expectation value in this local hidden variable (HV) theory for the *product* of the two measurements, by summing over all possible values for the hidden variables, weighted by some probability distribution for the hidden parameters (ρ):

(1.9) $E_{HV}(a,b) \equiv \left\langle \left(\vec{S}_1 \cdot a\right)\left(\vec{S}_2 \cdot b\right)\right\rangle_{HV} = \int d\lambda\, \rho(\lambda) A(a,\lambda)\, B(b,\lambda)$

We now show that the product of the hidden variable expectation values (where locality has been assumed) must obey an inequality that is violated by the predictions of quantum mechanics. We start by writing down the expression:

(1.10) $E_{HV}(a,b) - E_{HV}(a,c) = \int d\lambda\, \rho(\lambda)\left[A(a,\lambda)B(b,\lambda) - A(a,\lambda)B(c,\lambda)\right]$

where ***c*** is some other unit vector along which the spin projection might be measured. Using (1.8) this may be rewritten as

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[3] In this context, we are referring to a refusal to speculate on that which can’t be observed (measured).

(1.11) $E_{HV}(a,b) - E_{HV}(a,c) = -\int d\lambda\, \rho(\lambda)\left[A(a,\lambda)A(b,\lambda) - A(a,\lambda)A(c,\lambda)\right]$,

and then factored by recognizing that the square of any measurement outcome must be equal to +1, so that

(1.12) $E_{HV}(a,b) - E_{HV}(a,c) = -\int d\lambda\, \rho(\lambda)A(a,\lambda)A(b,\lambda)\left[1 - A(b,\lambda)A(c,\lambda)\right]$.

We must also have that:

(1.13) $\left|A(a,\lambda)A(b,\lambda)\right| \le +1$

so that taking absolute values in Eq. 1.12, and using the fact that $\rho(\lambda)$ is normalized, gives what is now known as *Bell's inequality*:

(1.14) $\left|E_{HV}(a,b) - E_{HV}(a,c)\right| \le 1 + E_{HV}(b,c)$.

The quantum mechanical expectation value for the product of spin measurements is

(1.15) $E_{QM}(a,b) \equiv \left\langle \left(\vec{S}_1 \cdot a\right)\left(\vec{S}_2 \cdot b\right)\right\rangle_{QM} = -\, a \cdot b = -\cos(\phi)$,

where ϕ is the angle between the unit vectors a and b. The equivalent expression for (1.14) in terms of the quantum mechanical (QM) expectation values is

(1.16) $\left|E_{QM}(a,b) - E_{QM}(a,c)\right| \le 1 + E_{QM}(b,c)$.

There are a variety of angles for which this quantum mechanical inequality holds, but for the simple case where the three vectors are situated at 60^0 to each other, so that $\hat{a} \cdot \hat{b} = \cos\left(60^0\right)$, $\hat{b} \cdot \hat{c} = \cos\left(60^0\right)$ & $\hat{a} \cdot \hat{c} = \cos\left(120^0\right)$ we find:

(1.17) $1 \;=\; \left|\frac{1}{2} - \left(-\frac{1}{2}\right)\right| \;\le\; 1 - \frac{1}{2} \;=\; \frac{1}{2}$

which clearly violates Bell's inequality. Because quantum mechanics correctly predicts the observed expectation values (see below), it follows that at least one of EPR's assumptions (realism and/or locality) is not valid when describing quantum phenomena. If locality is instead *not* assumed in the above argument:

(1.18) $A = A(a,b,\lambda) \quad \& \quad B = B(a,b,\lambda)$

(the outcome for each measurement depends on the orientation of *both* analyzers), there are many functions (A & B) for which the quantum mechanical expectation value (Eq. 1.15) is reproduced, [14] and so it is the assumption of locality that must be set aside, leaving open the possibility for *non-local* hidden variable theories [4]

In 1969, Clauser, et al. generalized Bell's theorem to realizable experiments by allowing for detector inefficiencies, and for the possibility that the measurement correlations are imperfect (less than 100%). [15, 16] From all of this we may conclude that: (A) No local hidden variable theory can reproduce all of the predictions of quantum mechanics; and (B) An experiment may now be devised to differentiate between the two. Various refinements have been made, and a number of loopholes closed over the years, [17-21] but the first definitive test of the assumptions of *Local Realism* was made in 1981 by Alain Aspect and colleagues, [22] when they measured the polarization correlation rate for entangled photon pairs emitted in a radiative atomic cascade. [Fig. 1.1]

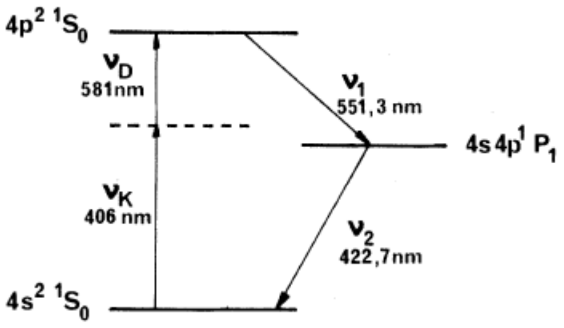

FIG. 1.1. Relevant energy levels of calcium. The atoms are excited by a two-photon absorption process (ν_K and ν_D), and then decay by the emission of two visible photons (ν_1 and ν_2) that are correlated in polarization. [22]

In this experiment, entangled photon pairs were created using a calcium 40 cascade that yields two visible photons (υ_1 and υ_2). The calcium atoms were pumped to the upper level of the cascade from the ground state by two-photon absorption; the average decay lifetime of the intermediate decay state is $\tau = 4.7$ ns. An atomic beam of calcium (with $\rho = 3\times 10^{10}$ atoms/cm^3) was irradiated at 90^0 by two laser beams with parallel polarizations, the first a krypton ion laser ($\lambda_K = 406.7$ nm), then with a Rhodamine 6G dye laser tuned to resonance for the two-photon process ($\lambda_D = 581$ nm). With each laser operating at 40mW, the typical cascade rate was $\sim 4\times 10^7$ per second. [22]

In its ground state, calcium 40 has two valence electrons outside a closed shell; with their spins oppositely aligned, the total angular momentum (spin plus orbital) of this state is J = 0. The upper level of the cascade is also a J = 0 state, and the intermediate state is J = 1, so that the excited atom has two possible decay paths (m = +1 or m = −1) on its way to the ground state. By conservation of angular momentum, any photon pair (υ_1 & υ_2) that *happen to be emitted back-to-back* in this process must therefore have the same circular polarization: either both right-handed (R) or both left-handed (L). The entangled state of the two photons may then be written as:

(1.19) $|\Psi_{12}\rangle = |R_1\rangle|R_2\rangle + |L_1\rangle|L_2\rangle$.

Einstein would argue that each atom always decays by either one path or the other, so that each photon pair is produced in just one of the two polarization states with equal probability (determined by some hidden parameter), but that we cannot know which one until the photon pair is observed. He would say that the superposition state describing each photon pair is a reflection of *classical ignorance* (a lack of knowledge regarding the true state of the photon pair). Bohr would argue that the superposition state is a reflection of a more *fundamental uncertainty*, and that each photon pair exists in an indeterminate superposition state until measured. Observing only one of the two photons instantly *collapses* the superposition at random into just one of the two definite states with equal probability. The collapse must be instantaneous if the two measurements occur at space-like separation, since there would be no time for a signal to travel between the two photons regarding how they should behave when they encounter a polarizer.

Aspect measured the rate of coincidental detection of back-to-back photon pairs with the same type of polarization along a variety of relative angles, and found that these measurements violated the generalized Bell's inequality by more than 13 standard deviations, providing strong evidence against *any* local hidden-variable theory. [Fig. 1.2]

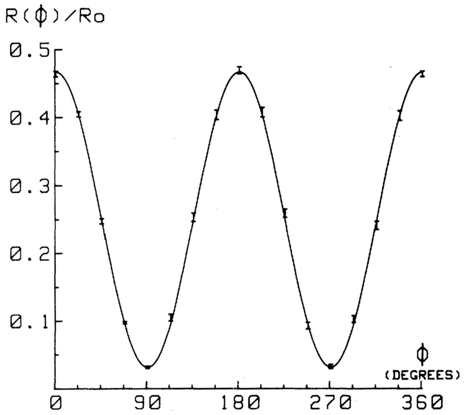

FIG. 1.2. Results of the first Aspect experiment testing Bell's inequality – normalized coincidence rate as a function of relative polarizer orientation. Error bars represent one standard deviation, and the curve drawn through the data points is not a best-fit curve, but rather what is predicted by quantum mechanics. [22]

I.C. Wave-Particle Duality and Ontological Flexibility

In arguing for the incompleteness of quantum mechanics, Einstein was essentially questioning whether quantum mechanics could be used to describe the real state of individual particles, or merely a statistical distribution of measurement outcomes for an ensemble of similarly prepared systems (e.g., a coherent beam of single photons or electrons), where the final distribution of results is determined by some set of unknown, hidden parameters (initial position and/or momentum, for example). Does the *instantaneous collapse of the wave function* represent a change in knowledge of the observer regarding the true state of an individual system, or does it represent a physical transition for that system from an indeterminate state to one that is definite? Erwin Schrödinger famously questioned exactly when this so-called *collapse* is supposed to take place, when he ironically proposed a thought-experiment in which a macroscopic object (in this case, a cat in a box) is imagined to be in a superposition of two states (dead or alive) right up until the moment it is observed (when we open the box). [23] By 1950, Einstein had few allies remaining in the assault on realism in physics, as he expressed in a letter to Schrödinger from that time:

