

CHAPTER 5

Teaching Quantum Interpretations – Curriculum Development and Implementation

“The tao that can be told is not the eternal Tao. The name that can be named is not the eternal Name.” – Lao-tzu, Tao Te Ching

I. Introduction

We wish to address one final question: Can the interpretive aspects of quantum mechanics be addressed at a level that is appropriate and meaningful for introductory modern physics students, without sacrificing traditional course content and learning goals? In fact, it would be hoped that an additional focus on interpretive topics (indeterminacy, the uncertainty principle, wave-particle duality, and the superposition of quantum states) would provide students with tools that would augment their overall understanding of traditional topics (quantum tunneling, atomic models); that discussions of the application of quantum mechanics could subsequently be framed in terms of language that has previously been unavailable to past instructors; and that students may develop more internal consistency in their interpretation of quantum phenomena.

The remainder of this dissertation will concern itself with the development of a modern physics curriculum designed to target these aspects of student thinking, and its recent implementation (Fall 2010) at the University of Colorado in the form of an introductory course for engineering majors. In this chapter, we discuss the guiding principles behind the development of this curriculum, and provide a detailed examination of specific, newly developed course materials designed to meet these goals. [A broader selection of relevant course materials can be found in Appendix C.] In doing so, we address the appropriateness and effectiveness of this curriculum by considering aggregate student responses to a subset of homework, exam, and survey items, as well as actual responses from four select students. [Appendix D contains a larger subset of complete responses from these particular four students.]

II. Curriculum Development and Implementation

It must be *strongly emphasized* from the outset that it is our aim to *improve* upon an already-existing body of work, which has seen contributions from over a dozen physics education researchers and modern physics instructors at the University of Colorado. As was the case for many of the modern physics offerings discussed in these studies, a substantial portion of the course materials we used should be credited to the original work of S. B. McKagan, K. K. Perkins, and C. E. Wieman. Their original course transformations, [1] which served as the basis for our course, incorporated a number of principles learned from physics education research, which include, but are not limited to:

1. Students' attitudes toward science tend to become less expert-like unless instructors are explicit in addressing student beliefs. [2, 3] The original course transformations were explicit in addressing scientific method and logical deduction; experimental evidence and real-world applications; and the uses and limitations of models. [4]
2. Interactive engagement during lecture can lead to higher learning gains than traditional lectures, [5] and can be useful in eliciting known student misconceptions. [6] Concept tests (clicker questions) provide real-time feedback from students, allowing instructors to gauge student understanding, as well as target common misconceptions. Peer discussion during concept tests gives students an opportunity to articulate their knowledge and engage in scientific argumentation in a low-stakes environment. Weekly collaborative homework sessions offer similar benefits for both students and instructors.
3. In order for students to best gain conceptual understanding and reasoning skills, all aspects of the course (including lecture, homework, and exams) should emphasize conceptual understanding alongside numerical problem solving. [1]
4. Interactive simulations used in and outside of the classroom can be useful in helping students to build models and intuition about quantum physics, by providing visual representations of abstract concepts and unobservable processes. [7]

We have argued [Chapter 3] that interpretive themes in quantum mechanics are an often *hidden* aspect of modern physics instruction, according to three criteria: A) These issues are frequently superficially addressed, and in a way that is not meaningful for students beyond the specific contexts in which they arise; B) Students often develop their own ideas regarding these interpretive themes, even when instructors do not adequately attend to them; and C) Those beliefs tend to be more novice-like (intuitively realist) in contexts where instruction is less explicit. We therefore chose to directly confront the kinds of realist beliefs and attitudes that are common to introductory modern physics students, as informed by our own research into quantum perspectives. Our aim was not only to make students consciously aware of their own (often intuitive and tacit) beliefs, but also for them

to acquire the necessary language and conceptual inventory to identify and articulate those beliefs (we are reminded that, even at post-instruction, most of the students in our interviews were not familiar with the word *determinism* in the context of physics, though they had certainly developed opinions about it).

We also chose to make the interpretation of quantum physics a course topic unto itself, primarily framing our discussions in terms of the historical back-and-forth between Albert Einstein and Niels Bohr. And though we decided to be explicit in promoting a matter-wave interpretation of quantum mechanics, our ultimate goal was for students to be able to distinguish between competing perspectives, to have the requisite tools for evaluating their advantages and limitations, and to be able to apply this knowledge in novel situations. In short, instead of trying to tell students what they should and shouldn't believe about quantum physics, we chose to engage them in an explicit, extended argument (with us and amongst themselves) against *Local Realism*. This argument was *extended* in two senses: 1) We were able to augment a number of standard topics (e.g., the uncertainty principle, atomic models) with discussions of interpretive themes; and 2) We introduced several entirely new topics (e.g., delayed-choice experiments) that created additional opportunities for students to explore the sometimes fluid boundaries between scientific interpretation and theory.

The entirety of our research has indicated that wave-particle duality is a particularly challenging topic for students, and wholly relevant to their beliefs regarding the physical meaning of quantum mechanics. Whether emphasized or not, *every* modern physics instructor considered in these studies made mention of the fact that double-slit experiments could be performed with single quanta, which are detected as localized particles, but which together form an interference pattern over time. This phenomenon was often (though not universally) demonstrated in class using the Quantum Wave Interference PhET simulation, [8] as seen in the post-instruction attitude surveys. Due to the distance scales involved, a true double-slit experiment was until recently only a thought experiment, crafted as a demonstration of principle; actual experiments had demonstrated the diffraction of electrons through periodic lattices (essentially, a many-slit experiment). [9] We sought in this course to emphasize connections between theory, interpretation, and experimental evidence, and so augmented these discussions with presentations on experimental realizations of these *Gedanken* experiments. In 2008, Frabboni, et al. employed nanofabrication techniques in the creation of a double-slit opening on a scale of tens of nanometers, which they then used to demonstrate electron diffraction, as well as the absence of interference after covering just one of the two slits (they also present in their paper STM images of the double-slits, formed by an ion beam in a gold foil, with both slits open and with one slit covered). [10] Tonomura, et al. have produced a movie that literally demonstrates single-electron detection and the gradual buildup of a fringe pattern. [11, 12] Students from prior courses were often skeptical as to whether such an experiment (where only a single electron passes through the apparatus at a time) could be done in practice – in this way, they can observe the phenomenon with their own eyes.

In addressing the tendency for students to interpret wave-particle duality as implying that quanta may act simultaneously as both particle and wave, we devoted

additional class time to a presentation of the single-photon experiments discussed in the first chapter, which are essentially isomorphic to the double-slit arrangement (the double-slit and the beam splitters play analogous roles). One of the guiding principles in the design of this curriculum was to avoid as much as possible the expectation for students to accept our assertions as a matter of faith. Rather than describing what the experimentalists had meant to demonstrate, and then simply asserting that they had been successful, we presented students with the actual reported data, which required the use of statistical arguments, and thereby afforded further opportunity to highlight the role of probability in quantum mechanics. These single-photon experiments demonstrate for students the dualistic nature of photons, and provide strong evidence against realist interpretations, but only if the details and results of the experiments are accessible to them, and so we omitted from our presentation extraneous technical details, while still focusing on the very process of designing the experiment and creating an adequate photon source. Devoting an entire class period to these experiments afforded us the time to walk students through each of the three experiments, and for them to debate the implications of each, while creating further opportunities to distinguish between a collection of data points, and an interpretation of their meaning.

Just as importantly, these experiments call for an explicit discussion of the need for ontological flexibility (without naming it as such) in the description of quanta, from which we may easily segue into a comparison of competing interpretations. Bohr has offered up *Complementarity* as a guide to making sense of this dualistic behavior (note that we refrain here from digressing into a full explication of the Copenhagen Interpretation for our students), but this interpretation can come across as more a philosophical sidestepping of the *measurement problem*, than its scientific resolution. Dirac's matter-wave interpretation allows for a consistent description of the behavior of photons at the beam splitters, but the physical collapse of the wave function is not described by any equation, and accepting it as physically real requires a fairly large leap of faith in itself. Moreover, these discussions allow for the explicit development of quantum epistemological tools [two paths = interference; one path = no interference] that may facilitate student understanding, and which may be applied to novel situations.

Before presenting and evaluating any newly developed course materials, some general comments on the structure of the course in which they were used are in order. As with other modern physics courses described here, our course spanned a 15-week academic semester, and consisted of large lectures ($N \sim 100$) meeting three times per week, together with weekly online and written homework assignments, and twice-weekly problem-solving sessions staffed by the instructors. Course transformations for this semester occurred primarily during Weeks 6-8, spanning a total of nine lectures. [13] Instruction was collaborative, with two lead co-instructors (one of them the author, the other a PER faculty member associated with our prior investigations into quantum perspectives), along with two undergraduate learning assistants, [14] who helped facilitate student discussion during lecture. As with the original course transformations, we omitted topics from special relativity in order to win time for the introduction of new material, without eating into the usual time at the end of the course devoted to applications.

We selected Knight's *Physics for Scientists and Engineers* [15] as a textbook (mostly for its readability), but the lectures did not follow the textbook very closely (if at all), and it was necessary to provide students with outside reading materials for many of the new topics (e.g., single-photon experiments [16] and Local Realism [17]); these *Scientific American* articles were chosen for their non-technical, but scientifically correct, treatment of interpretive ideas and foundational experiments in quantum mechanics. An online discussion board was created to provide students with a forum to anonymously ask questions about the readings, and to provide answers to each other; following these discussions granted us ample opportunity to assess how students were responding to many of the new ideas we were introducing.¹ A total of 13 weekly homework assignments consisted of online submissions and written, long-answer problems; there was a broad mixture of conceptual and calculation problems, both requiring short-essay, multiple-choice, and numerical answers. There were a total of three midterm exams (held outside of class) and the course ended with a cumulative final exam. In lieu of a long answer section on the final exam, students were asked to write a 2-3 page (minimum) final essay on a topic from quantum mechanics of their choosing, or to write a personal reflection on their experience of learning about quantum mechanics in our class (an option chosen by ~40% of students). As opposed to a formal term paper, this assignment was meant to give students the opportunity to explore an aspect of quantum mechanics that was of personal interest to them. The almost universally positive nature of the feedback provided by students in their personal reflections is evidence for the popularity and effectiveness of our transformed curriculum, and its practical implementation.

The progression of topics may be broken into three main parts: classical and semi-classical physics; the development of quantum theory; and its application to physical systems). A complete explication and analysis of the entirety of this new curriculum and associated course materials would be beyond the scope of this dissertation, and so we conclude this section with a summary overview of the progression of topics covered in this class. The remaining sections of this chapter will address specific lecture, homework and exam materials, alongside aggregate and individual student responses from the Fall 2010 semester.

PART I – Classical and Semi-Classical Physics (Weeks 1-5, Lectures 1-12): Introduction to the course and the philosophy behind its structure. Review relevant mathematics (complex exponentials, differential equations, wave equations); review classical electricity and magnetism, Maxwell's equations and how they lead to a wave description of light. [Lectures 1-3] Cover properties of waves (superposition, interference); address the wave properties of light through Young's double-slit experiment and Michelson interferometers. Introduce polarization and polarizing filters in anticipation of future topics concerning photon detection. [Lecture 4]

¹ Students were asked to make a contribution to the discussion board each week of the latter half of the course as part of their homework assignment, but no efforts were made to verify their participation, and students were free to put as little or as much effort as they liked into their postings.

