

CHAPTER 6

Teaching Quantum Interpretations – Comparative Outcomes and Curriculum Refinement

I. Introduction

In the previous chapter, we considered the design and implementation of a transformed modern physics curriculum for engineers, taught at the University of Colorado in the Fall 2010 semester. The accessibility and effectiveness of this new curriculum was discussed in terms of some measures that were entirely new and specific to that course – student responses to homework and exam questions relevant to the physical interpretation of quantum mechanics. But we have also gauged learning outcomes according to measures that had been employed in prior studies, (Chapter 3) and so we shall address in this chapter how some of the outcomes for this transformed course compare with three previous modern physics offerings.

Naturally, the outcomes from this course would be less significant if our learning goals had not represented a challenge for our students, or for ourselves as instructors and curriculum designers. Any newly implemented curriculum will certainly have need for refinement, requiring first the identification of specific student difficulties with the new material, which may then inform our suggestions for improvement. In light of our focus throughout this dissertation, it seems most appropriate to discuss problems students had in understanding the single-photon experiments, as revealed through their responses to another long-answer exam question from the second midterm. At the same time, we may also assess their use of some of the epistemological tools we had worked to establish in lecture. We will also consider aggregate and individual student responses to several of the multiple choice questions from our exams and the post-instruction content survey, which may indicate other student difficulties requiring future study.

II. Comparative Outcomes

We have already seen how certain instructional approaches with respect to interpretation can be associated with specific student outcomes (e.g., there is a greater prevalence of realist beliefs in contexts where instruction has been less explicit in promoting an alternative perspective, or in topic areas where realist/statistical interpretations were deliberately promoted). There are many similarities between our course from Fall 2010 and the four courses discussed in detail in Chapter 3, Section 3.II - they were all large-lecture courses ($N > 60$) where interactive engagement was employed during class, and covered roughly the same progression of topics from quantum mechanics and its applications. And all but the

course taught from a realist/statistical perspective utilized many of the same lecture materials that had been developed during the first round of course transformations in 2005-2007. Yet they all differed in their instructional approaches to interpretation, though we would say that Course B2 (as denoted in Section 3.II) was most similar to our own, in that the instructor was explicit in promoting a matter-wave interpretation of the double-slit experiment, and significant lecture time was given toward the very end of the semester to discussions of measurement and interpretation in quantum mechanics (but without specific reference to atomic systems). There were no significant differences in the wording or presentation of the online attitudes survey administered in each course. Before making direct comparisons of student outcomes, we first (briefly) remind ourselves of our characterizations of the courses with which we'll be making our comparisons, and establish how they will be denoted in this chapter. [Table 6.I]

TABLE 6.I Summary of the four courses to be compared in this section, including a characterization of each instructor's approach to interpretive themes. For reference, how each course was denoted in Chapter 3 is also included [n/a = not applicable].

STUDENT POPULATION	COURSE	INSTRUCTIONAL APPROACH	CH. 3 DENOTION
Engineering	ENG-FA10	<i>Matter-Wave</i>	n/a
	ENG-R/S	<i>Realist/Statistical</i>	A
	ENG-MW	<i>Matter-Wave</i>	B2
Physics	PHYS-C/A	<i>Copenhagen/Agnostic</i>	C

ENG-R/S is the only engineering class considered in our studies that was taught from a realist/statistical perspective. ENG-MW is the engineering course most similar to ours (ENG-FA10), in that similar lecture materials were used, a matter-wave perspective was promoted, and interpretive themes were discussed near the end. PHYS-C/A is a class for physics majors that also used many of the same lecture materials, but with less emphasis on interpretation.