> "You are the only contemporary physicist, besides Laue, who sees that one cannot get around the assumption of reality – if only one is honest. Most of them simply do not see what sort of risky game they are playing with reality – reality as something independent of what is experimentally established. They somehow believe that the quantum theory provides a description of reality, and even a complete description; this interpretation is, however, refuted most elegantly by your system of radioactive atom [plus] cat in a box, in which the Ψ-function of the system contains the cat both alive and [dead]. Is the state of the cat to be created only when a physicist investigates the situation at some definite time? Nobody really doubts that the presence or absence of a [dead] cat is something independent of observation. But then the description by means of the Ψ-function is certainly incomplete, and there must be a more complete description." [24]

The practical significance of EPR's argument (and its refutation via Bell's Theorem) was not truly realized until the mid-to-late 1970's – as reflected in how their paper had a total of only 36 citations in *Physical Review* before 1980, but added 456 more citations in the period from 1980 to June 2003. [25] A similar trend can be seen [Fig. 1.3] in the belated, sudden increase in citations of Bell's paper, "On the Einstein-Podolsky-Rosen Paradox." [26]

It was the development during the 1970's and onward of experimental techniques for isolating and observing single quantum objects like photons, electrons, and atoms that caused physicists to take ideas about "quantum weirdness" seriously. According to Aspect: "I think it is not an exaggeration to say that the realization of the importance of entanglement and the clarification of the quantum description of single objects have been at the root of a *second quantum revolution*, and that John Bell was its prophet." [27]

Long before any such experiments were possible, physicists were already arguing for their preferred interpretations of quantum mechanics in terms of the individual behavior of quanta. In his own book on quantum mechanics, Dirac considers a thought experiment wherein individual photons are directed toward a beam splitter, and have equal probability of being transmitted or reflected. The quantum mechanical wave describing the probability for detecting the photon coherently splits at the beam splitter (it is both reflected and transmitted), but the result of any detection "must be either the whole photon or nothing at all. Thus the photon must suddenly change from being partly in one beam and partly in the other to being entirely in one of the beams." [28] Dirac argued this as a point of principle, despite there being no specific experimental evidence at the time for this assertion.

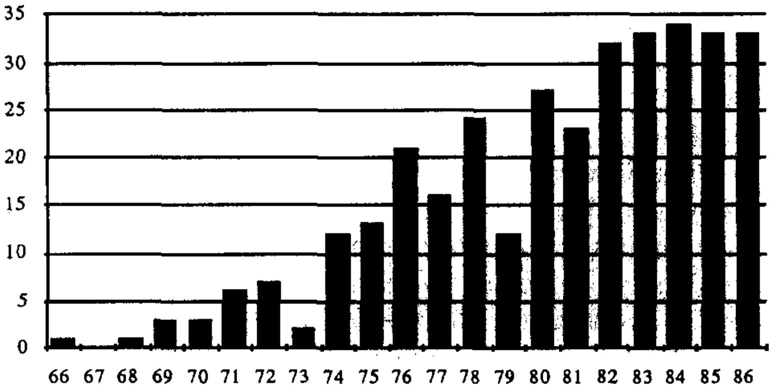

FIG. 1.3. Number of annual citations [1966-1986] of "On the Einstein-Podolsky-Rosen Paradox," by J. S. Bell, *Physics* **1**, 195 (1964). [26]

Definitive evidence for such behavior was most elegantly demonstrated by Grangier, Roger and Aspect in 1986 [29] using the same calcium 40 cascade photon source used in Aspect's first experiments testing the assumptions of Local Realism. [Fig. 1.4] Their first experiment was designed to demonstrate the particle-like behavior of photons; the second was meant to demonstrate the wave-like behavior of photons in a nearly identical situation. The experimental setup was along the lines proposed by Dirac in the thought experiment described above.

In each of these two experiments, the first photon (ν_1) emitted in the calcium cascade serves as a trigger when detected in PM1, and the electronics opens a gate for a time equal to twice the lifetime of the intermediate state ($2\tau \sim 10$ ns), telling counters N_A & N_B to expect a second photon (ν_2); a coincidence counter (N_C) is triggered if both photomultipliers fire during the short time the gate is open. The path to the beam splitter (BS1) from the source is collimated such that the second photon must have been one that was emitted back-to-back with the first, which greatly reduces the luminosity of this "single-photon" source. A set of mirrors (M_A & M_B) direct the second photon toward either PMA (it was reflected at BS1) or PMB (it was transmitted at BS1).

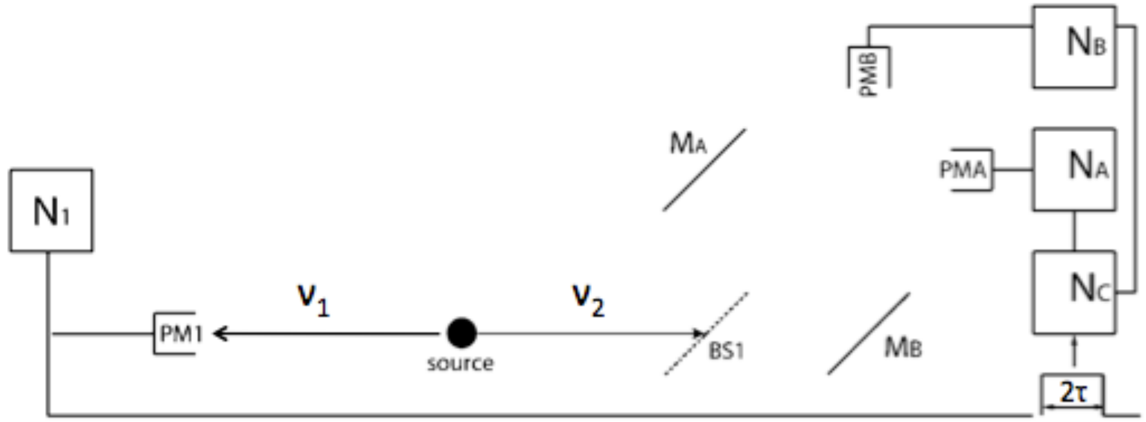

FIG. 1.4. Schematic diagram for the first anticoincidence experiment by Grangier, et al. PM1, PMA & PMB are photomultipliers; N_1, N_A, N_B &N_C are counters; BS1 is a beam splitter; M_A and M_B are mirrors. [29]

If the photon energy were coherently split at the beam splitter (wave-like behavior) it would be expected that energy would be deposited into the photomultipliers coincidentally, and that they would therefore fire together more often than separately. If the photon were instead either transmitted or reflected at the beam splitter (but not both; particle-like behavior) we expect the photomultipliers to always be triggered separately, so long as only one photon is in the apparatus at a time. We can quantify how often this is happening by defining an *anticorrelation parameter* (α):

$$(1.20)\quad \alpha \equiv \frac{P_C}{P_A \cdot P_B}$$

where P_A is the probability for PMA to fire, P_B is the same for PMB, and P_C the probability for both to fire during the time the gate is open.

- If individual photons are always detected in only one photomultiplier or the other (particle-like behavior), then $\alpha = 0$ since P_C must be zero (there is zero probability that the two detectors click together during the time the gate is open).
- If the detectors are firing randomly and independently, then $\alpha = 1$, since P_C is just the product of P_A and P_B. This would be consistent with either many photons being present in the apparatus at once, or with waves depositing energy over time and randomly triggering the detectors.
- If there is a clustering of counts (higher than random probability that both detectors click together; consistent with wave-like behavior), then $\alpha > 1$ (i.e. P_C is greater than just the product of P_A and P_B).

The results for this first experiment show that, more often than not, photons are being detected in either one photomultiplier or the other during the time the gate is open, which is consistent with the predictions for particle-like behavior ($\alpha \geq 0$), while being inconsistent with the predictions for wave-like behavior ($\alpha \geq 1$). [Fig. 1.5 – the solid curve represents the predictions of quantum mechanics; error bars represent one standard deviation. It is necessary to extrapolate the measurements to "single-photon" intensity ($\alpha = 0$) since the apparatus has a *dark rate* of ~300 counts/second.] We *interpret* these results as meaning that each photon must always take one path or the other on its way to detection – it is either reflected at the beam splitter or transmitted (but not both).