Discuss photoelectric effect experiment in terms of classical wave predictions, contrasted with a particle description of light. Photomultiplier tubes are introduced as an application of the photoelectric effect, but also so as to not be unfamiliar to students when they arise in the future. An emphasis on the physical meaning of the work function foreshadows applications of the Schrödinger equation to square well potentials. [Lectures 4-5] Review potential energy curves and explicitly relate them to models of physical systems. Discuss modeling in physics, and lead discussions on the differences between observation, interpretation, and theory. [Lectures 6-7] Relate spectral lines (Balmer series) to atomic energy levels via the energy-frequency relationship established in the photoelectric effect, and use them to make inferences about quantized atomic energy levels. Emphasize the differences between photon absorption (an all-or-nothing process) and collisional excitation of atoms (discharge tubes). [Lectures 8-9] Apply knowledge of photon absorption and emission processes to the construction of lasers. Compare and contrast wave and particle descriptions of light, and address their ranges of applicability. Relate wave intensity to the probability for photon detection in the context of a single-photon double-slit experiment (simulated). [Lectures 10-11] Review for the first exam. [Lecture 12]

PART II - Development of Quantum Theory (Weeks 5-8, Lectures 13-24): Review potential and kinetic energy of electrons in a Coulomb potential, then introduce the semi-classical Bohr model of hydrogen. Discuss the ad-hoc mixture of classical and quantum rules, along with the strengths and weaknesses of the model. Introduce de Broglie waves and his atomic model as an explanation for quantized energy levels. [Lectures 13-14] Review the behavior of magnets in response to homogeneous and inhomogeneous magnetic fields; employ a Bohr-like model for atomic magnetic moments, and explicitly address classical expectations for their behavior in a Stern-Gerlach type apparatus.² [Lecture 15] Use repeated spin-projection measurements to introduce ideas of: quantization of atomic spin (two-state systems); definite versus indefinite states; state preparation; and probabilistic descriptions of measurement outcomes. Digress briefly to cover classical probability, statistical distributions, and the calculation of expectation values. [Lectures 16-17] Offer multiple interpretations of repeated spin measurements for future evaluation, and discuss the differences between classical ignorance and quantum uncertainty. Introduce *entanglement* in the context of distant, correlated atomic spin measurements, and relate to topics in quantum cryptography. Make explicit definitions of *hidden variables*, *locality*, *completeness* and *Local Realism*, followed by a discussion of the EPR argument and its implications for the nature of quantum superpositions. Use the notion of *instruction sets* as a first pass deterministic model, and reveal its limitations in the face of observation.³ [Lectures 18-19] Use the single-photon experiments by Aspect, et al. as an argument against simultaneous wave and particle descriptions of photons. Invoke *Complementarity*

² Much of the lecture and homework material on magnetic moments and repeated spin measurements was inspired by D. F. Styer. [18]

³ The “Local Reality Machine” argument is due to N. D. Mermin. [17]

and other interpretive stances in the establishment of quantum epistemological tools. [Lectures 20-21] Relate conclusions drawn from single-photon experiments to an understanding of the double-slit experiment performed with single electrons. Plane wave descriptions of single particles lead to more generalized notions of quantum wave functions and their probabilistic interpretation. Introduce the Heisenberg uncertainty principle, its mathematical expression, and various interpretations of its physical meaning. [Lectures 22-23] Review for second exam. [Lecture 24]

PART III - Applications of Quantum Mechanics (Weeks 9-15, Lectures 25-44): Motivate the Schrödinger equation through analogies with electromagnetic waves and solve for free particles in terms of plane waves. [Lectures 25-26] Introduce square well potentials (infinite and finite) and use them to model electrons in wires. [Lectures 27-28] Frame discussions of quantum tunneling as a consequence of the wave behavior of matter, then apply tunneling to scanning tunneling microscopes, and a description of alpha-decay. [Lectures 29-31] Apply the Schrödinger equation to an electron in a 3-D Coulomb potential and develop the Schrödinger model of hydrogen. Generalize to multi-electron atoms and account for the periodicity of elements. [Lectures 32-35] Review for the third exam. [Lecture 36] Explain molecular bonding and conduction banding in terms of the superposition of atomic potentials and electron wave functions. [Lectures 37-39] Apply these concepts to the theory of transistors and diodes. [Lecture 40] Finish with a foray into radioactivity, nuclear energy, and nuclear weapons (at student request) [Lectures 41-42] Review for the final exam. [Lectures 43-44]

II.A. Assessing Incoming Student Perspectives and Conceptual Understanding

Developing pre-instruction content surveys for modern physics students is more difficult than assessing incoming student beliefs about classical physics, for several reasons. First, it is expected that introductory students with little knowledge of Newtonian mechanics will have already developed intuitions (right or wrong) through their everyday experiences about the motion of macroscopic objects; in contrast, our everyday experiences with applied quantum physics (e.g. computers) provide little insight into the rules governing the behavior of quantum entities. Second, many of the learning goals for modern physics courses concern topics, such as quantum tunneling, that are entirely foreign to introductory students; and so, for example, it is practically meaningless to discuss incoming student responses to questions regarding deBroglie wavelengths and transmission probabilities, since the distributions of responses are often statistically indistinguishable from guessing.⁴ Third, the broad variation in learning goals

⁴ For example, an (unpublished) analysis by this author of pre-instruction QMCS scores from several modern physics courses showed them to be normally distributed about an average consistent with random guessing.

among modern physics instructors indicates a lack of consensus in the physics community regarding canonical course content, making it difficult to develop general assessment instruments that would be appropriate for a range of course offerings and student populations.

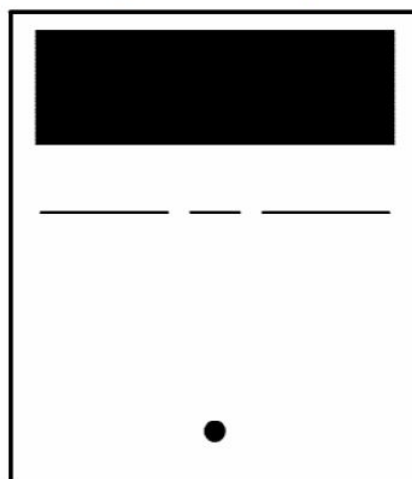
We therefore constructed a content survey (administered in the first week of the semester) that would be appropriate for the specific learning goals of this course, by culling questions from a variety of previously validated assessment instruments, [19-21] and then limiting pre-instruction items to ones where it could be reasonably expected that students would have specific reasons for responding as they do beyond random guessing (i.e., prior content knowledge or intuitive expectations). So, for example, even if students have never heard of a double-slit experiment performed with electrons, their intuitive notions of particles might still lead them expect a pattern that would be consistent with their expectations for macroscopic particles in an analogous situation (these questions taken from the QPCS; [21] student responses are given in Table 5.I):

The following questions refer to the following three experiments:

In one experiment electrons pass through a double-slit as they travel from a source to a detecting screen. In a second experiment light passes through a double-slit as it travels from a source to a photographic plate. In a third experiment marbles pass through two slit-like openings as they travel from a source to an array of collecting bins, side-by-side.

The right-hand figure diagrams the experimental setup, and the figures below show roughly the possible patterns that could be detected on the various screens.

Top view of experimental set-up (not to scale)



Possible patterns (not to scale)



A through C represent some patterns which might be observed. If you think none is appropriate, answer D. Which pattern would you expect to observe when...

6. ...*marbles* pass through the double opening?
7. ...*electrons* pass through the double slit?

TABLE 5.I. Pre- and post-instruction student responses (in percent) to items 6 & 7 from the content survey used in the modern physics course from Fall 2010. The standard error on the proportion for all cases was ~5% (Pre: N=110; Post: N=88). Students shift from expecting *similar* behavior for marbles and electrons, to expecting *different* behavior.

PRE (N=110)	A	B	C	D
Marbles	15%	60%	21%	5%
Electrons	14%	51%	35%	1%
POST (N=88)	A	B	C	D
Marbles	9%	86%	2%	2%
Electrons	0%	12%	88%	0%

We note first that, prior to instruction, the most popular response to both items was the same (B), indicating that most students expected *similar* behavior for both electrons and marbles in similar situations. These responses are consistent with our hypothesis that incoming students have particle-like expectations for the behavior of all matter. These items saw dramatic shifts in post-instruction student responses, indicating that most students expected *different* behavior for macroscopic marbles and microscopic electrons by the end of the course. The class average on the pre-instruction content survey was 46% (+/- 2%), and the average for post-instruction items common to both surveys was 80% (+/- 3%), for a normalized gain of 0.63. [See Appendix C for a complete list of pre- and post-instruction items from the content survey, with an item-by-item summary of student responses.]

As part of their first homework assignment, students were also asked to complete the same online attitudes survey administered in other courses. We summarize below the distribution of pre-instruction student responses (in terms of agree/neutral/disagree) for the entire class, along with the full responses of four select students. These four students (denoted as A, B, C & D) were not selected in order to be representative of any one group of students; their responses instead serve to demonstrate typical pre/post differences in student reasoning, even when overall responses to survey items (agreement or disagreement) had not changed. Their specific homework submissions and exam responses will later serve to address the question of whether topics that are new to the curriculum are accessible to students. Closely following these four students also allows for a more detailed exploration of the curriculum's influence on some of the aspects of student thinking that had been targeted, without making unnecessary extrapolations to the entire class population. Together, these two types of pre-instruction data will allow us to establish a baseline on incoming student perspectives.

1. It is possible for physicists to carefully perform the same measurement and get two very different results that are both correct.

PRE	Agree	Neutral	Disagree
Class (N=94)	0.65	0.13	0.22

Student A: (Agree) I feel that no matter how much technology advances or how much we learn, we can never fully understand how the world works and in many cases, we use outcomes of experiments to look at phenomena in different ways that may or may not be entirely correct in the real world. For instance, looking at the behavior of light as both a particle and wave. So, yes, I believe that an experiment can be conducted twice with different outcomes.

Student B: (Agree) I don't know of any examples, but the fact that quantum physics has some things that seem counter-intuitive and contradict classical physics, it seems that this could be a possibility.

Student C: (Strongly Agree) What the two physicists are measuring could be highly unstable and sensitive to multiple external stimulus.

Student D: (Strongly Agree) It is possible for identical measurements to produce different results if that which is being measured can exist in more than one state at the same time. Thus, one would not know whether the subject of the measurement is the object in one state or the other. Interpreting this question differently, one could comment on the fact that the very act of measuring itself introduces new elements into a system, and thus actually changes the outcome of the measurement.

Overall class responses are consistent with prior results, with a strong majority of students agreeing with this statement, though it should be cautioned that students vary greatly in the reasoning behind their responses, as seen in Chapter 2. Students A, B & D have all invoked quantum phenomena in their agreement with this statement, with varying degrees of sophistication. Student D speaks of quantum superposition and the physical influence of observation; Student A notes that light may be described as both particle and wave; Student B simply states his impression that quantum mechanics will challenge his intuition, so perhaps this statement might be true. Student C's reasoning is more consistent with the idea that chaotic, hidden variables may randomly influence the outcomes of similar measurements – an attitude commonly seen in pre-instruction responses.

2. The probabilistic nature of quantum mechanics is mostly due to physical limitations of our measurement instruments.

PRE	Agree	Neutral	Disagree
Class (N=94)	0.46	0.32	0.22

Student A: (Neutral) I really don't know enough about quantum theory to make a guess on that. However, even our most basic assumptions about the world have sometimes proven to be incorrect and quantum seems to involve so much theory that we can never really be sure if it actually functions the way physicists think it does or if we are coming up with theories that just fit what we find without even seeing the entire picture.

Student B: (Strongly Agree) I believe that in the future, we would be able to make more accurate and exact assertions due to technological advances and would not need to rely on probability.

Student C: (Neutral) I don't know what quantum mechanics is yet.

Student D: (Strongly Disagree) The probabilistic nature of quantum mechanics is a fundamental property of the system. For example: it is impossible to define (not just measure) the position and momentum of an electron at the same instant in time (Heisenberg's uncertainty principle). Thus, the uncertainty exists outside of the instruments used to try to measure those properties. (I would really, really like to learn the math behind these statements!)

Responses here were more varied than with the first statement, though agreement amongst the class is moderately favored; the individual responses range from strong agreement to strong disagreement. The two neutral responses from Students A & C indicate a similar tentativeness due to a lack of knowledge about quantum mechanics; Students A & B both echo a common perception that knowledge in science is itself tentative, and that profound progress (technological or theoretical) often upends previously held beliefs. In contrast, Student D identifies quantum uncertainty as fundamentally different from experimental uncertainty, explicitly stating there are limits not only on the precision of simultaneous measurements, but also on simultaneous quantum descriptions of incompatible observables (position and momentum, specifically).

3. When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time.

PRE	Agree	Neutral	Disagree
Class (N=94)	0.72	0.09	0.19

Student A: (Strongly Agree) An electron is a fundamental piece of an atom, though it moves extremely fast, so at any point in time, yes it does occupy a position being that it is matter.

Student B: (Strongly Agree) An electron is a particle, and every particle has a definite position at each moment in time.

Student C: (Agree) Because I have been told this since 9th grade.

Student D: (Agree) An electron occupies a single definite position at any given point in time. It is only our measurement (and thus knowledge) of that position at any given point in time that is subject to the Heisenberg uncertainty principle, where either the position or the momentum of the electron may be measured to a high level of precision, but not both.

As expected, a strong majority of incoming students chose to respond in a manner that would be consistent with realist expectations; all four of our individual students were in agreement that atomic electrons should exist as localized particles. The reasoning invoked by Students A & B is consistent with our hypothesis of *classical attribute inheritance* – electrons, as a form of matter, have the same properties as macroscopic particles, including a localized position at all times; Student A further implies that the uncertainty in an electron’s position can be attributed to its swift, chaotic motion about the nucleus – similar to the hidden-variable style reasoning of Student C in response to the first survey item. Here, Student C makes an appeal to authority: the idea of localized electrons conforms to what he has been told in school since (presumably) first learning about the structure of atoms. Most interestingly, Student D is explicit in asserting the realist belief that electrons always exist as localized particles; he claims it is our simultaneous *knowledge* of incompatible observables that is constrained by the uncertainty principle.