II.A. Student Interest in Quantum Mechanics

It is now well known in physics education research that student attitudes toward physics have a tendency to become less positive after instruction in introductory courses of all kinds, including ones where specific attention had been paid to student attitudes and beliefs. [3, 4] Similar effects have been seen in modern physics courses; one study showed that traditional modern physics instruction typically led to significant negative shifts in student attitudes (as measured by the CLASS [4]), while a curriculum transformed using principles from PER saw no significant pre/post-instruction shifts, meaning overall student attitudes at least did not get worse. [1] By combining pre-instruction survey responses on their reported

interest in quantum mechanics from six semesters of engineering courses (including the Fall 2010 semester), we see that incoming interest for engineers is on average between 75-80% favorable. [Fig. 6.1] The average post-instruction interest among engineering students from five of these course offerings dropped to below 70%, while negative responses increased significantly ($p < 0.001$) – approximately 1/3 of engineering students would not agree that quantum mechanics is an interesting subject after having learned about it in our modern physics courses! Students from the Fall 2010 semester were nearly unanimous (98%) in their reported interest in quantum physics, and not one student responded with a negative opinion. [Relative to the number of students who completed the final exam, the response rate for our post-survey was $\sim 90\%$.]

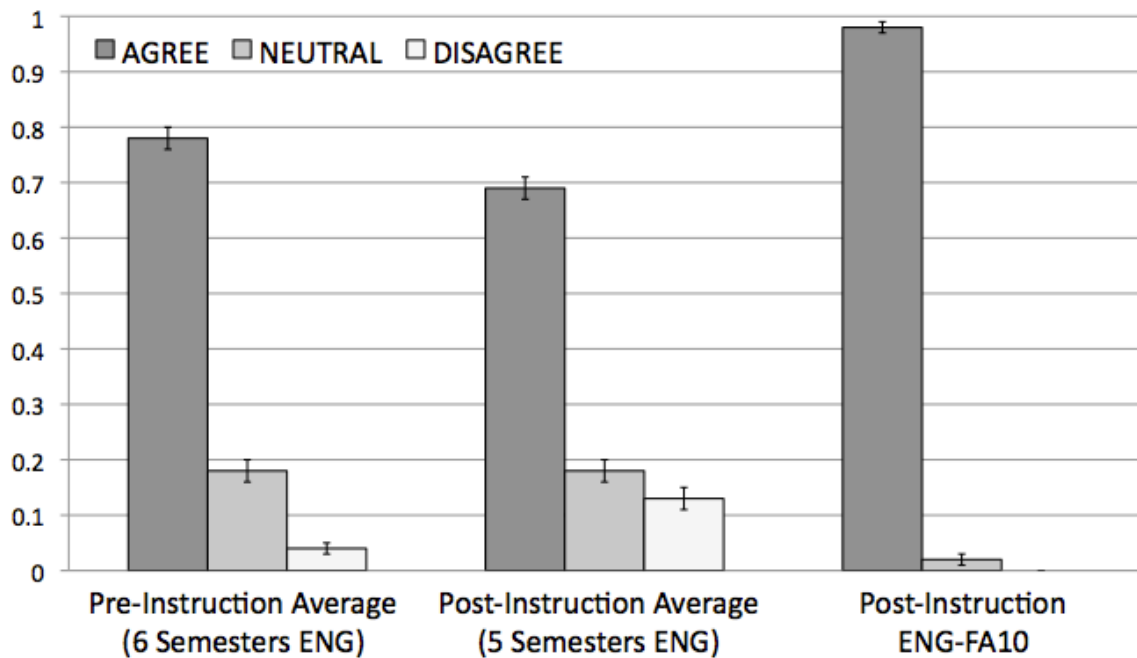


FIG. 6.1. Average pre- and post-instruction student responses to the statement: *I think quantum mechanics is an interesting subject*, from five modern physics courses for engineers, plus the FA10 semester. [Error bars represent the standard error on the proportion; $N \sim 50-100$ for each course].