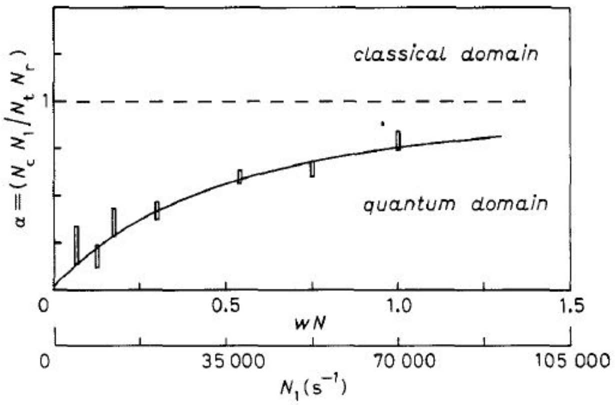

FIG. 1.5. Results from the first photon anticoincidence experiment performed by Grangier, et al. The anticorrelation parameter plotted as a function of the counting rate in PM1 (equivalently, the luminosity of the "single-photon" source). [29]

The experiment can be run a second time after a slight modification is made: inserting a second beam splitter into the paths taken by the photons (BS2). [Fig. 1.6] With BS2 in place, a photon might reach PMA by transmission at BS2 (Path A – it was reflected at BS1) or by reflection at BS2 (Path B – it was transmitted at BS1). Either way, a detection in PMA or PMB yields no information about the path taken by a photon to get there. According to quantum mechanics, the probabilities for photon detection in either PMA or PMB are oppositely modulated, as a function of the pathlength difference between Paths A & B. This means that, for certain pathlength differences (δ), *all* of the photons are detected in PMA and *none* are detected in PMB; and there are intermediate phases where detection in either photomultiplier is equally likely. [Fig. 1.7]

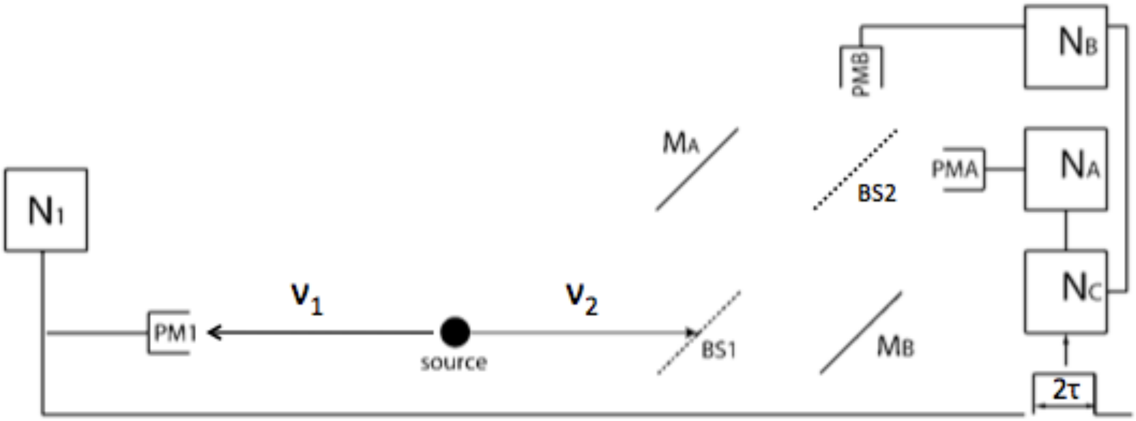

FIG. 1.6. Schematic diagram for the second anticoincidence experiment by Gragier, et al. PMA, PMB & PM1 are photomultipliers; N_1, N_A, N_B &N_C are counters; BS1 and BS2 are beam splitters; M_A and M_B are mirrors. [29]

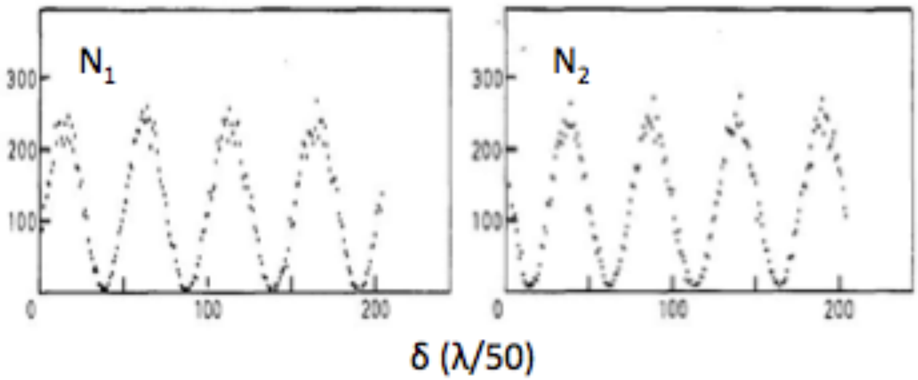

FIG. 1.7. Results from the second photon anticoincidence experiment performed by Grangier, et al. Counting rates at 15-second intervals for each of the two counters N_1 (left) and N_2 (right) as a function of path length difference (δ - in units of $\lambda/50$). For this experiment, $\alpha = 0.18$. [29]

We *interpret* these results as meaning that each photon is coherently split at each beam splitter – it is both reflected *and* transmitted at BS1 (wave-like behavior, in contradiction with our conclusions from the first experiment) for, as the argument goes, how else could changing something about Path B affect the behavior of the photons that were supposed to have only taken Path A? For this second experiment, the anticorrelation parameter was small ($\alpha = 0.18$), and so we must conclude that each photon is interfering with itself along the two paths (as opposed to many photons interfering with each other).

How are we to make sense of these two experiments, when the results seem to indicate contradictory behavior for the photons at BS1? How does each photon know whether BS2 is in place or not (whether we are conducting the first experiment or the second) when it first encounters BS1? Dirac would argue that every photon is coherently split as a delocalized wave at each beam splitter in *both experiments*, and that in each case the wave instantly collapsed down to a point when interacting with a detector.[4] Bohr would argue (more philosophically) that each photon is, from the very beginning, interacting with the entire apparatus as a whole, and that it behaves as it does at the first beam splitter (particle-like or wave-like) according to which type of behavior is allowed for which type of experiment.

In the end, these are all questions of *ontological category attribution* – it is clear that photons sometimes exhibit particle-like behavior, and sometimes exhibit wave-like behavior, depending on the experiment. Is it possible for photons to simultaneously behave as both particle and wave, for them to simultaneously *straddle* two (classically) distinct ontological categories?[5] A famous thought-experiment was proposed by Wheeler in 1978 [30] (and realized by Hellmuth, et al. in 1987 [31]) to test for this possibility. Imagine a photon entering the apparatus when only one path is available (the photon must take a single, definite path) from source to detector, but then a second path is opened up at the last moment (suddenly, two paths are available). If the photon had already “chosen” to take a single path at the first beam splitter, there should be no opportunity for the photon to interfere with itself, and no interference should be visible in the detectors.

In the actual experiment, [Fig. 1.8] a short-pulsed laser (less than a billionth of a second, with an average of one photon per pulse) was directed at a beam splitter, and the light then passed through 10-meter long optical fibers (in order to increase the transit time by ~30 ns). A Pockels cell (PC-A) in conjunction with a Glans prism was used to effectively insert and remove a path. When a voltage is applied to the Pockels cell, it rotates the plane of polarization of the light within five nanoseconds; the Glans prism then deflects away photons whose polarization has been altered, while transmitting unrotated photons. Therefore, when a voltage is

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[4] Dirac did not take this “mental model” as a literal description of what was happening, but instead considered it to be a picture that helps to make sense of the situation: “One may extend the meaning of the word ‘*picture*’ to include any way of looking at the fundamental laws which makes their self-consistency obvious.” [28]

[5] Ontology concerns itself with the categorization of concepts, physical entities and processes according to their fundamental properties. Entities with similar characteristics belong to similar categories or sub-categories.

applied to the Pockels cell, there is only one path by which a photon could reach the second beam splitter; with no voltage applied, both paths are possible. By randomly applying and removing voltages to the Pockels cell at the required frequency, it was possible to change the nature of the experimental setup after each photon had encountered the first beam splitter.

They found that when the experiment was run with initially only one path open (voltage applied), but then switched to both paths open after the photon had already encountered the first beam splitter (voltage removed), the photon still behaved as though two paths had been available all along, and interference was observed in the detectors. [Fig. 1.9] Wheeler argued that the photon's "choice" as to how to behave at BS1 (like a particle or a wave) must have been made *after the fact* (hence the term *delayed-choice experiment*, which may also refer to the delayed choice made by the observer of which experiment to conduct). [30]

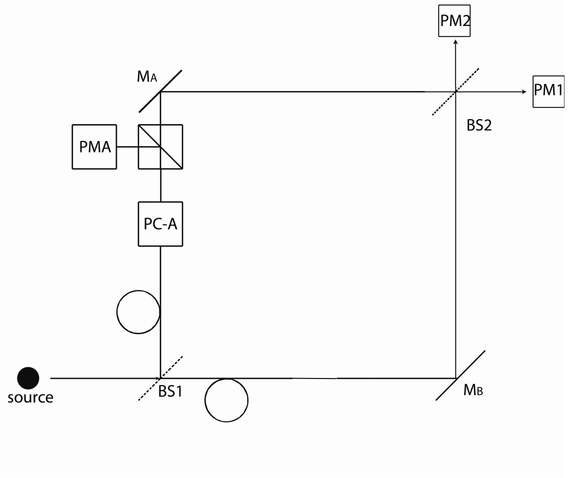

FIG. 1.8. Schematic diagram of the delayed-choice experiment conducted by Hellmuth, et al. PC-A is a Pockels cell used to rotate the plane of photon polarization when a voltage is applied; a Glans prism is used to pass unrotated photons, and to reflect away rotated photons. Only one path to BS2 is available with the voltage applied. [31]