4. I think quantum mechanics is an interesting subject.

PRE	Agree	Neutral	Disagree
Class (N=94)	0.85	0.13	0.02

Student A: (Strongly Agree) From the examples I have heard and some of the theory, I think quantum mechanic is very interesting.

Student B: (Strongly Agree) I think that I'm going to learn that what I would think is correct is actually completely incorrect. Plus, it just sounds cool.

Student C: (Neutral) I don't know yet.

Student D: (Strongly Agree) Quantum mechanics fascinates me precisely because it is so counterintuitive. I want to challenge my perception of the world, and there are few better ways to do that than QM. It is also interesting to me because I am much more used to physics on very large, indeed cosmic scales. It is especially interesting to see how the world of the unimaginably tiny and the world of the unimaginably large interact...

5. I have heard about quantum mechanics through popular venues (books, films, websites, etc...)

PRE	Agree	Neutral	Disagree
Class (N=94)	0.61	0.19	0.20

Student A: (Strongly Agree) [BLANK]

Student B: (Strongly Disagree) I'm completely out of the "physics loop" and hope to get more into it in this class!

Student C: (Agree) I read part of the book In Search Of Schrodinger's Cat by John Gribbin

Student D: (Agree) In high school, I got a taster of quantum mechanics through generalized physics books, but nothing more in depth. Beyond that, my knowledge of quantum mechanics is limited, and comes primarily from several online lectures by MIT (through itunes U) and several from the University of Madras (posted on youtube).

The reported incoming interest in quantum mechanics for these students is somewhat higher (85%) than is usually seen in a course for engineering majors (~75%; and comparable with typical incoming attitudes among physics majors; see Chapter 6). Because we have no other reason to believe that students from this semester would be any different from previous populations for this course, we can only speculate that this is what resulted from all four members of the instruction team hyping the excitement of quantum physics on the first day of lecture. And as with previous introductory modern physics courses, a majority of students reported having heard *something* about quantum mechanics before enrolling in the course, which underscores the fact that incoming students are not entirely blank slates when it comes to quantum physics, and will certainly bring *some* preconceived notions into the course – incoming students will have impressions about the nature of quantum mechanics, positive or negative.

With these considerations in mind, it seems reasonable to conclude that this particular group of students held incoming attitudes and beliefs that were typical of similar student populations (as measured by these specific assessments), and to assert that any aggregate student outcomes associated with the implementation of this curriculum should not be attributed to there being anything unique about this particular class. We have no means of objectively assessing just how representative Students A – D are of the overall student population, but it is our subjective opinion (based on the experience of studying a wide variety of modern physics offerings over the span of several academic years) that Students A, B & C represent several points of view that are common among incoming engineering students. It is also our subjective assessment that Student D holds a relatively sophisticated view on quantum mechanics for an incoming student, but one that could be categorized as *Realist/Statistical* in light of his explicit belief in the localized nature of electrons, and his assertion that the uncertainty principle constrains simultaneous *knowledge* of incompatible observables.

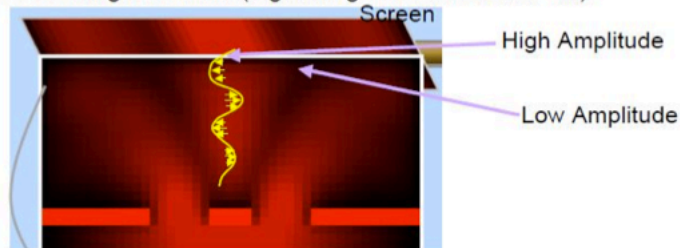
II.B. Lecture Materials

In their end-of-term reflective essays, the topics most frequently cited by students as having influenced their perspectives on quantum physics were the single-quanta experiments with light and/or matter, and so we focus our attention here on one lecture (#20) primarily devoted to the experiments performed by Aspect, et al. (as described in Chapter 1). Topics from immediately prior to this lecture included: hidden variables, Local Realism, and indeterminacy in quantum mechanics. [Lectures 18-19] Our primary objectives for this lecture were for students to understand how two similar experimental setups can lead to dramatically different observations; to highlight the differences between observation and inference (interpretation of experimental facts); and to provide experimental evidence that contradicts the simultaneous attribution of particle and wave characteristics to photons.

Recall the Double-Slit Experiment

E-field describes probability of finding light there

Electromagnetic wave (e.g. hitting screen of double slit)



Describe EM wave spread out in space.

Probability of detection (peak / trough)
 $\sim (\text{Amplitude of EM wave})^2$

L20.S01. Students are reminded that the double-slit experiment can be performed with single photons, which are detected individually. Wave intensity is associated with the probability for detection, which is greater in locations where there is constructive interference.

What is a Photon?

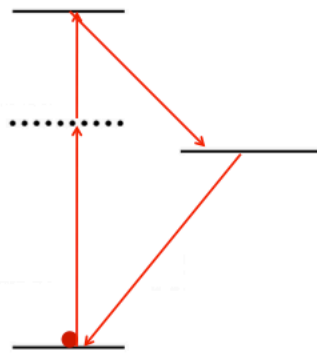


"...each photon interferes only with itself. Interference between different photons never occurs.

P. A. M. Dirac, *The Principles of Quantum Mechanics* (1947).

L20.S02. Dirac offered his interpretation of these kinds of experiments long before they could be realized: each photon must pass through both slits as a delocalized wave and interfere with itself; interference with other photons does not occur.

Single Photon Source (1986)



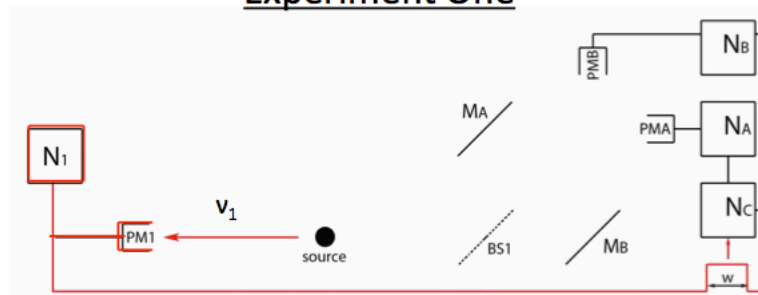
ν_1 and ν_2 are emitted back-to-back.

Why two-photon excitation? Why not a single laser pulse of 5.18 eV?

- Calcium atoms are excited by a two-photon absorption process ($E_K = 3.05 \text{ eV}$) + ($E_D = 2.13 \text{ eV}$).
- The excited state first decays by single photon emission ($E_1 = 2.25 \text{ eV}$).
- The lifetime of the intermediate state is $\tau \sim 5 \text{ ns}$.
- High probability the second photon ($E_2 = 2.93 \text{ eV}$) is emitted within $t = 2\tau$

L20.S03. A “single-photon source” was employed by Aspect in 1986 to explore the wave-particle duality of photons. The two-step excitation process greatly reduces the intensity of the source, where the goal is to detect only specific photons: ones emitted in a two-step, back-to-back de-excitation process.

Experiment One



- Detection of first photon (ν_1) is counted by N_1 .
- A signal is sent to tell the counters (N_A , N_B & N_C) to expect a second photon (ν_2) within a time $w = 2\tau$.

L20.S04. Detection of the first photon (ν_1) in PM1 signals the counters to await the detection of the second photon (ν_2). The gate is open for a time equal to twice the lifetime of the intermediate state, making it highly probable that a second photon was emitted during that time period.

Experiment One

If the second photon (v_2) is detected by PMA, then the photon must have been...

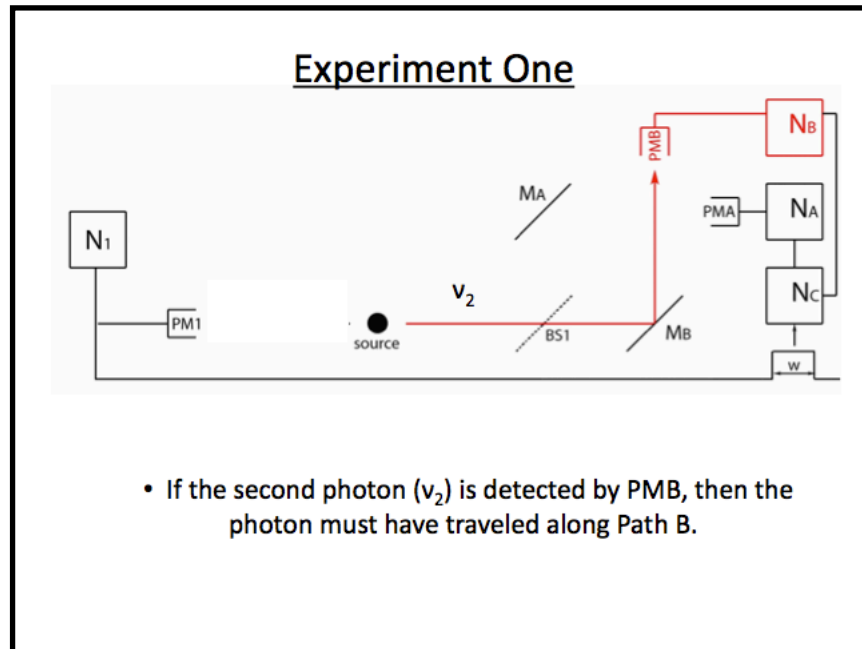
A) ...reflected at BS1.
 B) ...transmitted at BS1
 C) ...both reflected and transmitted at BS1.
 D) Not enough information.

L20.S06. With a little discussion, students quickly converged on (A). The greatest student confusion arose from the schematic nature of the diagram, which implies there is open space between BS1 and the two photomultipliers, which might allow for a photon reflected at BS1 to reach PMB. This question helps check that students understand the purpose of each element of the experimental setup (beamsplitter, mirror, detector, counter).

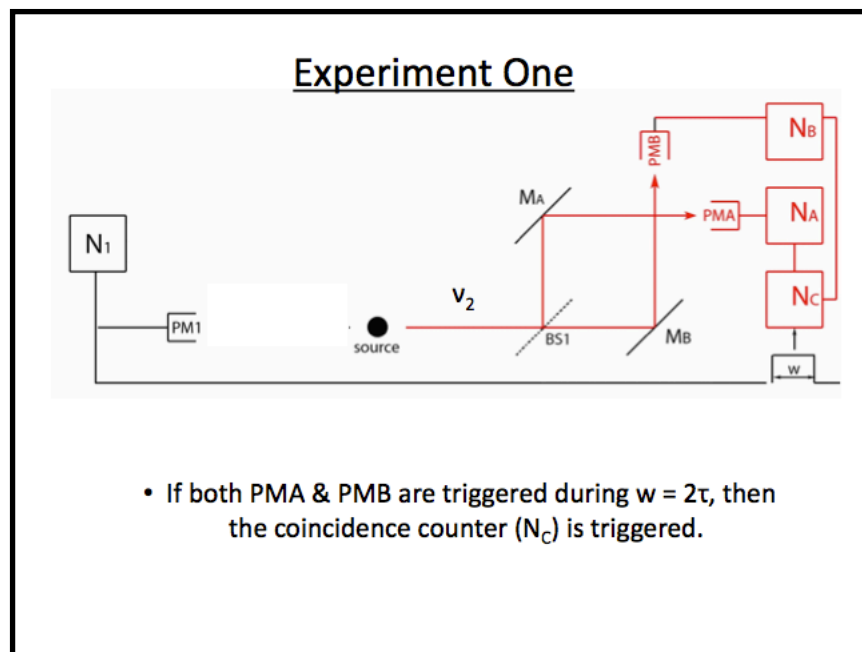
Experiment One

• If the second photon (v_2) is detected by PMA, then the photon must have traveled along Path A.

L20.S08. Following the previous concept test and subsequent discussion, it should now be clear there is only one path by which a photon might reach PMA: it must have traveled along Path A, by reflection at BS1, and reflection again at M_A .



L20.S09. The same is true for a detection in PMB: the photon can only have traveled via Path B, by transmission at BS1, and reflection at M_B .



L20.S10. It is still possible to record a detection in both photomultipliers during the short time the gate is open – when this happens, the coincidence counter (N_C) is triggered. How often this happens has implications for how we interpret the behavior of photons.

Anti-Correlation Parameter

- Need some kind of measure of how often PMA & PMB are being triggered at the same time.
- Let $\alpha \equiv \frac{P_C}{P_A P_B}$
- P_A is the probability for N_A to be triggered.
- P_B is the probability for N_B to be triggered.
- P_C is the probability for the coincidence counter (N_C) to be triggered (both N_A and N_B during $t = 2\tau$).