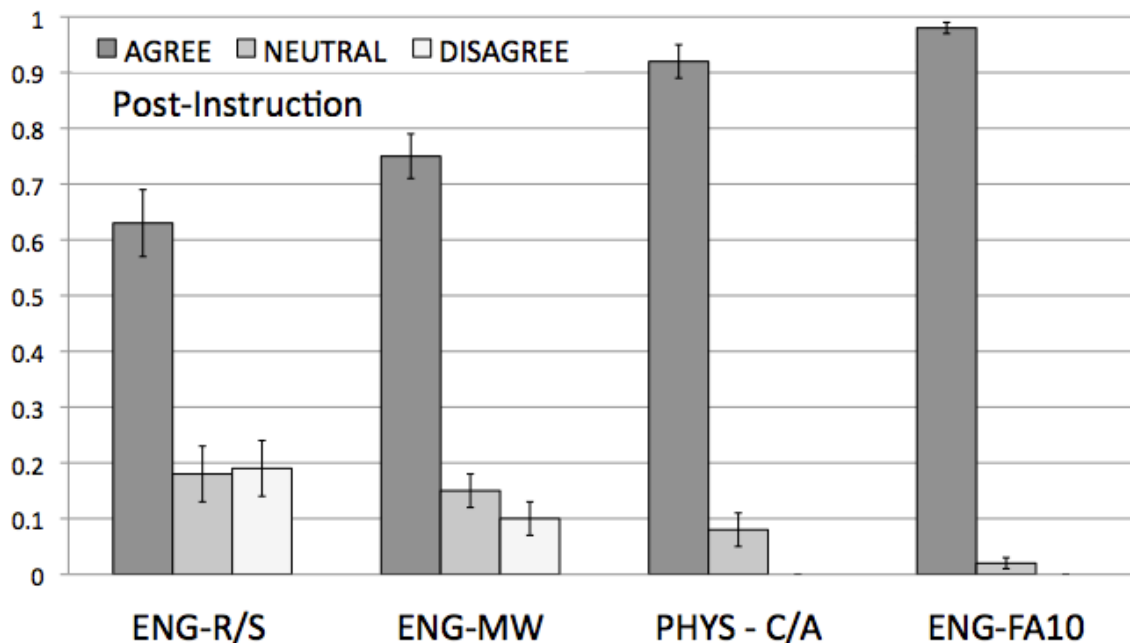


FIG. 6.2. Post-instruction student responses to the statement: *I think quantum mechanics is an interesting subject*, from four modern physics offerings, as denoted in Table 6.I. [Error bars represent the standard error on the proportion; $N \sim 50$ -100 for each course]

It would be too great an assumption to conclude that shifts in student interest are necessarily directly correlated with the interpretive approach of the instructor, or with the student population. There are surely myriad other affective considerations, such as instructor popularity or choice of textbook, and we have seen courses for physics majors where overall interest in quantum mechanics declined. Nonetheless, we note that the Fall 2010 course had the greatest proportion of students reporting positive post-instruction attitudes towards quantum mechanics, including the course for physics majors; [Fig. 3.2] and that end-of-term student evaluations from ENG-MW, PHYS-C/A and ENG-FA10 ranked all of those instructors in the top 25%, relative to departmental averages (the instructor for ENG-R/S was ranked lower, at 32%). Different results were achieved by instructors of comparable popularity, and the responses from students to the newly introduced topics were overwhelmingly positive, which leads us to conclude that the new curriculum was at least partly responsible for the increased popularity of the course.

II.B. Interpretive Attitudes

We may assess the relative impact of our transformed curriculum on student perspectives by further considering their post-instruction survey responses in relation to outcomes from previous modern physics offerings. The overall

distribution of student responses from our course to the double-slit essay question is consistent with prior results, which had shown them to be generally reflective of each instructor’s specific approach to that particular topic, whether *Realist*, *Quantum* or *Agnostic*. [Fig. 6.3] Considering this question had been adapted for use on the second exam, and that exam solutions detailing “acceptable” responses were later available online, it might be reasonably argued that the near absence of student preference for a realist interpretation of this experiment is mere confirmation of the effect of explicit instruction in that context.