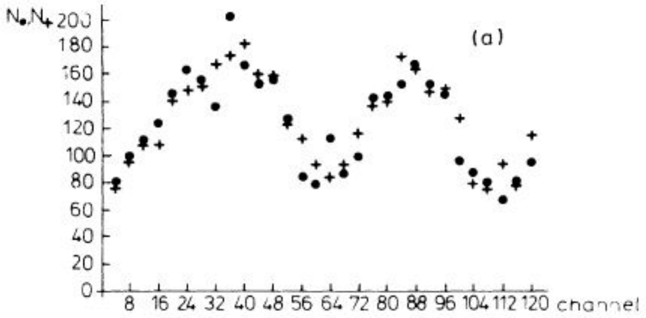

FIG. 1.9. Counting rates in "normal" mode [dots; no voltage applied throughout] and in "delayed-choice" mode [crosses; second path is unblocked after photon encounters BS1] as a function of path length difference. A clear interference pattern is observed in both data sets. [31]

It seems that no matter how an experiment is devised, we observe the behavior of quanta to be particle-like in some circumstances, and consistent with our expectations for classical waves in others, but we cannot demonstrate both types of behaviors simultaneously. Dirac has preemptively offered his interpretation of these experiments: each photon coherently divides at each beam splitter as a delocalized wave, interferes with itself when more than one path is available, and then instantly collapses to a point when interacting with a detector. Niels Bohr would characterize this *dual wave-particle* behavior as *complementary* (but exclusive) features of our ultimately classical understanding of an abstract quantum world. No single classical ontological category (particle or wave) can account for all the results of quantum experiments, but the union of these two complementary concepts allows for a generalized description of the whole. Like the *Yin* and *Yang* of Chinese philosophy, Bohr saw *Complementarity* as an epistemological[6] tool with broader implications; for example, he considered *truth* and *brevity* to be complementary concepts (the more you have of one, the less you have of the other). [1]

This complementary wave-particle duality is not limited to massless photons, but can be seen in the behavior of all kinds of matter. [32-34] A double-slit experiment performed with single electrons [35] is isomorphic to the experiments described above involving single photons. In this experiment, single electrons are passed through two slits and detected one at a time at seemingly random places, yet an interference pattern still builds up over time. [Fig. 1.10] A *matter-wave* interpretation of this result would insist that each electron propagates as a delocalized wave and is coherently split at both slits, interferes with itself, then becomes instantly localized in its interaction with the detecting screen. The *Copenhagen Interpretation* would say each electron's behavior at the two slits must

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[6] Epistemology concerns itself with the nature of knowledge, and how it is acquired. In simplest terms, it addresses the question: How do we know what we know?

be understood in terms of classical waves, and the nature of the detecting apparatus reveals a *complementary* electron behavior that can only be understood in terms of classical particles. Changing the nature of the experimental setup (e.g., blocking one of the slits, removing one of the paths) changes how the behavior of each electron is to be described over the course of the experiment.

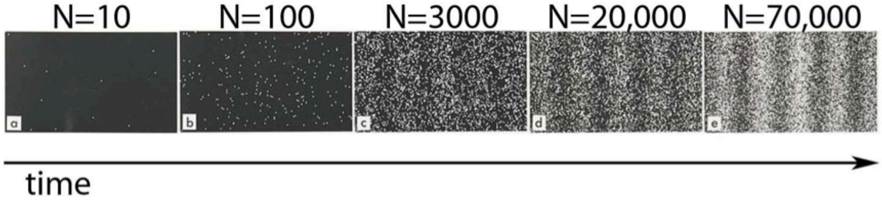

FIG. 1.10. Buildup of a single electron interference pattern. Single electrons are initially detected at seemingly random places, yet an interference pattern is still observed to build up after detecting many electrons. [35]

As epistemological tools, both interpretations are of similar use, in that we may employ either to decide which type of behavior will be observed in a given situation, without actually conducting the experiment: there should be no interference effects when only one path from source to detector is available; when two (or more) paths are allowed, interference will be observed. The two interpretations differ, however, in the physical meaning behind any switch between ontological descriptions of the behavior of quantum entities. In a *matter-wave* interpretation, each electron is viewed as a quantized excitation of a matter-field that (randomly) deposits its energy at a single point in its interaction with a detector. This *instantaneous collapse of the wave function* is viewed as a physical process by which these quantized excitations transition from a delocalized state (wave category) to one that is localized in space (particle category). *Complementarity* views this collapse as a moment when new information is available to the observer regarding the state of the quantum system in its interaction with the measurement apparatus (the line between which is arbitrarily drawn). The experiment reveals two sides of a more abstract quantum whole, each in analogy to classical behavior (particle- or wave-like), but any switch between ontological categories occurs only in the mind of the observer describing the system. Dividing the behavior of quantum entities into (classically) separate ontological categories is seen as a method for making sense of the decidedly nonclassical behavior of quantum entities, in terms of classical concepts intuitively associated with particles and waves.

However you choose to look at it, it should be clear that a proper understanding of quantum physics requires some degree of flexibility in the assignment of ontological categories when describing the behavior of quantum systems, and epistemological tools must be developed for understanding when each type of assignment is (or is not) appropriate for a given situation. A variety of formal interpretations of quantum theory may then be regarded in terms of coherent *epistemological and ontological framings* that guide the process of category assignment according to context. In this way, many difficulties in the conceptual understanding of quantum mechanics may be understood as stemming from varying degrees of commitment to *epistemological and ontological resources* that are in themselves neither right nor wrong, but which lead to incorrect or paradoxical conclusions when inappropriately applied to the description of quantum phenomena. We will see how this view has implications for the teaching and learning of quantum mechanics among introductory modern physics students.

II. Epistemology and Ontology in Physics Instruction

Research into student learning has shown that, in contrast to the straightforward acquisition of facts or skills, there are particular topics in science that are notoriously difficult for students, and where traditional modes of instruction have been demonstrated to be ineffective. Such difficulties in student learning are most generally thought of as stemming from any number of *prior ideas* held by students, which mediate the learning process, and which in some way or other must *change* before a proper (scientifically normative) understanding can be achieved. Precisely what it is that must change during this process of learning, whether it be concepts, beliefs, epistemological framings, or ontologies, is where education researchers primarily diverge. [36]

One line of research posits that many of the conceptual barriers faced by students in learning classical physics can be traced to unproductive or inappropriate degrees of commitment to *ontological category assignments*, and issues of *category inheritance*. It has been noted, for example, that *emergent processes* (such as electric current, resulting from the net motion of individual charged particles) are often alternatively conceptualized by students as *material substances* (electric current as a fluid that can be stored and consumed). [37] The general idea is that, whenever learners encounter some unfamiliar concept, they engage in a (conscious or unconscious) process of ontological categorization, whereby they sort the concept according to whatever information is available at the time. This information may include (but is not limited to) the context in which the concept is introduced, its similarity or co-occurrence with other concepts, or language patterns that give indications to its ontological nature. Once an ontological category (or sub-category) for that concept has been decided upon, it is believed that learners will then automatically associate with that concept the attributes of other concepts that fall within that same category – the new concept *inherits* the characteristics of other concepts that are ontologically similar in the mind of the learner. Many student difficulties in understanding emergent processes in classical physics can then be viewed as arising from the misattribution of properties intuitively associated with material substances. According to Chi, when the category assignment held by the learner is sufficiently distinct from the targeted (scientifically accepted) category, the process of reassignment cannot come about in gradual steps, and the learner must set aside their initial conceptualization in favor of a new conceptualization with other attributes. This *incompatibility hypothesis* motivates Chi's description of *radical conceptual change* in novice learners. [38]

A key question surrounding Chi's hypothesis is: What happens with the original ontology that is to be replaced? In their empirical work, Slotta and Chi make no real assertions regarding the ultimate fate of the original ontologies that are to be ignored by novice students, [39] though they have mentioned that

> "…physics experts do maintain substance-based conceptualizations in parallel with their more normative *process-like* views. In their everyday reasoning, physics experts often use substance-like models of heat, light, and electricity, although they are well aware of the limitations of such models, including when the models should be abandoned. Thus, if the early *substance-like* conceptions are not actually removed or replaced, we can interpret conceptual change as a matter of developing new conceptualizations alongside existing ones and understanding how and when to differentiate between alternatives." [40]

Slotta and Chi are therefore not only allowing for the possibility of *parallel ontologies* in student and expert thinking, they are insisting that productive use can be made of them by experts with a certain amount of sophistication in the flexible use of multiple ontological attributions for a single concept. [39]

Gupta, et al. have recently taken issue with the views of Slotta and Chi on ontologies in student and expert thinking, [41] most specifically with their delineation of ontologies into distinct, normative categories that remain *static*. [42] Gupta, et al. assert that not only do experts and novices often bridge between parallel ontologies, but that in many situations, clear distinctions between ontological categories don't even apply. Their view on *dynamic ontologies* claims that delineations between ontological categories and their associated attributes are not necessarily *rigid* in the minds of both experts and novices, and that they often *blend* material and process conceptualizations in their reasoning. They further take issue with the assumption that any one "scientific concept *correctly* belongs to a single ontological category." [42]

The differences in these two models of learning and cognition can be seen as analogous to the differences between material substances and emergent processes as ontological categories. A view of ontologies as distinct and stable *structures* (which is one way of accounting for the observed robustness of common student misconceptions) is contrasted with a dynamic view of flexible and adaptive ontologies that emerge in real time through the *coordinated activation* of cognitive resources (that are in themselves neither right nor wrong). In this way, the stability of misconceptions observed by Reiner, et al. [37] may be understood as resulting from contextually stable and coherent patterns of resource activation. [43] It is therefore the pattern of resource activation within a given context that must change in the minds of learners, and Gupta, et al. argue this may come about in gradual steps, so that matter-based reasoning can slowly lead to process-based reasoning. [41, 42]

It is possible these two perspectives are not entirely incompatible in the context of classical physics instruction; they may disagree on questions of meta-ontology (ontological attributions as stable cognitive structures versus emergent cognitive processes), but both agree that the learning of new concepts is mediated (and sometimes hindered) by prior knowledge (students do not enter the learning environment as blank slates), and that conceptual difficulties in learning physics often arise from the misattribution of ontological characteristics to unfamiliar concepts. And both agree that a degree of flexibility in switching between ontological attributions is not only possible, but also a *desirable* aspect of expert-like thinking. In the context of quantum physics, however, the wave-particle duality in the behavior of light and matter makes this flexibility *necessary* for a proper understanding of quantum mechanics.