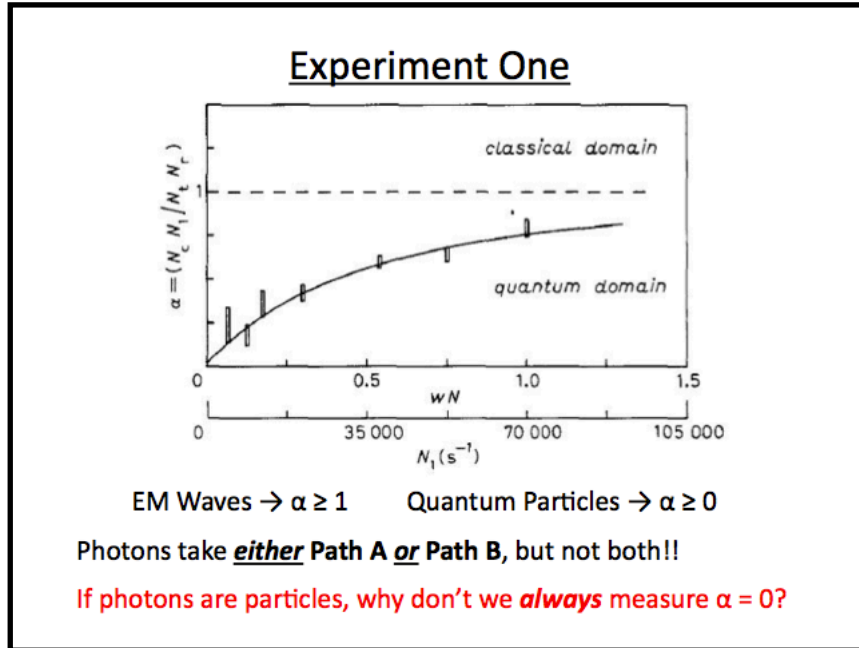
L20.S11. We first require some kind of statistical measure of how often the two photomultipliers are firing together versus firing separately. This can be defined in terms of a ratio of the counting rates per unit time for each of the three counters, or equivalently, in terms of the probability for each of the counters to be triggered during the short time the gate is open.

Anti-Correlation Parameter

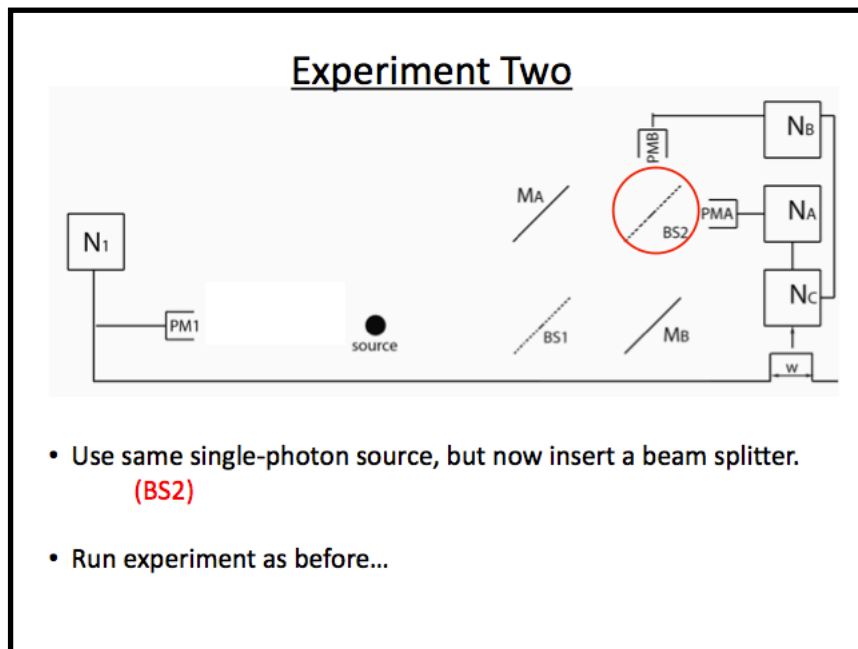
$$\alpha \equiv \frac{P_C}{P_A P_B}$$

- If N_A and N_B are being triggered randomly and independently, then $\alpha = 1$.
 $P_C = P_A \times P_B$ which is consistent with:
 - Many photons present at once
 - EM waves triggering N_A & N_B at random.
- If photons act like particles, then $\alpha \geq 0$.
 $P_C = 0$ when particles are detected by PMA or by PMB, but not both simultaneously.
- If photons act like waves, then $\alpha \geq 1$.
 $P_C > P_A \times P_B$ means PMA and PMB are firing together more often than by themselves ("clustered").

L20.S12. If the detectors were to fire together more often than not (implying that the photon energy is coherently split at BS1 and deposited equally in both detectors – wave behavior), then α should be ≥ 1 . It will be less than one if the detectors tend to fire independently (implying each detection corresponds to a single photon following a single path – particle behavior).



L20.S13. At all intensities (but particularly at low counting rates), the two photomultipliers fire independently more often than not. Since only a single path leads to either of the two detectors, we interpret these results as indicating that each photon is either reflected or transmitted at BS1, but not both.



L20.S14. The experiment is run again as before, except that now a second beam splitter (BS2) is inserted into the path. It is impossible to determine which-path information through a detection in either one of the photomultipliers.

Experiment Two

If the photon is detected in **PMA**, then it must have been...

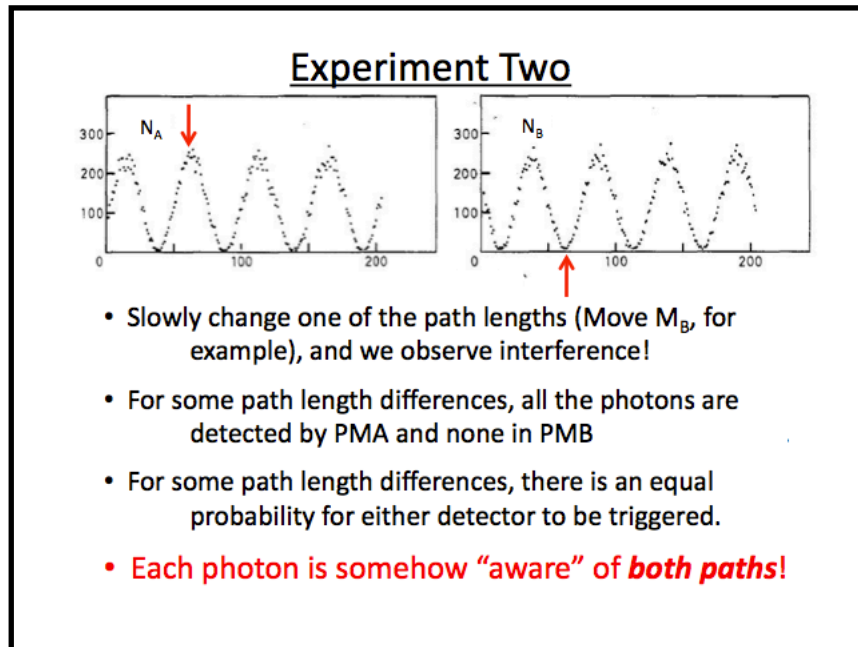
A) ...reflected at BS2.
 B) ...transmitted at BS2
C) ...either reflected or transmitted at BS2.
 D) Not enough information.

L20.S15. With the second beam splitter in place, there are now multiple paths a photon could take to be detected in a given photomultiplier. Students were quick to converge on (C) as the correct answer, with less discussion than was required for the first concept test.

Experiment Two

- Whether the photon is detected in PMA or PMB, we have **no information** about which path (**A or B**) any photon took.
- What do we observe when we compare data from PMA & PMB?

L20.S16. Detection in either of the photomultipliers yields no information about which path a photon must have taken to get there. With multiple possible paths, interference effects are expected, though not of a kind previously encountered by students. In this case, interference is observed by comparing the counting rates in the two detectors.



L20.S17. According to quantum mechanics, the counting rates in the two detectors are oppositely modulated according to the difference in path lengths between A & B. Photons that had only taken Path A should not be affected by any changes made to Path B, yet their behavior at BS2 is determined entirely by the relative lengths of **both paths**.

Experiments One & Two

- Photons in **Experiment One** took only Path A or Path B.
(which-path information – a particle encounters BS1 and takes either one path or the other)
- Photons in **Experiment Two** take both Path A and Path B.
(no path information – a wave encounters BS1 and splits equally to take both paths)

Experiment One says photons behave like *particles*.

Experiment Two says photons behave like *waves*.

Can a photon be **both** at once?
A) Yes B) No C) Maybe?

L20.S18. An explicit connection is made between the interpretation of a photon’s behavior at BS1 and the which-path information available to the experimenter. There was no favored response to this moderately rhetorical clicker question, which was meant more to get students thinking and talking about the validity of our interpretations, and to prime them for the delayed-choice experiment.

The “Conspiracy” Theory

How can the photon “know” whether we are conducting Experiment One or Experiment Two when it encounters BS1?

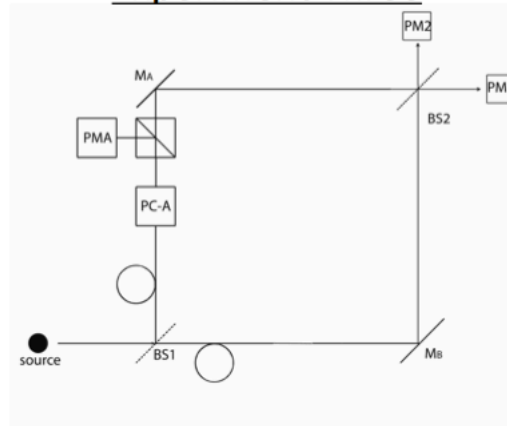
Perhaps each photon “senses” the entire experimental apparatus and always behaves accordingly.

Can we “trick” a photon into acting like a particle when it should act like a wave, or the other way around?

Suppose we let the photon enter the apparatus when the second beam splitter is absent (particles take one path or the other), but then insert the beam splitter at the last moment.

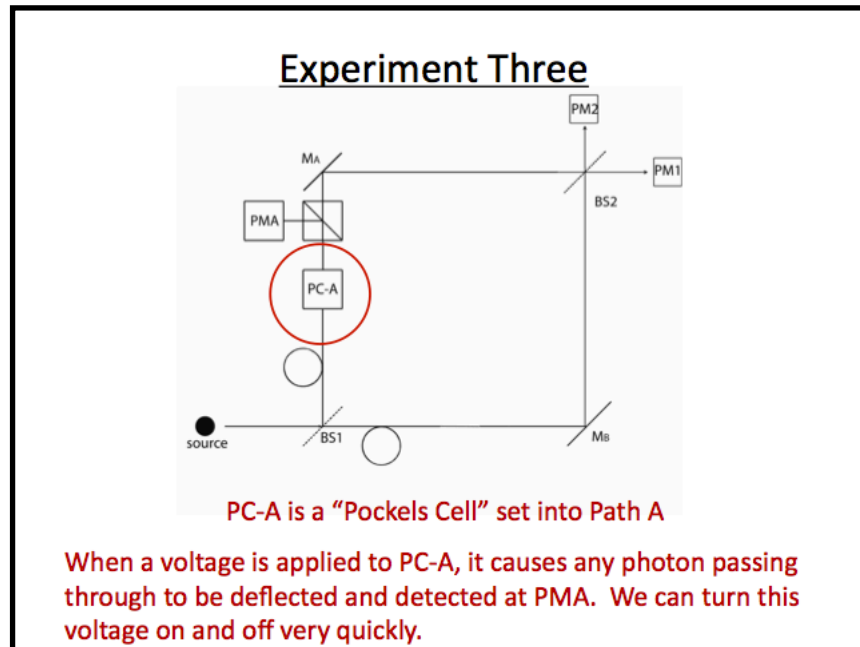
L20.S19. The question is now whether we can make a change in the experimental apparatus after the photon has encountered the first beam splitter; in such a way that we go from conducting Exp. 1 to Exp. 2 (or vice-versa) after the photon has already “decided” how to behave when it encounters BS1.