We wish to extend these views on learning to the context of quantum physics in a way that would similarly address difficulties students have with changing their classical conceptions of light and matter. We first hypothesize that the intuitively *realist* perspectives of introductory physics students are reinforced by classical physics instruction, and that instruction in quantum physics can lead to measureable changes in student thinking. [Chapter 2] We will find that the highly contextual nature of student conceptions of light and matter are differentially influenced by the myriad ways in which instructors may choose (or choose not) to address interpretive themes in quantum mechanics, and that these instructional choices manifest themselves both explicitly and implicitly in the classroom. [Chapter 3] We further hypothesize that *realist* expectations among novices and experts in quantum physics are a manifestation of *classical ontological attribute inheritance*; in other words, quantum particles (at least initially, and despite evidence to the contrary) *inherit* many of their classical attributes, which can lead to incorrect or contradictory interpretations of quantum phenomena. We will demonstrate that novice quantum physics students exhibit varying degrees of flexibility in the ontological categorization of the behavior of quanta, and present evidence of students not only switching between ontological attributions both within and across contexts, but also creating a *blended* ontological category for quantum entities, simultaneously classifying them as both particle and wave (most consistent with a *pilot-wave* interpretation of quantum mechanics [4]). Moreover, it will be seen that ontological category reassignment among students can occur piecewise, context by context (particularly in cases where instruction is explicit), and that our findings are not reflective of some sudden, wholesale change in student perspectives on the ontological nature of quanta. [Chapter 4]

III. Motivation and Overview of Dissertation Project

A detailed exploration of student perspectives on the physical interpretation of quantum mechanics is necessary, since these perspectives are an aspect of understanding physics, and have implications for how traditional content might be taught. Introductory modern physics courses are of particular interest since they often represent a first opportunity to transition students away from classical epistemologies and ontologies, to ones that are more aligned with those of practicing physicists.

In terms of assessing student difficulties in quantum mechanics, several conceptual surveys have been developed, [44-50] though most are appropriate for advanced undergraduate and beginning graduate students, since they address such advanced topics as the calculation of expectation values, or the time-evolution of quantum states. Because there does not seem to be a canonical curriculum for modern physics courses, the applicability of assessment instruments designed specifically for this kind of student population must be evaluated course-by-course. The Quantum Physics Conceptual Survey (QPCS) [51] is a recent example of an assessment instrument developed for introductory modern physics students. The authors of the QPCS found that students had the most difficulty with six questions they had classified as *interpretive*; for example, the two survey items with the lowest percentage of correct responses (~20% for each) ask whether, "according to the standard (Copenhagen) interpretation of quantum mechanics," light (or an electron) is behaving like a wave or a particle when traveling from source to detector. These authors also found that not only do a significant number of students perform reasonably well on non-interpretative questions while still scoring low on the interpretative items, there were no students who scored high on the interpretative questions but scored low on the non-interpretative ones. As the authors note, this parallels findings from Mazur [52] when comparing student performance on conventional classical physics problems versus ones requiring a solid conceptual understanding. Their results suggest that many introductory modern physics students may grasp how to use the computational tools of quantum mechanics, without a corresponding facility with notions (such as wave-particle duality) that are at odds with their classical intuitions.

Mannila, et al. [53] have previously explored student perspectives on particle-wave duality and the probabilistic nature of quantum mechanics within the context of a double-slit experiment, where a low intensity beam of quanta passes through a two-slit system and gradually forms a fringe pattern on a detecting screen. Their analysis of open-ended written student responses to a series of questions found they were dominated by "semi-classical" or "trajectory-based" ontologies, and that very few students expressed perspectives that were aligned with expert models, or even productive transitional models.[7] These authors also reported many instances of *mixed* student ontologies within that single context of a double-slit experiment, yet the design of their study provided no opportunity to further question students on any apparent inconsistencies. Our studies have

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[7] Non-local and/or statistical (probabilistic) perspectives, by their standards.

demonstrated that student perspectives on quantum phenomena can vary significantly by context, [54–56, Chapters 2-4] so that it may not always be possible to make generalizations about student beliefs based on investigations within a single context.

This dissertation concerns itself with a detailed exploration and characterization of student perspectives on the physical interpretation of quantum mechanics, and how these perspectives develop within the context of an introductory modern physics course. [Chapter 2] In doing so, we identify variations in teaching approaches with respect to interpretation, and their associated impacts on student thinking. [Chapters 3 & 4] These studies serve to inform the development of instructional materials designed to positively influence student perspectives on quantum physics. [Chapter 5] Further research conducted during the implementation of these materials in a modern physics course for engineering majors allow for an assessment of their effectiveness in influencing student perspectives, and inform their refinement for future use. [Chapter 6]

Chapter 2: Development of Quantum Perspectives – Initial Studies

The first indication that student perspectives are being significantly influenced through formal instruction came from an analysis of student responses to a particular statement on the Colorado Learning Attitudes about Science Survey (CLASS) [57]: *It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct.* There is a clear trend in how student responses to this statement change over the course of a three-semester introductory sequence of physics courses. In a cross-sectional study of student responses (PHYS1 [classical mechanics], N=2200; PHYS2 [classical electrodynamics], N=1650; PHYS3 [modern physics], N=730) we see a shift first from agreement to disagreement, and then back to agreement with this statement. [Fig. 1.11] At the beginning of instruction in classical mechanics (A), more students will agree (40%) with this statement than disagree (26%); yet the number in agreement decreases significantly (B) following instruction in classical physics (to 30%, $p<0.001$), while an increasing number of students disagree (to 39%, $p<0.001$). This trend then reverses itself over a single semester of modern physics (C), at the end of which a greater percentage of students agree with this statement (46%) than at the beginning of classical physics instruction.

We then analyzed the reasoning provided by approximately 600 students in an optional text box following the multiple choice response, in order to establish if their reasons for agreeing or disagreeing had changed. We find that, among students of introductory classical physics, those who disagree with this statement primarily concern themselves with the idea that there can be only one correct result for any physical measurement, while those in agreement are more conscious of the possibility for random, hidden variables to influence the outcomes of two otherwise identical experiments. Few students invoke quantum phenomena when responding before any formal instruction in modern physics; however, a single semester of modern physics instruction results in a significant increase in the percentage of students who believe that quantum phenomena would allow for two valid (but different) experimental results.

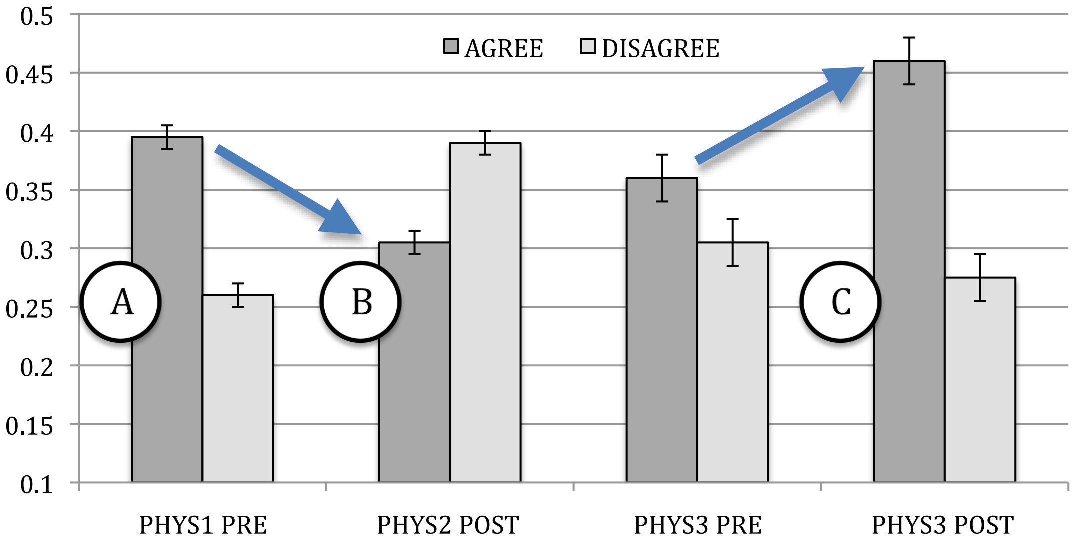

FIG. 1.11. Cross-sectional analysis of student responses to the statement: *It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct* (expressed as a fraction of total responses: PHYS1, N=2200; PHYS2, N=1650; PHYS3, N=730). Error bars represent the standard error on the proportion.