Experiment Three

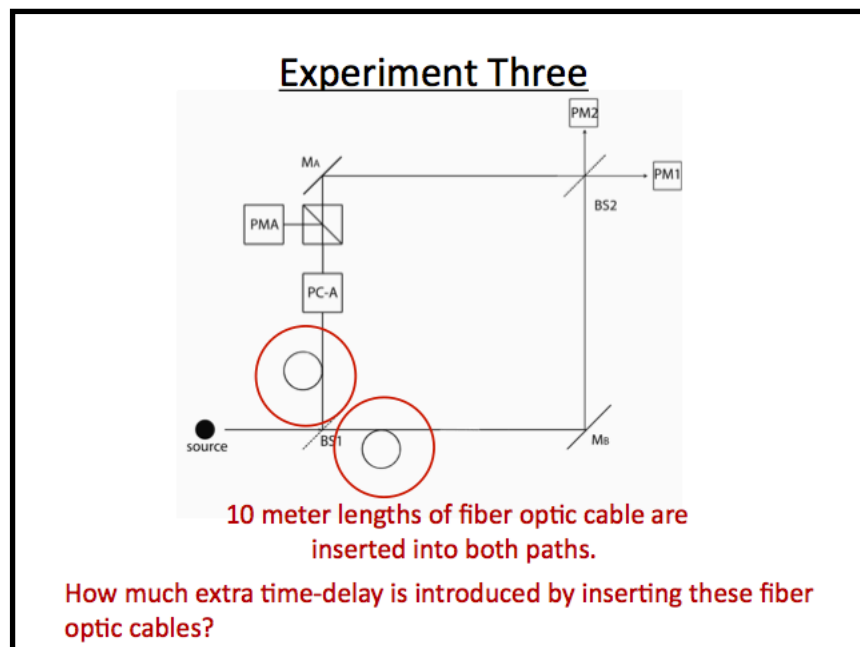


Impossible to physically remove actual beam splitter at the necessary speed, but this type of experimental setup is equivalent to what we just described.

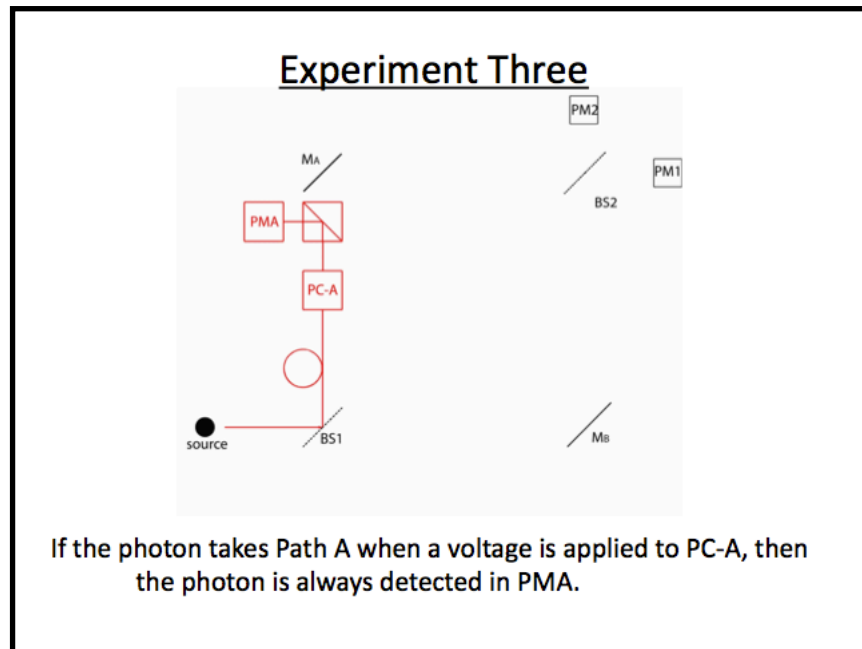
L20.S20. While structurally similar to the first experiment, this one utilizes a laser tuned to such low intensity that there is, on average, only one photon per pulse.



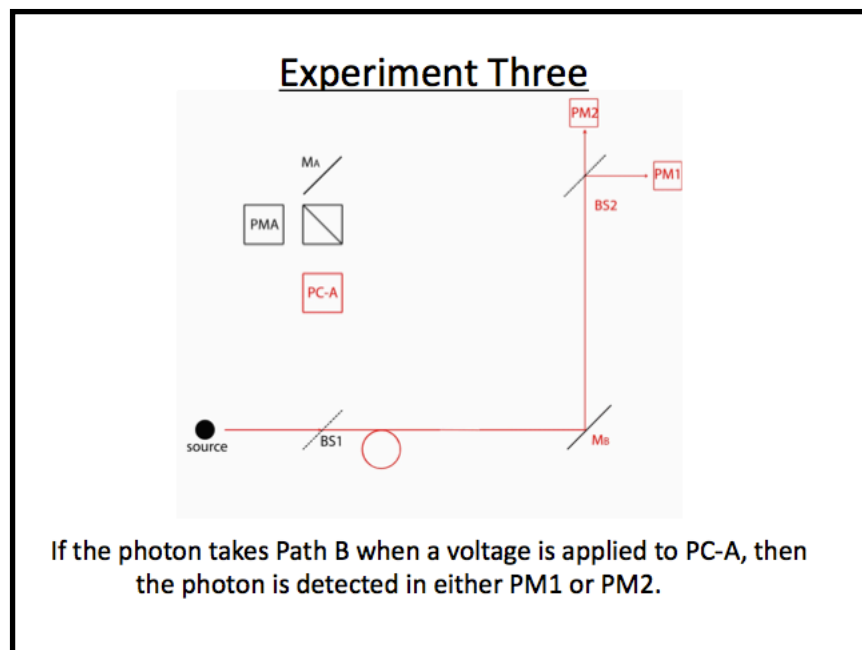
L20.S21. When a voltage is applied to the Pockels cell it rotates the plane of polarization of a photon such that it is always reflected by the Glans prism into PMA. This voltage can be turned on and off with a frequency that is sufficient for the time resolution of this experiment.



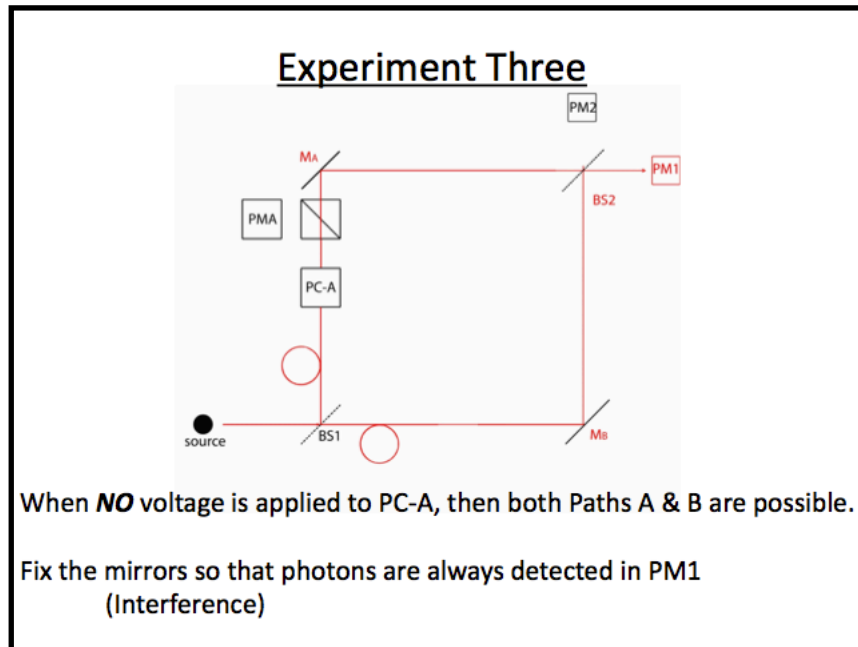
L20.S22. Two 10-meter lengths of fiber optic cable introduce a transit delay time of about 30 nanoseconds after the photon has encountered the first beam splitter.



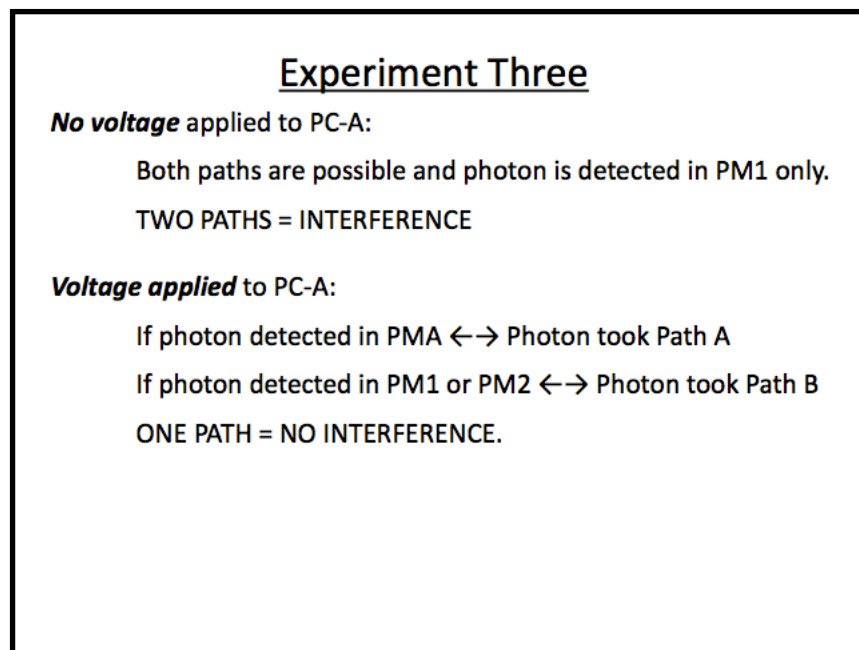
L20.S22. With a voltage applied to the Pockels cell (PC-A), any photon reflected at BS1 will be detected in PMA with 100% probability.



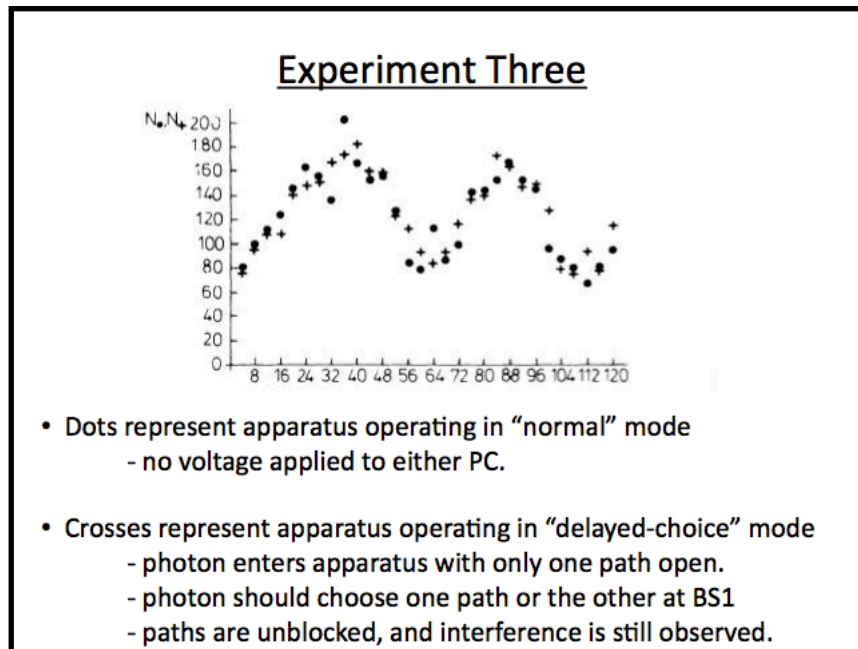
L20.S24. With a voltage applied to the Pockels cell, any photon transmitted at BS1 will have an equal likelihood of being detected in either PM1 or PM2.



L20.S25. With no voltage applied to the Pockels cell, both Path A and Path B are open to the photon. Since self-interference is possible in this case, we may fix the mirrors so that every photon is detected only in PM1 when no voltage is applied.



L20.S26. This may form the basis of a quantum epistemological tool for students. With only one path possible, no interference effects should be seen (photons behave like particles); two (or more) paths means interference should be visible (photons behave like waves).



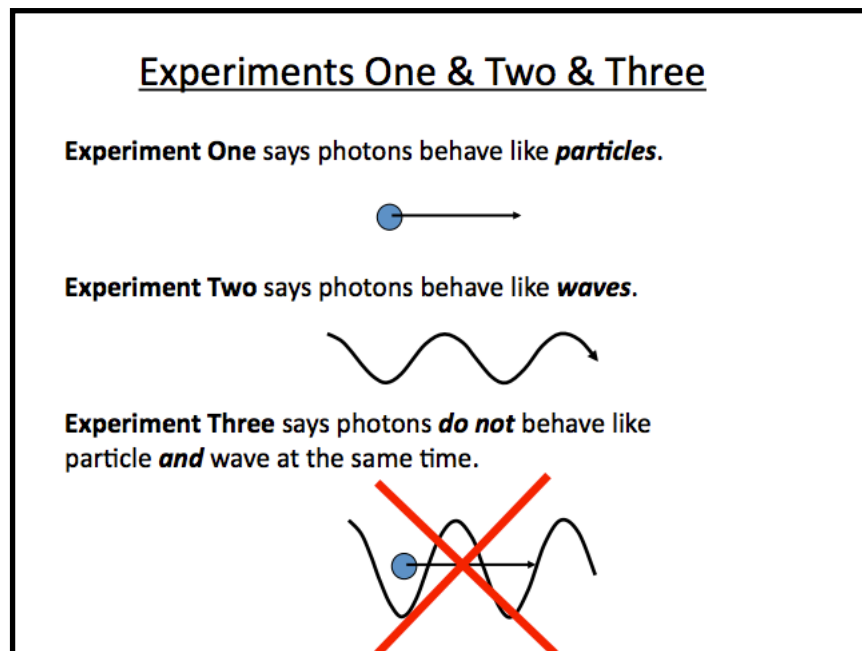
L20.S27. When the experiment is run, interference is seen whenever two paths were open to the photon, and absent when only one path was open, regardless of which was the case at the time the photon encountered the first beam splitter.

What is a Photon?

“The result of [the detection] must be either the whole photon or nothing at all. Thus the photon must change suddenly from being partly in one beam and partly in the other to being entirely in one of the beams.”

P. A. M. Dirac, *The Principles of Quantum Mechanics* (1947).

L20.S28. Dirac’s interpretation suggests the photon is coherently split into a superposition state at the first beam splitter in all three experiments, and then collapses to a point when (randomly) interacting with a detector.



L20.S29. It is hoped that, by this point, students will not just accept, but conclude for themselves that photons never exhibit both types of behaviors simultaneously.

II.C. Homework

Informal interviews with modern physics instructors have revealed a common concern that a proper treatment of the interpretive aspects of quantum theory requires an understanding and knowledge base that is beyond the reach of most introductory students, and may only open a Pandora's Box of unanswerable questions that could ultimately lead to more confusion. We believe, however, that this end result is more likely in a course where students are not given the requisite tools, including language, to fully appreciate the arguments against classical thinking in quantum contexts; and that it is precisely these kinds of open questions in physics that inspire the excitement and imagination of our students. We also believe that realist preferences are common, and so intuitive to students that many are simply lacking a name for beliefs they had already articulated in their pre-instruction survey responses. The full implications of nonlocality in quantum phenomena might not be appreciated by every student, but most will readily agree that a measurement performed on one of two physically separated systems should have no influence on the outcome of a measurement performed on the second. We wish to address here just how accessible some of the formal definitions of concepts associated with *Local Realism* are to students, following their discussion in class and in the assigned reading. [16]

One of the homework essay questions from Week 7 asks students to articulate their own understanding of the terms *realism*, *locality*, and *completeness*, and to provide some examples of *hidden variables*:

Student A: To me, realism can be described as the idea that things happen whether someone is there to witness it. For example, if a tree falls in the middle of the woods and there is nothing around to hear it, does it still make a sound? Locality represents an intuition that objects around us can only be directly influenced by other objects in its immediate surrounding. Completeness is a description of the world that is represented by the smallest physical attributes such as particles, electrons, waves, atoms, etc. Completeness describes the complete world as one. A great example of hidden variables is the example referred to in class about 2 socks being put into different boxes, mixed up and sent to opposite sides of the universe. Once you discover the color of one sock, you know the color of the other one... entanglement. These socks are hidden variables until one sock's color is discovered.

Student B: Realism is a property in which every measurable quantity exists. In other words, everything is definite, and there is no superposition. The only thing that keeps us from knowing what all the quantities are is our ignorance. Completeness refers to a theory that can describe everything without leaving anything unknown. By this definition, quantum physics is not complete because when we measure a certain quantity such as the projection of the atom in the Z direction, then we can't know its projection in the X direction.

Locality is the concept of being able to relate all actions to actions that occurred before them. For example, locality can describe a car accident – all the events that lead up to the car accident are clear and relate to one another. Bohr's interpretation of entanglement is not local, because we have no way of explaining how the observation of one atom collapses the wave such that the other atom (which would be miles apart) instantaneously is affected.

Student C: Locality: Locality of the two particles that are being separated and measured means that in some way the particles are linked to each other. These two linked particles are then able to influence each other without traveling faster than the speed of light.

Realism: Realism suggests that no quantum superposition exists. If I see a red sock in the classic two socks in box experiment, the sock was red all along and the other sock was blue all along.

Completeness: If the sum total parts of any experiment is known, the outcome can be predicted. There is completeness to an experiment that can always be predicted. Quantum mechanics suggests otherwise.

Hidden Variables: A hidden variable could influence the outcome of an experiment and explain the non-locality of entangled particles. A tachyon is an example of a hidden variable, it is something that can travel faster than the speed of light.

Student D: Realism states that a quantity in a measured system has an objectively real value, even if it isn't known. For example, under a realist interpretation, an atom always has a particular spin, we are simply unable to know that spin before we measure it (it is "hidden"). Locality is the concept that there must always be a causative chain in the real world linking two events, in other words, that one object may only effect another by causing a change in its local surroundings that may eventually propagate to cause a change in the second object through its local surroundings. Entanglement appears to violate this principle by allowing two particles to influence the state of each other regardless of their physical separation or the material in-between them. For a physical theory to be "Complete" according to the guidelines set by EPR, it must be able to explain the nature and behavior of everything in physical reality. In this sense, quantum mechanics is not complete; if locality is not to be violated quantum mechanics cannot explain all of the physical properties of a system at the most basic level.

Not surprisingly, the coherence of Student D's overall response indicates a solid understanding of each of these terms, not only individually, but also in how they relate to each other in making up EPR's argument for the incompleteness of quantum mechanics. Student B's responses are also satisfactory, and a careful reading reveals his continued preference for realist notions: his specific choice of language implies that an atom can indeed have a definite spin projection along multiple axes, and that our quantum mechanical knowledge of the system is therefore incomplete. Student A's definition of *completeness* seems not far off the mark, though his last statement on the matter is somewhat vague – does he mean that a complete theory consists of a complete description of everything in the universe, or that a complete theory describes everything as a complete and undivided whole? Student C's ideas about *completeness* are linked with determinism: knowing all of the relevant variables would make the outcomes of measurements predictable. In defining *locality*, Student C actually describes a state of *entanglement*, though he later correctly refers to entanglement as being *non-local* in his description of hidden variables. He is also correct in asserting that, should tachyons exist, their unknown presence may have some hidden influence on the outcome of measurements, but we consider it preferable that students focus their attention on more concrete examples of hidden variables (such as position or momentum), as opposed to exotic, hypothetical phenomena.

Fortunately, this was not the last opportunity for students to wrestle with the meaning of these terms, and all that they imply. During Weeks 6-8, students responded each week to an online reading quiz, which merely asked them to pose (at least) one question about something (anything) from the reading assignments for that week. These questions were then compiled and used as seeds for an online class discussion forum. For each of the subsequent five weeks, students were asked to make a contribution to the discussion board as part of their weekly homework assignments, but no efforts were made to verify their participation, and students were free to put as little or as much effort as they liked into their postings. Student postings were anonymous (even to the instructors), though we could verify at the

end of the semester how many postings a student had made. Figure 5.1 shows how a large majority (> 75%) of students made at least four contributions to the discussion board during the course of the semester (the few students who made zero contributions are not shown).

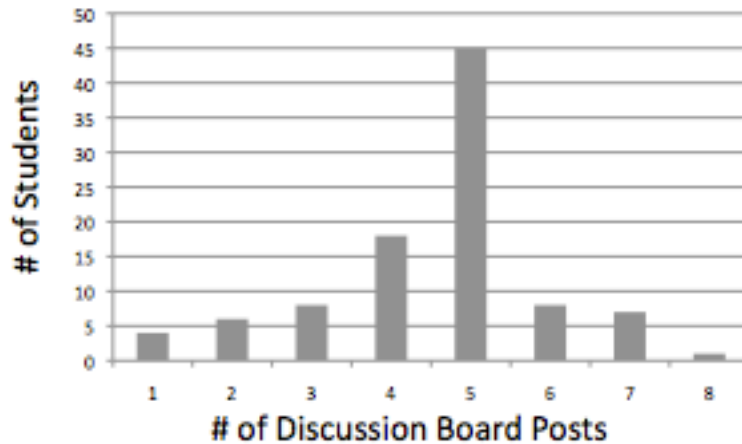


FIG. 5.1. Total number of postings made by students by the end of the Fall 2010 semester. Well over 3/4 of the enrolled students made at least four contributions to the discussion board over the course of the semester.

Our overall assessment would be that students engaged each other in a thoughtful and creative exchange of ideas, sometimes within topics that were fairly removed from our immediate focus (tachyons, time travel, warped space, and the like...). Many of the discussion threads centered on students clarifying their understanding of specific concepts (with the *occasional* intervention of an instructor, in order to stem the propagation of misconceptions), but a good deal more showed how many of the students didn't struggle so much with understanding what the interpretations were about; they struggled more with what they implied about the nature of science and reality. In just one excerpt from a discussion thread, [see Appendix F for a larger selection] we see how students are troubled by the idea of collapsing wave functions – is it some ad hoc rule invented to make the theory conform with observation? We see opposing views on questions of ontology: a literal switch between categories, or a switch between descriptions, or do photons belong to a category all their own? What are our everyday experiences with quantum phenomena, and where do we draw the line between the classical and the quantum world?

Subject: Delayed-Choice Experiments

Date: October 12, 2010 10:53 PM

[...] It seems that what's important for the argument is what's going on at the first beamsplitter. I think Dirac is saying that we can think of each photon always taking both paths and then the collapse of the wavefunction forces the photon to suddenly go from being in both paths to being in just one?

Date: October 17, 2010 7:46 PM

I got the same message from Dirac's statement that "each photon interferes only with itself" and that the photon is wavelike until observed as a particle. Or innocent until proven guilty if you will ;)

Still, riddle me this, how can a propagating wave suddenly switch to particle like behavior?

And the weirdness of quantum mechanics persists.

Date: October 19, 2010 3:34 AM

That has been tough to grasp for me as well, how do we understand that there is some mechanism for the wave to switch to particle behavior?

We have only the wave equation collapse and probability which seem like the algorithms we discarded earlier in the semester for the "farmer and the seed". I know there isn't an answer yet of the process its what me have to accept for now since the math coincides with experimentation so perfectly. (My observations thus far)

Date: October 19, 2010 9:56 AM

I've been thinking about the nature of photons and the like, and I've decided that "behaving like a particle/wave" doesn't say anything about what the photon actually is. These comparisons just give us something to relate them to, at certain times. Photons are in a category all their own, and behave like nothing we know classically.

Date: November 3, 2010 12:39 AM

Like so much in our world: words can never suffice.

It's just so very perturbing to me: the idea a wave acts like a wave when we want it to and vice versa with the particle. Why is the measurement so important? Have particles such as photons always acted this way even when we were ignorant of things not just at the quantum level, but at simply the cellular level? I sometimes wonder if the world behaves in a quantum manner just because we are observing it behave in a quantum manner, like the whole of existence is just a hypothetical wave in someone's photon experiment and there's a whole other particle-side out there which we don't know about. Is it just a question of making an effort to find it?

Date: November 9, 2010 7:14 PM

I wholeheartedly agree. Light quanta is a concept used to explain certain phenomena we perceive in certain experiments, not the absolute truth. What the photon actually is can only be described in partially complete terms "wave or particle" that end up confusing the people.

But light behaves in a so called "classical" manner, does it not? You perceive light all the time. As you are reading this light is stimulating nerves in your eyes. You know the effects of light well. So, do photons truly behave like nothing we know classically?

Date: November 15, 2010 9:49 PM

We've discussed plenty of times that objects that were previously believed to have only "classical" properties behave in a quantum manner. Bucky balls for instance are quite "large" especially compared to an electron or photon and in general I would say that we would think of the Bucky ball behaving "classically." That said, we've seen interference patterns from them which is strictly a quantum behavior. What is your justification for light behaving "classically"? Remember that your retina is a measurement device and will destructively alter the quantum state of a photon.