Chapter 3: Quantum Interpretation as Hidden Curriculum – Variations in Instructional Approaches and Associated Student Outcomes

Our efforts to characterize student perspectives on quantum physics were initially limited to the application of coarse labels (discussed below) to student responses to a post-instruction online essay question on interpretations of the double-slit experiment, coupled with responses to a survey statement concerning the existence of an electron's position within an atom. Students from courses that emphasized a *matter-wave* interpretation overwhelmingly preferred a wave description of electrons in the double-slit experiment (each electron passes through both slits and interferes with itself), while responses from courses taught from a *realist/statistical* perspective were dominated by realist interpretations (each electron goes through either one slit or the other, but not both).

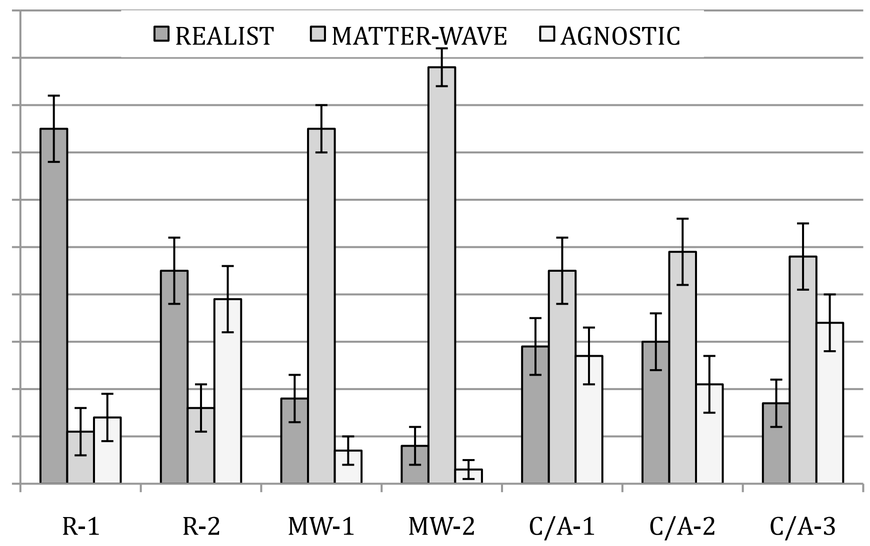

FIG. 1.12. Post-instruction student responses to the double-slit essay question, from seven different modern physics offerings of various instructional approaches [R = *Realist*; MW = *Matter-Wave*; C/A = *Copenhagen/Agnostic*]. Error bars represent the standard error on the proportion; N ~ 50-100 for each course.

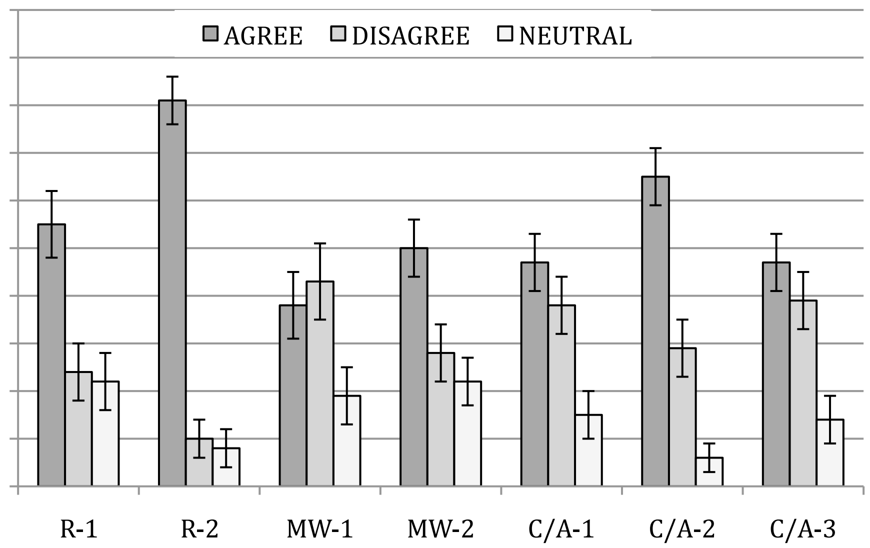

FIG. 1.13. Post-instruction student responses to the statement: *An electron in an atom exists at a definite (but unknown) position at each moment in time*, from seven different modern physics offerings of various instructional approaches [R = *Realist*; MW = *Matter-Wave*; C/A = *Copenhagen/Agnostic*]. Error bars represent the standard error on the proportion; N ~ 50-100 for each course.

Students from courses taught from a *Copenhagen* perspective (or ones that de-emphasized interpretation) offered more varied responses. These latter students were not only more likely to prefer an agnostic stance (quantum mechanics is about predicting the interference pattern, not discussing what happens in between), they were also more likely to align themselves with a realist interpretation. [Fig. 1.12] Of particular interest is how these same students responded to the statement: *An electron in an atom has a definite but unknown position at each moment in time*; [Fig. 1.13] Agreement with this statement would be most consistent with a realist perspective. Students from all of these types of modern physics courses were generally most likely to agree with this statement, including students from courses emphasizing a matter-wave interpretation.

When aggregate student responses from four modern physics offerings are combined so that responses to this statement on atomic electrons are grouped by how those same students responded to the essay question on the double-slit experiment, [Fig. 1.14] we see that students in the (double-slit) *Realist* category were the most consistent, with most preferring realist interpretations in both contexts. However, nearly half of the students who preferred a wave-packet description of electrons in the double-slit experiment would still agree that electrons in atoms exist as localized particles. Only those students who preferred an agnostic stance on the double-slit question were more likely to disagree with the statement than agree, and none of these students felt neutrally about whether atomic electrons are always localized. In addition, a small number of students from all courses (~5%, not shown) chose to agree with both *Matter-Wave* and *Realist* interpretations of the double-slit experiment. These findings indicate a need for more detailed characterizations of student perspectives on quantum phenomena.

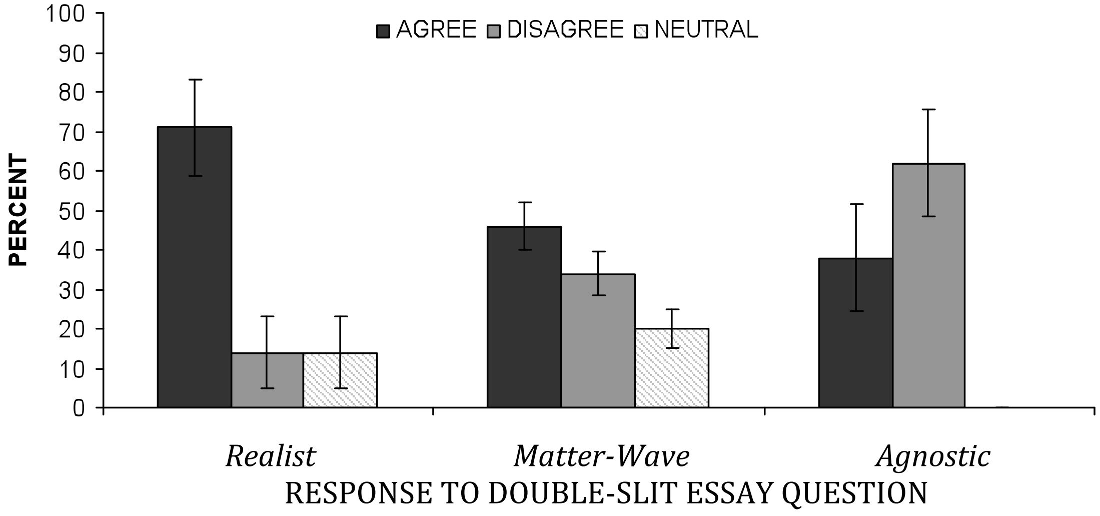

FIG. 1.14. Combined student responses from both PHYS3 courses to the statement: *An electron in an atom has a definite but unknown position at each moment of time*, grouped by how those students responded to the double-slit essay question. Error bars represent the standard error on the proportion (N~60).

Chapter 4: Refined Characterization of Student Perspectives on Quantum Physics

A total of nineteen post-instruction interviews with students from four recent introductory modern physics courses taught at the University of Colorado have demonstrated that, though they may not employ the same formal language as expert physicists, students often invoke concepts and beliefs that parallel those invoked by expert physicists when arguing for their preferred interpretations of quantum mechanics. These parallels allow us to characterize student perspectives on quantum physics in terms of some of the same themes that distinguish these formal interpretations from each other. Of particular significance is the finding that students develop attitudes and opinions regarding these various themes of interpretation, regardless of whether these themes had been explicitly addressed by their instructors in class.