II.D. Exam Materials

One learning goal for this section of the course was for students to be able to identify a perspective as being realist, and to have some facility with the arguments in favor or against any particular interpretation. Since our usual post-instruction essay question on the double-slit experiment had proven useful in our interviews (in terms of eliciting students' attitudes toward some interpretive themes), we thought it appropriate to adapt this question for the second midterm exam. The problem statement for the exam question was identical to its presentation in the post-instruction online survey, but here students were asked first to identify and characterize the assumptions of Student One in terms of the interpretations of quantum mechanics we had discussed in class:

- Student A:** Student One interprets this sequence of screen shots classically, he obviously is thinking of this problem not quantum mechanically because if he did he would think the electron is going through both slits at the same time although he is thinking of this in terms of the Bohr model a bit. I think this is because he knows that we don't know the true position of the electron which means he is also thinking of it in terms of the uncertainty principle too. He thinks classically because he thinks it can't go through 2 slits at the same time.
- Student B:** Student One believes that the electron is indeed just a particle the whole time, but is moving around so fast in a random way that we can't detect it. He does not believe in wave-particle duality of electrons. He does believe that there are hidden variables (i.e., position). He also does not believe that there is a superposition. Overall, he has a realist point of view that the electron has a specific path but we just don't know it.
- Student C:** Student 1 is taking a somewhat realist perspective. They are assuming the electron traveled through one slit or the other. They claim the reality of the situation is the particle-like electron existed in a cloud of probability, and passes through one slit or the other as the cloud moved through the double slits. This explanation does not mention the probability density predicted by the wave equation.
- Student D:** Student 1's statement is consistent with that of someone who holds realism to be true. He/she assumes that: 1) The electron was always a particle with a fixed position in space and time; and 2) The only reason that the probability field is so large is because we are unable to determine its position (a "hidden variable") prior to it striking the screen. Thus, he believes that the properties of the electron are always the same, but we (the observer) are only able to observe those properties under a given set of circumstances (when the particle hits the screen).

Like Student A, there were some students who didn't utilize the specific terminology we had developed in class (e.g., distinguishing only between *classical* and *quantum* thinking, or *particle* and *wave* perspectives, without employing terms

like *realism*); virtually every single student was regardless able to recognize that Student One's belief in localized electrons was an assumption. The second part of the essay question asks students to list any rationale or evidence that favors or refutes the first two statements; and to explain whether the third statement is claiming the first two are wrong, and why such a stance might or might not be favored by practicing physicists:

Student A: For Student 1, I agree that the prob. density is large because we don't know position of the electron – we never do. I disagree that this can't be represented quantum mechanically. From experiments in the past it is proven that we get fringes (pattern).

For Student 2, I disagree that the electron is the blob because in the brighter part of the blob there is a higher probability that an electron will be detected than in the dimmer part. Although I agree the electron acts as a wave, I disagree that a single electron can be described as a wave packet.

The third student isn't saying the first 2 are wrong. All he is saying is that the interference patterns are a result of probability not classical physics and that both are right. We don't know how we get the results we do so we work with probabilities.

Student B: Since Student One believes that the electron was traveling within the blob and went through only one slit, he believes that electrons act as particles. This would mean that he would never observe interference. This is not true though because the experiment shows that over a long time, interference is observed. (Even the nickel atoms in a crystal lattice experiment shows this too.) Since Student 2 believes that the electron acts as a wave packet, he suggests that we have a small uncertainty in its position (and large uncertainty in its momentum). However, if we had a small uncertainty in its position, then we could later predict where it would show up on the screen. The double-slit experiment shows this. In other words, the blob doesn't represent the electron, but rather the probability density of the electron to be detected. Experiments show that we don't really know what the electron is doing before we detect it. Student 3 is indeed disagreeing with Students 1 & 2 by saying that Students 1 & 2 can't make some of their claims, as we really just can't tell what the electron is doing between being emitted from the gun and being detected on the screen. He might not be stating that Students 1 & 2 are necessarily wrong, but he says that quantum mechanics can't conclude their conclusions. A practicing physicist would most likely agree with Student 3 because it is consistent with the Aspect experiment for photons.

Student C: Student 2 describes the electron as a wave packet. When a double slit experiment is performed, the interference pattern that is observed corresponds to a probability density that can be described by a wave-packet equation. A packet of waves would interfere with itself, creating a probability of the electron to pass through both slits. Also, which slit the electron went through cannot be measured without altering the uncertainty in the momentum.

Student D: Rationale/Evidence for Student 1 (aka EPR):

Realism argument: all objects must have definite properties within the system regardless of observation. Location is real but hidden variable. Makes intuitive sense.

Against Student 1:

Idea of definite quantities for all states (Local Realism) does not hold to experiment. Probabilistic provides correct explanation, deterministic does not. Single-photon interference experiments.

Rationale/Evidence for Student 2 (aka Bohr):

Electron is a wave function that collapses to a determinate state at plate. Consistent with matter waves argument put forward by deBroglie. Allows for interference with only one electron.

Against Student 2:

Fails when applied quantitatively; no mechanism for wave collapse yet developed.

No, Student Three is simply stating the theory behind the interpretations put forth by the first two students. In other words, he is limiting his assessment of the experiment to what can be predicted and explained through existing QM theory. A practicing physicist would tend to agree with Student 3 because his description requires the least assumptions and adheres to what we know as opposed to what we postulate.

Once again, Student D offers a near textbook response. Student B employs standard arguments against a strictly particle view of electrons, and in favor of a wave representation, but is explicit in saying that the wave corresponds to the probability for where an electron might be found, and not the electron itself. He is also cognizant of the incompatibility of the two statements – it is not possible for both of the fictional students to be correct. Not every student saw these two views as contradictory, in the sense that they reduced the two statements down to simply representing either a particle view or a wave view, without considering how each statement makes an explicit assertion regarding the behavior of the electron at the slits – it either goes through one slit or it goes through both. In other words, not every student took a definitive stance on the question of whether an electron always passes through one slit or both, focusing more on the legitimacy of particle or wave views in this context.

Interestingly, Student A's response is an almost exact recapitulation of Student R3's reasoning in Chapter 4: they both agree the electron is somehow behaving like a wave in this experiment, but object to the idea that a wave packet can describe an individual particle. Student A also indicates a belief that we can never know the true position of an electron, hence the large probability density. At this stage, it seems that Student A is not yet *split* in his beliefs – he hasn't conceded that an authoritative stance trumps his intuitive views, and indeed implies that scientists might believe that Students One & Two are both right, and that we can't really know why we observe what we do. Student C is not explicit in arguing against

Student One, but instead explains why Student Two's description conforms to observation. As we shall see in the final portion of this exam question, Student C still believes in a continuously localized existence for electrons in this experiment:

(Part III) Which student(s) (if any) do you *personally* agree with? If you have a different interpretation of what is happening in this experiment, then say what that is. Would it be reasonable or not to agree with *both* Student 1 & Student 2? This question is about your personal beliefs, and so there is no "correct" or "incorrect" answer, but you will be graded on making a reasonable effort in explaining why you believe what you do.

Student A: I think from what I have learned in this class that Student 3 is correct. Probability can show us patterns but we really don't know what's going on before. It is reasonable to agree with both Student One who thinks classically and Student 2 who thinks quantum mechanically because that allows you to form your own ideas about what is going on but the truth is that we don't know what's going on between emission and the screen.

Student B: I personally believe that the electron acts like a wave until we observe it. This is Dirac's interpretation. Student 1 & Student 2 can't both be right because that would suggest that the electron acts like a wave and particle at the same time, and there is experimental evidence that refutes this.

Student C: Since electrons show both wave and particle like behavior, it would be reasonable to side with either Student 1 or 2. Student 2 used a more wave-like interpretation, Student 1 used a more particle like interpretation.

I personally visualize the situation as a flow of some fluid that travels through the two slits in waves. It appears through all space as soon as the electron is fired. The electron then rides this chaotic fluid toward the screen and strikes in a location that is somewhat determined by the interference patterns of the fluid. Trying to measure this fluid flow collapses the waves created.

Student D: I personally agree with Student 3. I see no reason to jump to a conclusion regarding the electron's behavior without a quantitative mechanism to explain its behavior between source and the plate. We know from this experiment that an electron exhibits behavior consistent with that of a wave, but we do not know exactly why or how that is so. That being said, I find Student 2's statement a more convenient way to think about the electron's behavior.

Student A merely restates his earlier stance: we require probabilistic descriptions because we can't really know what is going on between source and detection, and so either point of view might be equally legitimate. In the end, it seems this student is asserting his right to believe as he chooses when science has

no definitive answer. At this point, we would characterize Student A as *Agnostic* – he recognizes the implications of competing perspectives, but is unwilling to take a stance on which might best describe reality.

Student B does not explicitly say which student he agrees with, but reports his belief in Dirac’s matter-wave interpretation. Notice, however, that he says the electron *acts* like a wave, and not that an electron *is* a wave. Without further information from Student B, his views at this point might be consistent with either a *Quantum* or a *Copenhagen* perspective, since his stance on the reality of the wave function, and the nature of its collapse, is unclear.

We may easily place Student C within the *Pilot-Wave* category; indeed, his response sounds eerily similar to Student P3 (from Chapter 4) – the interference of nonlocal quantum waves determines the trajectories of localized particles. These two students arrived at the same conclusions independently; we made only cursory mention of Bohm’s interpretation in our class, and it was not discussed at all in Student P3’s class. This suggests that such ideas may be more prevalent among students than it seemed at first glance.

Student D’s sentiments are not so different from Student A – it isn’t known why quanta behave as they do, and so being agnostic requires the fewest assumptions (though he does mention that he finds it useful to employ a wave description in this situation). It seems reasonable to characterize Student D as subscribing to a *Copenhagen/Agnostic* perspective at this stage of the course.

The class as a whole performed well on this exam question: ~75% of students received full credit for their responses; the remaining students primarily lost one or more points (usually not more than three, from a total of ten points) for providing incomplete responses (very few students made any assertions that were unequivocally false). Overall, we would say that several of our learning goals surrounding this material were met by the majority of our students: they were able to identify the realist assumptions of the first fictional student, and to contrast them with an alternative perspective; they could provide evidence that favors or refutes competing points of view; and they were able to articulate their own beliefs regarding the interpretation of this quantum experiment. All of this regardless of whether they actually employed the exact terminology that had been developed in class (though most students did indeed use terms like *realism* and *hidden variables* in their argumentation). 18% of students chose to explicitly agree with Student One, though only one of them agreed with this statement exclusively; the remaining students were split between agreeing with both of the first two statements, or agreeing with all three. 46% of students said they agree with Student Two, or with both of the last two statements, while 36% preferred Student Three’s statement exclusively.

II.E. Assessing Outgoing Perspectives

As part of their final homework assignment, students were asked to respond to the same post-instruction attitudes survey that had been administered in other courses. We report here the final class wide responses to each survey item, juxtaposed with how they responded at the beginning of the semester. We similarly

offer complete responses from Students A, B & C. Student D did not respond to this final survey, but we shall hear from him again in our discussion of the final essay assignment below. [Section II.F]

1. It is possible for physicists to carefully perform the same measurement and get two very different results that are both correct.

	Agree	Neutral	Disagree
POST (N=90)	0.78	0.06	0.17
PRE (N=94)	0.65	0.13	0.22

Student A: (Disagree) Take the example of hidden variables. If you put one red sock and one blue sock into identical boxes and both socks are identical beside their color, and you send them across the universe, then your technically performing the same measurement. When you open one box you find out what color the sock is in that box and it can be either red or blue, two different results. At the same time you also know what is in the other box every time you perform the experiment, in that respect, you are kinda getting the same result.

(PRE: Agree)

Student B: (Strongly Agree) This is possible especially when it comes to measuring the position of an electron. This is because there is no definite position to begin with. All we can know is the probability of finding the electron in a particular position, but probability does not determine where the electron will be when we measure it.

(PRE: Agree)

Student C: (Strongly Agree) Two very different results could confirm the same fact. Being correct is nothing more than confirming a fact.

(PRE: Strongly Agree)

Students shifted towards more agreement with this question (and less neutrality), but drawing conclusions from overall agreement or disagreement should be done with caution, for there are quantum mechanical reasons for disagreeing with this statement. For example, it has been argued by students that, in practice, scientists perform a number of measurements in any given experiment, and it is the statistical distribution of data that is the final result, which should be always be the same for similar experiments:

“...if we are measuring the position of an electron, we will measure a different position each time. But if we compile all our results we will find positions that correspond to the wave function. I strongly disagree with the above statement because if an experiment is performed correctly it should produce the same results!”

The distribution in Table 5.II of the kinds of reasoning invoked by students at pre- and post-instruction (by the same categorization scheme employed in Chapter 2)

shows that students shifted dramatically in their preferences for deterministic and hidden-variable style thinking (Categories D & E). Students shifted from 47% to 17% in providing Category D & E responses (whether in agreement or disagreement). And while only 17% of students invoked quantum phenomena (Category A) at the outset of the course, 65% of post-instruction responses made reference to quantum systems. Most students agreed with this statement before and after instruction, but learning about quantum mechanics caused most of them to consider it in a new light. For example, Student B has confirmed his pre-instruction suspicion that quantum mechanics might allow for this statement to be true. Student A originally agreed because of wave-particle duality, but now disagrees through an example of hidden variables and classical ignorance. Student C strongly agreed in both cases, first providing a Category D response, and then one more consistent with Category C.

TABLE 5.II. Categorization (as in Chapter 2) and distribution of reasoning provided at pre- and post-instruction, in agreement or disagreement with the statement: *It is possible for physicists to carefully perform the same measurement and get two very different results that are both correct*; standard error on the proportion $\leq 5\%$ in each case.

CATEGORY DESCRIPTION				
A	Quantum theory/phenomena			
B	Relativity/different frames of reference			
C	There can be more than one correct answer to a physics problem. Experimental results are open to interpretation.			
D	Experimental/random/human error Hidden variables, chaotic systems			
E	There can be only one correct answer to a physics problem. Experimental results should be repeatable.			
CATEGORY	PRE-INSTRUCTION (N=94)		POST-INSTRUCTION (N=90)	
	AGREE	DISAGREE	AGREE	DISAGREE
A	15%	2%	58%	7%
B	4%	0	0	0
C	13%	0	10%	0
D	29%	3%	9%	4%
E	1%	14%	0	4%
TOTAL	62%	19%	77%	15%

2. The probabilistic nature of quantum mechanics is mostly due to physical limitations of our measurement instruments.

	Agree	Neutral	Disagree
POST (N=90)	0.18	0.21	0.61
PRE (N=94)	0.46	0.32	0.22

Student A: (Strongly Agree) The probabilistic nature of quantum mechanics comes from the fact that there are aspects of quantum mechanics that can't be measured due to physical limitations of our measurement instruments. For instance how the uncertainty principle interacts with electrons orbiting a nucleus. Electrons are too small and move too fast for humans to know exactly where an electron is at a certain moment, so we can only perform one measurement at a time. Position and momentum of a particle can't be known at the same time, we can only calculate the probability of finding them there.

(PRE: Neutral)

Student B: (Strongly Disagree) It seems that the probabilistic nature of quantum mechanics is mostly due to the nature of sub-atomic particles rather than the limitations of our measurement instruments. If the particles were in definite states and definite positions to begin with, or even if there were a wave function that could define the exact state of the particles at any time, then one could argue that the problem is our measurement instruments. Perhaps such a formula will exist in the future, but that would mean that the limitation is our knowledge, not our instruments.

(PRE: Strongly Agree)

Student C: (Neutral) I have no idea.

(PRE: Neutral)

There was a strong shift away from agreement and in favor of disagreement by the end of the class; without passing judgment on students who feel neutrally towards this statement (after all, we do not consider agnosticism to be unsophisticated), we would at least like for our student to *not agree* with the notion that technology might one day reduce the need for probabilistic descriptions of quantum phenomena. Student B's response is desirable, in that he identifies uncertainty in quantum mechanics as fundamental, and not a consequence of experimental uncertainty. Student A's response is consistent with his reasoning on atomic electrons at the beginning of the course: their chaotic, rapid motion precludes knowledge of their true positions. We placed Student A in the *Agnostic* category at the time of the second exam, but we shall now see his explicit preference for realism:

3. When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time.

	Agree	Neutral	Disagree
POST (N=90)	0.26	0.18	0.57
PRE (N=94)	0.72	0.09	0.19

Student A: (Strongly Agree) Every physical thing exists whether it is being observed or not. This is the idea of realism, and I completely agree with it. An electron is a particle therefore I believe that it has a physical manifestation. An electron will definitely still exist at a definite position at every moment in time. This correlates with my answer above.
(PRE: Strongly Agree)

Student B: (Disagree) This thought process only makes sense if one were to view electrons as particles (like billiard balls). However, we know from experimentation that the electron has wave-like properties and can be described in the form of an electron cloud (Schrodinger's model). Thus, we can have an idea of where we are likely to find the electron if we make a measurement, but when we don't make a measurement, the electron should not be acting like a particle. But then again, we can't be 100% sure of what's happening when we aren't measuring...
(PRE: Strongly Agree)

Student C: (Neutral) If an electron orbits a nucleus in a forest and no physicist is there to observe it, does it obey the uncertainty principle?
(PRE: Agree)

As with the second survey item, we would have liked for our students to *not* choose to agree with this statement, and only 26% of them did by the end of the semester. We may not infer too much from Student C's tongue-in-cheek response, except to suggest his neutral attitude implies this question may now have as little (or as much) meaning to him as considering the sound of one hand clapping – at a minimum, his response has shifted away from agreement. In his disagreement, Student B explicitly addresses the wave-like properties of atomic electrons, though he also expresses a modicum of tentativeness in his beliefs.

Even though Student A has come through this course with explicitly realist notions intact (perhaps even reinforced), we would still consider his response to be in keeping with at least some of our learning goals: he has given conscious consideration to his intuitive beliefs and confirmed them to himself, and he can now articulate those beliefs in terms of language that been previously unavailable to him. At the very least, he did not use such language in his pre-instruction responses, which focused more on the tentativeness of scientific knowledge. Let us consider these students' last thoughts on the double-slit experiment before drawing any final conclusions on their overall outgoing perspectives:

- Student A:** I agree with Student 1 mostly except for the fact that the electron could be going through both slits at the same time for all we know. I also agree with student 2 because I think that the electron is acting as a wave and again possibly go through both slits at the same time. Therefore I agree more with student 3 because we really don't know what is happening between the moment the electron is shot from the gun and it hits the detection screen.
- Student B:** I agree with student three because it seems that the electron can act as a wave until we observe it. Even if this isn't the reality, there's nothing we can know about it from when the electron is emitted to when it is detected. However, student one and student two cannot be both correct because the electron cannot act like a wave (student 2) and a particle (student 1) at the same time, because there is experimental evidence that refutes this.
- Student C:** Student One is assuming the electron is always a particle. Student Two is assuming that the electron is pretty much a wave until it gets smushed by the screen. Student three is sticking to the fact that the electron has a probability of going in certain places on the screen. I think there will always be a more accurate description of observations and quantum mechanics is, for now, an accurate description of reality.

And so it would have been premature to consider Student A to be a confirmed *Realist*, seeing how he maintains an explicit tentativeness regarding what can actually be known in this experiment, and so we might best characterize his overall final responses as *Realist/Agnostic*. Student B's earlier exam responses placed him somewhere between the *Quantum* and *Copenhagen* categories, but his overall language has consistently referred to the *behavior* of quanta, and he has explicitly refused to equate the wave with the particle it describes. Considering his final agreement with Student Three, and his concession that a wave description of quanta may ultimately not conform to reality, Student B's outgoing perspective on quantum mechanics is most consistent with the *Copenhagen* category. Student C's final response requires some thought: we believe he is suggesting there will one day be a *more accurate* description of reality, but that quantum mechanics is currently a *sufficiently accurate* description of that reality, and so we don't interpret his response as implying that quantum mechanics is necessarily incomplete. Student C expressed beliefs in non-local realism at mid-semester, and we did not ask him for his own interpretation of the double-slit experiment in the post-instruction survey, but his overall final response indicate he would be best described as being in the *Agnostic* category.

A final look at the overall class responses to this post-instruction essay question, in conjunction with their responses on atomic electrons, provides some insight into the consistency of student perspectives, which was part of our original motivations for our investigations. [Chapter 2] Only five of the 87 students who provided clear responses to this survey item explicitly agreed with Student One, and

three of them did so in their expression of agreement with all three statements. Of these five students, three of them agreed with the statement on atomic electrons, one was neutral, and the other replied in disagreement. This means that 23% of students who chose to *not agree* with Student One in the double-slit experiment essay question offered a response to the statement on atomic electrons that would be consistent with realist expectations. Even though we are only considering five students here (meaning there is significant statistical error), we note that this distribution of responses on atomic electrons for students who had expressed realist preferences in the double-slit experiment matches our findings in Chapter 2 exactly. We also note that this 23% ($\pm 4\%$) of students evidencing inconsistent thinking across these two contexts is significantly less than the 33% ($\pm 6\%$) found in our initial studies ($p < 0.001$, by a one-tailed t-test). We believe these results allow us to conclude that another of our learning goals had been achieved for a majority of our students – the consistency of student perspectives between these two contexts has been significantly increased over prior incarnations of modern physics courses.

We conclude this section by considering the level of personal interest in quantum mechanics expressed by students at the end of the semester:

4. I think quantum mechanics is an interesting subject.

	Agree	Neutral	Disagree
POST (N=90)	0.98	0.02	0.0
PRE (N=94)	0.85	0.13	0.02

Student A: **(Strongly Agree)** I found quantum mechanics to be an interesting subject because the concepts around it are not proven. A lot of what is behind quantum mechanics is qualitative which is very different than most physics classes which are quantitative. It is nice to look at a complex subject such as physics from a qualitative manner because for the past two years I've been taking all engineering classes which are all involving math significantly.

(PRE: Strongly Agree)

Student B: **(Strongly Agree)** The fact that there are truths associated with quantum mechanics that still can't be explained is a very interesting concept. I have never been taught something in school that is proven in experiments but still lacks a proper reasoning (such as entanglement). I also think it's very interesting to learn how sub-atomic particles behave so differently than macroscopic particles.

(PRE: Strongly Agree)

Student C: **(Strongly Agree)** Quantum mechanics is strange and interesting and mind stretching. This has been a great course.

(PRE: Neutral)

We find it remarkable that virtually every student expressed an interest in quantum mechanics by the end of the course, and that only two students responded neutrally – these final numbers are contrary to the usual decrease in interest among engineering students, and are on par with what is typically seen in a course populated with physics majors, where it is fairly safe to assume that nearly every student is already interested in learning about quantum mechanics coming into the course. [Chapter 6.] Still, considering the relatively high rate of incoming interest in quantum mechanics for students from our course, it is not entirely clear how effective we were in influencing student attitudes without considering a more detailed breakdown of their responses. In all other cases, *agreement* and *strong agreement* had been collapsed into a single category, and similarly for *disagreement* and *strong disagreement*; we therefore consider the number of students who became *more emphatic* in their agreement. Initially, 32% of students merely agreed that quantum mechanics is an interesting subject, and 53% were in strong agreement – these numbers shifted by the end of the course to 20% and 78%, respectively. We may therefore conclude that this curriculum, as implemented, was successful in not only maintaining student interest in physics, but in promoting it as well. As a final comment, we note that Students A, B & C all express a strong interest in the subject, and their responses suggest that it is precisely the still-open questions in quantum mechanics that inspire their fascination – Pandora’s Box has been opened, and we don’t have to be afraid!

II.E. Final Essay

In lieu of a long answer section on the final exam, students were asked to write a 2-3 page (minimum) final essay on a topic from quantum mechanics of their choosing, or to write a personal reflection on their experience of learning about quantum mechanics in our class (an option chosen by ~40% of students). As opposed to a formal term paper, this assignment was meant to give students the opportunity to explore an aspect of quantum mechanics that was of personal interest to them. Topics selected by students for their final essays (ones that were not personal reflections) included: quantum cryptography; quantum computing; enzymatic quantum tunneling; bosons and fermions; the Quantum Zeno Effect; string theory; atomic transistors; quantum mechanics in science fiction; and more... The nearly universally positive nature of the feedback provided by students in their personal reflections is evidence for the popularity and effectiveness of our transformed curriculum, and its practical implementation. [Excerpts from *each* of the submitted personal reflections from the Fall 2010 semester are collected in Appendix E.]

We recall from earlier in this chapter that Student D had entered this course with a relatively sophisticated view on quantum mechanics, but one that was explicitly realist/statistical. We are interested, of course, in whether this curriculum has something new to offer students with a high degree of background knowledge coming into the semester. Though he did not complete the end-of-term attitudes

survey, we may still draw some conclusions regarding the effectiveness of this curriculum at influencing Student D's interpretive stances:

“Upon entering the class, I was most excited to learn about the various interpretations put forth to explain quantum mechanical phenomena. I already had a fairly strong footing in the actual mathematics of the material, both from my own independent studies and from an exceptional AP Physics course I had taken in my senior year in high school. However, neither of those pursuits had given me a strong grounding in the overarching theoretical principles behind the material, especially when it came to interpreting the experimental data in the more recent work such as Aspect's single photon experiments and electron diffraction. I came in understanding the results of those experiments, but not their implications for the nature of light and matter. This class did a fantastic job of patching those holes in my understanding. [...] Although this class has not significantly changed my ideas about physics and the practice of science, it has been one of the few courses I have taken that accurately portrays the scientific method of careful observation. The course was exceptional in how it handled conclusions drawn from experimental results, the most memorable example being the refutation of the “hidden variable” interpretation. The class was at its best when discussing the interpretations of experiments and the implications of their results; Aspect's single photon experiments were explained with particular clarity and care.”

We may not know precisely how Student D would have responded to the post-instruction survey, but we may infer from his statements that he no longer personally subscribes to the notion of *hidden variables*. We assert that Student D successfully transitioned from a *Realist/Statistical* perspective on quantum mechanics, to one that is more aligned with the beliefs of practicing physicists (*Copenhagen*).

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