Results from these interviews show that, even when modern physics students have learned about “correct” responses from their instructors (or elsewhere), their classical intuitions may still influence their responses. Similar findings among classical physics students [58, 59] have shown that students most often explained differences between their *personal* and *public* perspectives in terms of responses that made intuitive sense to them (*personal*), versus ones based on their perceptions of scientists’ beliefs (*public*), having not yet reconciled that knowledge with their own intuition. The inconsistent responses of some modern physics students may be similarly understood in terms of competing *personal* and *public* perspectives on quantum physics – when responding in interviews or surveys, some students frequently vacillated between what they personally believed and the answer they felt an expert physicist would give, without always explicitly distinguishing between the two.

A significant number of students from our interviews (ten of nineteen) demonstrated a preference for realist interpretations of quantum phenomena; however, only three of these students expressed personal confidence in the correctness of their perspectives, whereas four others differentiated between what made intuitive sense to them (*Realist*) and what they perceived to be correct responses (*Matter-Wave*). In addition to splits between intuition and authority, some of the seemingly contradictory responses from students may also be explained by their preferences for a *mixed* wave-particle ontology (a *pilot-wave* interpretation, wherein quanta are simultaneously *both* particle *and* wave). The realist and nonlocal beliefs of these three students were at odds with how wave-particle duality was addressed in class by their instructors (i.e. quanta are sometimes described by waves, and sometimes as particles, but never both simultaneously). The remaining nine of nineteen students seemed to express fairly consistent views that could be seen as in agreement with the instructional goals of their instructors. In other words, these students seemed to have successfully incorporated probabilistic and nonlocal views of quanta and quantum measurements into their personal perspectives.

Chapter 5: Teaching Quantum Interpretations – Curriculum Development and Implementation

In exploring student perspectives on quantum physics, it seems natural that students should have attitudes regarding some themes of interpretation, in that these stances are reflections of each student's ideas about the very nature of reality, and the role of science in describing it. Is the universe deterministic or inherently probabilistic? When is a particle a particle, and when is it a wave? Is it unscientific to talk about that which can't be measured? A modern physics curriculum aimed at positively influencing student perspectives should provide students with the tools to formulate answers to such questions for themselves, since simply telling students about "scientifically accepted" answers does not seem to impact students at more than a superficial level.

The question remains: In what ways can student perspectives be addressed at a level appropriate for introductory modern physics students, without sacrificing traditional course content and learning goals? Although many instructors may feel that introductory students do not have the requisite sophistication to appreciate matters of interpretation in quantum mechanics, several authors have developed discussions of EPR correlations and Bell inequalities that are appropriate for the introductory level; [60, 61] relevant experimental tests of the foundations of quantum theory [5, 20, 22, 29, 31-35] may be addressed in a non-technical way. [17-19, 21, 61-64] Questions of interpretation may also be framed in terms of *scientific modeling*, an aspect of epistemological sophistication that is often emphasized in physics education research as a goal of instruction. [65] Moreover, a common lament among physics education researchers is that we are losing physics majors in the first years of their studies by only teaching them 19th-century physics in our introductory courses. Similar issues may arise when modern physics instructors limit course content mostly to the state of knowledge at the first half of the last century, or are reluctant to address questions that are clearly of personal and academic interest to students.

A modern physics course that specifically addresses student perspectives might do so within the following topics (among others):

EPR Correlations/Entanglement: Make explicit the assumptions of determinism and locality in the context of classical physics. The notion of atomic spin may be built up from a semi-classical (Bohr-like) atomic model; the limitations of this deterministic model become evident as it leads to predictions in conflict with experimental observation. Issues of measurement, quantum states and state preparation, and interpretation arise naturally. Indeterminacy and non-local aspects of quantum phenomena are demonstrated with simple probability arguments (thought experiments) [60, 61] and experimental evidence. [5, 17-22, 29, 31-36, 61-64] Address implications for quantum information theory (cryptography, computing, etc…). [5]

Single-Quanta and Delayed-Choice Experiments: The experiments of Aspect et al. demonstrate the complementary particle- and wave-like behavior of quanta, [29] providing opportunities to address various aspects of student perspectives on quantum mechanics enumerated in previous studies. [54-56, 65, 69, 70] Delayed-choice experiments [31] demonstrate the limitations of realist/statistical and pilot-wave interpretations. The basics of these experiments requires a simple understanding of atomic spectra and lasers, polarization and polarizers, beam-splitters [interferometry experiments] and photon detectors [photoelectric effect]. Discussion of these experiments can be facilitated by pointing students to non-technical articles. [17-19, 21, 61-64] Address complementarity as a general principle; help students develop an intuition for when interference effects should be visible, and when not.

The Uncertainty Principle: Discussions of the Uncertainty Principle (UP) follow naturally as a mathematical expression of complementarity. The UP can be framed in terms of Fourier decomposition and the properties of wave-packets. It may also be framed in terms of explicit formal interpretations. A realist/statistical interpretation is embodied in Heisenberg's Microscope. [71] A statistical interpretation concerns separate measurements performed on an ensemble of identically prepared system. [72, 73] Matter-wave and Copenhagen interpretations confront issues of indeterminacy in quantum measurement. Order-of-magnitude estimates can be made using simple models and assumptions, indicating a deeper physical meaning behind the UP beyond simple peculiarities of the measurement process. [5]

Such a curriculum has been implemented in the form of an introductory modern physics course for engineers in the Fall 2010 semester at the University of Colorado. Quantitative and qualitative data have been collected in the form of student responses to questions from previously validated instruments such as the CLASS [57], QMCS [49] and QPCS [51], as well as the same survey items and essay questions employed in our previous studies. [54-56] In this chapter, we discuss the guiding principles behind the development of this curriculum, and provide a detailed examination of specific, newly developed course materials designed to meet these goals. [A broader selection of relevant course materials can be found in Appendix C.] In doing so, we address the appropriateness and effectiveness of this curriculum by considering aggregate student responses to a subset of homework, exam, and survey items, as well as actual responses from four select students. We may employ the framework developed in Chapter 4 to characterize the perspectives of these four students as they progress through the course, and compare their incoming reasoning with how they responded at the end of the semester.

Chapter 6: Teaching Quantum Interpretations – Comparative Outcomes and Curriculum Refinement

Results from these data collections may then be compared with previous incarnations of modern physics courses at the University of Colorado where similar data are available. We also examine student responses to specific exam questions and post-instruction content survey items, in an effort to identify which aspects of the new curriculum were most challenging for students, and propose refinements for the sake of potential future implementations and studies. Course materials specific to interpretation will be compiled and archived in a way that allows future instructors to incorporate them into their own curricula.

References (Chapter 1)

1. A. Pais, *Niels Bohr's Times, in Physics, Philosophy and Polity* (Clarendon Press, Oxford, 1991), p. 23.

2. A. Einstein, "Autobiographical Notes" in *Albert Einstein: Philosopher-Scientist*, P. A. Schillp (Ed.) (Open Court Publishing, Peru, Illinois 1949).

3. A. Einstein, B. Podolsky and N. Rosen, Can quantum mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777 (1935).

4. D. Bohm and Y. Aharonov, Discussion of Experimental Proof for the Paradox of Einstein, Rosen and Podolsky, *Phys. Rev.* **108**, 1070 (1957).

5. G. Greenstein and A. J. Zajonc, The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics, 2nd ed. (Jones & Bartlett, Sudbury, MA, 2006).

6. N. Bohr, Can quantum mechanical description of physical reality be considered complete? *Phys. Rev.* **48**, 696 (1935).

7. Quoted in: A. Pais, *Niels Bohr's Times, in Physics, Philosophy and Polity* (Clarendon Press, Oxford, 1991), p. 314.

8. R. G. Newton, *How Physics Confronts Reality: Einstein was correct, but Bohr won the game.* (World Scientific, Singapore, 2009).

9. W. H. Stapp, The Copenhagen Interpretation, *Am. J. Phys.* **40**, 1098 (1972).

10. See, for example: D. J. Griffiths, *Introduction to Quantum Mechanics, 2nd Ed.* (Prentice Hall, Upper Saddle River, New Jersey, 2004), p. 3.

11. N. D. Mermin, What's Wrong with this Pillow? *Phys. Today* **42**, 9 (1989).

12. J. von Neumann, "Measurement and Reversibility" & "The Measuring Process" in *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1955), pp. 347-445.

13. J. S. Bell, On the Einstein-Podolsky-Rosen paradox, reprinted in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), pp. 14-21.

14. J. S. Bell, "Locality in quantum mechanics: reply to critics," reprinted in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), pp. 63-66.

15. J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Proposed Experiment to Test Local Hidden-Variable Theories, *Phys. Rev. Lett.* **23**, 880 (1969).

16. S. J. Freedman and J. F. Clauser, Experimental test of local hidden-variable theories, *Phys. Rev. Lett.* **28**, 938 (1972).

17. A. L. Robinson, Quantum Mechanics Passes Another Test, *Science* **217**, 435 (1982).

18. A. L. Robinson, Loophole Closed in Quantum Mechanics Test, *Science* **219**, 40 (1983).

19. A. Shimony, The Reality of the Quantum World, *Scientific American* (January 1988, pp. 46-53).

20. W. Tittel, J. Brendel, B. Gisin, T. Herzog, H Zbinden and N. Gisin, Experimental demonstration of quantum correlations over more than 10 km, *Phys. Rev. A* **57** (5), 3229 (1998).

21. A. Watson, Quantum Spookiness Wins, Einstein Loses in Photon Test, *Science* **277**, 481 (1997).

22. A. Aspect, P. Grangier and G. Roger, Experimental Tests of Realistic Local Theories via Bell's Theorem, *Phys. Rev. Letters* **47**, 460 (1981).

23. E. Schrödinger, "The Present Situation in Quantum Mechanics," in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek (Eds.) (Princeton, NJ, 1983) p. 152. Originally published under the title, "Die gegenwärtige Situation in der Quantenmechanik," *Die Naturwissenschaften*, **23**, 807 (1935); translation into English by J. D. Trimmer, 1980.

24. A. Einstein, in *Letters on Wave Mechanics*, K. Przibram (Ed.) (Philosophical Library, New York, NY, 1986), p. 39.

25. S. Redner, Citation Statistics from 110 Years of Physical Review, *Phys. Today* **58**, 52 (2005).

26. L. E. Ballentine, Resource letter IQM-2: Foundations of quantum mechanics since the Bell inequalities, *Am. J. Phys.* **55**, 785 (1987).

27. A. Aspect, "John Bell and the second quantum revolution," in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), p. xix.

28. P. A. M. Dirac, *The Principles of Quantum Mechanics, 3rd ed.*, (Clarendon Press, Oxford, 1947), p. 9.

29. P. Grangier, G. Roger, and A. Aspect, Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences, *Europhysics Letters* **1**, 173 (1986).

30. J. A. Wheeler, in *Mathematical Foundations of Quantum Mechanics*, A. R. Marlow (Ed.) (Academic Press, New York, NY, 1978), pp. 9-48.

31. T. Hellmuth, H. Walther, A. Zajonc and W. Schleich, Delayed-choice experiments in quantum interference, *Phys. Rev. A* **35**, 2532 (1987).

32. R. Gaehler and A. Zeilinger, Wave-optical experiments with very cold neutrons, *Am. J. Phys.* **59** (4), 316 (1991).

33. O. Carnal and J. Mlynek, Young's Double-Slit Experiment with Atoms: A Simple Atom Interferometer, *Phys. Rev. Letters* **66** (21), 2689 (1991).

34. S. Frabboni, G. C. Gazzadi, and G. Pozzi, Nanofabrication and the realization of Feynman's two-slit experiment, *App. Phys. Letters* **93**, 073108 (2008).

35. A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki and H. Exawa, Demonstration of single-electron buildup of an interference pattern, *Am. J. Phys.* **57**, 117 (1989).

36. A. A. diSessa, "A history of conceptual change research: Threads and fault lines," in *Cambridge Handbook of the Learning Sciences*, K. Sawyer (Ed.) (Cambridge University Press, Cambridge, 2006), pp. 265-281.

37. M. Reiner, J. D. Slotta, M. T. H. Chi and L. B. Resnick, Naïve physics reasoning: A commitment to substance-based conceptions, *Cognition and Instruction* **18**, 1 (2000).

38. M. T. H. Chi, Common sense misconceptions of emergent processes: Why some misconceptions are robust, *Journal of the Learning Sciences* **14**, 161 (2005).

39. J. D. Slotta, In defense of Chi's Ontological Incompatibility Hypothesis, *Journal of the Learning Sciences* **20**, 151 (2011).

40. J. D. Slotta and M. T. H. Chi, The impact of ontology training on conceptual change: Helping students understand the challenging topics in science, *Cognition and Instruction* **24,** 261 (2006).

41. A. Gupta, D. Hammer and E. F. Redish, The case for dynamic models of learners' ontologies in physics, *J. Learning Sciences* **19**, 285 (2010).

42. D. Hammer, A. Gupta and E. F. Redish, On Static and Dynamic Intuitive Ontologies, *J. Learning Sciences* **20**, 163 (2011).

43. D. Hammer, A. Elby, R. E. Scherr and E. F. Redish, "Resources, Framing and Transfer" in *Transfer of Learning*, J. Mestre (Ed.) (Information Age Publishing, 2005) pp. 89-119.

44. E. Cataloglu and R. Robinett, Testing the development of student conceptual and visualization understanding in quantum mechanics through the undergraduate career, *Am. J. Phys.* **70**, 238 (2002).

45. C. Singh, Student understanding of quantum mechanics, *Am. J. Phys.* **69**, 8 (2001).

46. C. Singh, Assessing and improving student understanding of quantum mechanics, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

47. C. Singh, Student understanding of quantum mechanics at the beginning of graduate instruction, *Am. J. Phys.* **76**, 3 (2008).

48. J. Falk, Developing a quantum mechanics concept inventory, unpublished master thesis, Uppsala University, Uppsala, Sweden (2005). Available at: http://johanfalk.net/node/87 (Retrieved January, 2011).

49. S. B. McKagan and C. E. Wieman, Exploring Student Understanding of Energy Through the Quantum Mechanics Conceptual Survey, *PERC Proceedings 2005* (AIP Press, Melville, NY, 2006).

50. S. Goldhaber, S. Pollock, M. Dubson, P. Beale and K. Perkins, Transforming Upper-Division Quantum Mechanics: Learning Goals and Assessment, *PERC Proceedings 2009* (AIP, Melville, NY, 2009).

51. S. Wuttiprom, M. D. Sharma, I. D. Johnston, R. Chitaree and C. Chernchok, Development and Use of a Conceptual Survey in Introductory Quantum Physics, *Int. J. Sci. Educ.* **31 (5)**, 631 (2009).

52. E. Mazur, *Peer instruction: A user's manual* (Prentice Hall, New York, NY, 1997).

53. K. Mannila, I. T. Koponen and J. A. Niskanen, Building a picture of students' conceptions of wave- and particle-like properties of quantum entities, *Euro. J. Phys.* **23**, 45-53 (2002).

54. C. Baily and N. D. Finkelstein, Development of quantum perspectives in modern physics, *Phys. Rev. ST: Physics Education Research* **5**, 010106 (2009).

55. C. Baily and N. D. Finkelstein, Teaching and understanding of quantum interpretations in modern physics courses, *Phys. Rev. ST: Physics Education Research* **6**, 010101 (2010).

56. C. Baily and N. D. Finkelstein, Refined characterization of student perspectives on quantum physics, *Phys. Rev. ST: Physics Education Research* **6**, 020113 (2010).

57. W. K. Adams, K. K. Perkins, N. Podolefsky, M. Dubson, N. D. Finkelstein and C. E. Wieman, A new instrument for measuring student beliefs about physics and learning physics: the Colorado Learning Attitudes about Science Survey, *Phys. Rev. ST: Physics Education Research* **2**, 1, 010101 (2006).

58. T. L. McCaskey, M. H. Dancy and A. Elby, Effects on assessment caused by splits between belief and understanding, *PERC Proceedings 2003* **720**, 37 (AIP, Melville, NY, 2004).

59. T. L. McCaskey and A. Elby, Probing Students' Epistemologies Using Split Tasks, *PERC Proceedings 2004* **790**, 57 (AIP, Melville, NY, 2005).

60. D. F. Styer, *The Strange World of Quantum Mechanics* (Cambridge University Press, Cambridge, 2000).

61. N. D. Mermin, Is the moon there when nobody looks? Reality and the quantum theory, *Phys. Today* **38** (4), 38 (1985).

62. M. Tegmark and J. A. Wheeler, 100 Years of Quantum Mysteries, *Scientific American* (February 2001, pp. 68-75).

63. D. Z. Albert and R. Galchen, A Quantum Threat to Special Relativity, *Scientific American* (March 2009, pp. 32-39).

64. B.-G. Englert, M. O. Scully and H. Walther, The Duality in Matter and Light, *Scientific American* (December 1994, pp. 86-92).

65. S. B. McKagan, K. K. Perkins and C. E. Wieman, Why we should teach the Bohr model and how to teach it effectively, *Phys. Rev. ST: Physics Education Research* **4**, 010103 (2008).

66. E. Redish, J. Saul and R. Steinberg, Student expectations in introductory physics, *Am. J. Phys.* **66**, 212 (1998).

67. R. Hake, Interactive-Engagement Versus Traditional Methods: A Six-Thousand-Student Survey of Mechanics Test Data for Introductory Physics Courses, *Am. J. Phys.* **66** (1), 64 (1998).

68. L. C. McDermott, Oersted Medal Lecture 2001: "Physics Education Research – The Key to Student Learning," *Am. J. Phys.* **69**, 1127 (2001).

69. S. B. McKagan, K. K. Perkins and C. E. Wieman, Reforming a large lecture modern physics course for engineering majors using a PER-based design, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

70. S. B. McKagan, K. K. Perkins, M. Dubson, C. Malley, S. Reid, R. LeMaster and C. E. Wieman, Developing and Researching PhET simulations for Teaching Quantum Mechanics, *Am. J. Phys.* **76**, 406 (2008).

71. W. Heisenberg, "The Physical Content of Quantum Kinematics and Mechanics," in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek (Eds.) (Princeton, 1983) p. 62. Originally published under the title, "Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik," Zeitschrift fur Physik, **43**, 172 (1927); translation into English by J. A. Wheeler and W. H. Zurek, 1981.

72. L. E. Ballentine, The Statistical Interpretation of Quantum Mechanics, *Rev. Mod. Phys.* **42**, 358 (1970).

73. L. E. Ballentine, Quantum Mechanics: A Modern Development (World Scientific Publishing, Singapore, 1998).