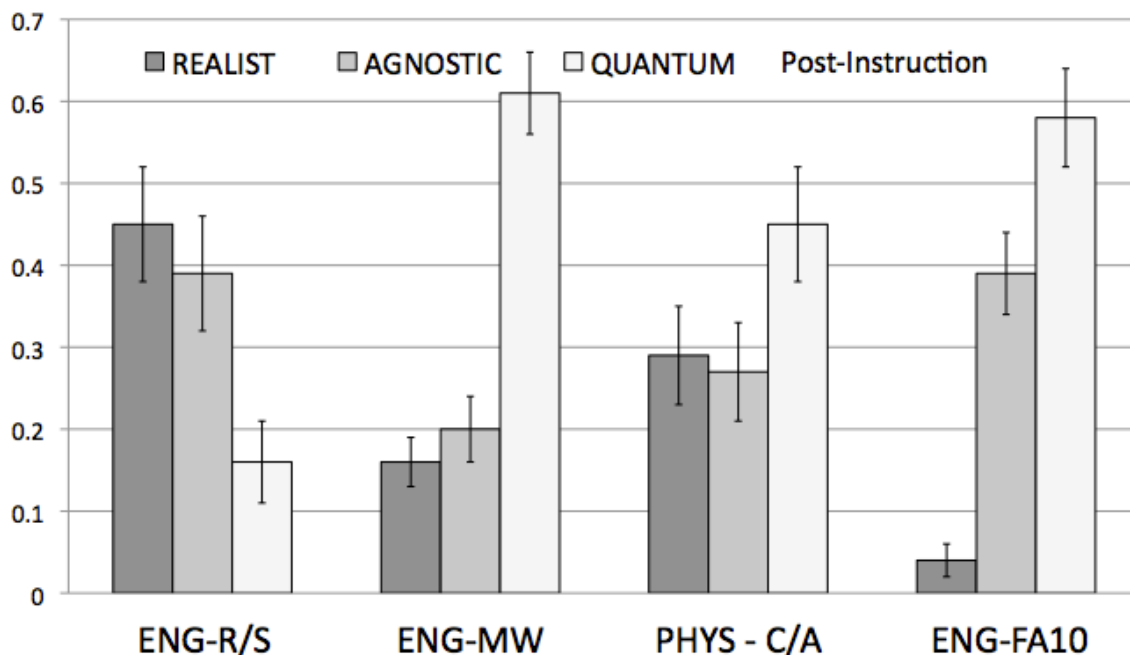


FIG. 6.3. Post-instruction student responses to the double-slit essay question, from four different modern physics courses, as denoted in Table 3.I. [Error bars represent the standard error on the proportion; $N \sim 50-100$ for each course.]

However, we made no mention during the entire semester of student responses to the pre-instruction attitudes survey, and did not give students any indication they would be revisiting these questions at the end of the course. We offered no explicit instruction as to what kinds of responses would be considered “acceptable”, and repeatedly emphasized in the survey and in the homework assignments that we were most interested in what students actually believed. The lecture materials used during our treatment of the Schrödinger model of hydrogen were essentially the same as those used in ENG-MW and PHYS-C/A, with a few notable exceptions. Like the instructors for those two courses, we showed students how the Schrödinger model predicts zero orbital angular momentum for an electron in the ground state, and contrasted this result with the predictions of Bohr and de Broglie. But we continued by explicitly arguing how this result has implications for

the physical interpretation of the wave function – for how could conservation of angular momentum allow for a localized particle to exist in a state of zero angular momentum in its orbit about the nucleus? This difficulty is removed when we choose to view atomic electrons as delocalized standing waves in quantized modes of vibration. More importantly, having already established language and concepts specific to interpretive themes, we were able to explicitly identify the position of an atomic electron as yet another example of a hidden variable, which we had argued couldn't exist as a matter of principle. Ours is the only course among these four where a significant majority of students chose to disagree with the idea of localized atomic electrons at the end of the semester. [Fig. 6.4]

The instructor for ENG-R/S told students during lecture that they *should* think of atomic electrons as localized, and overall responses from his course reflect this instruction. More specifically, he explained that quantized energy levels represent the average behavior of electrons over a time scale that is long relative to their orbital frequency, and that atomic electrons may be found to have a continuous range of energies when the time scale of the energy measurement is short (as enforced by the time-energy uncertainty relation); hence the broadening of spectral lines. This kind of reasoning is not unique among physicists, [5] and has therefore likely been utilized by modern physics instructors elsewhere.

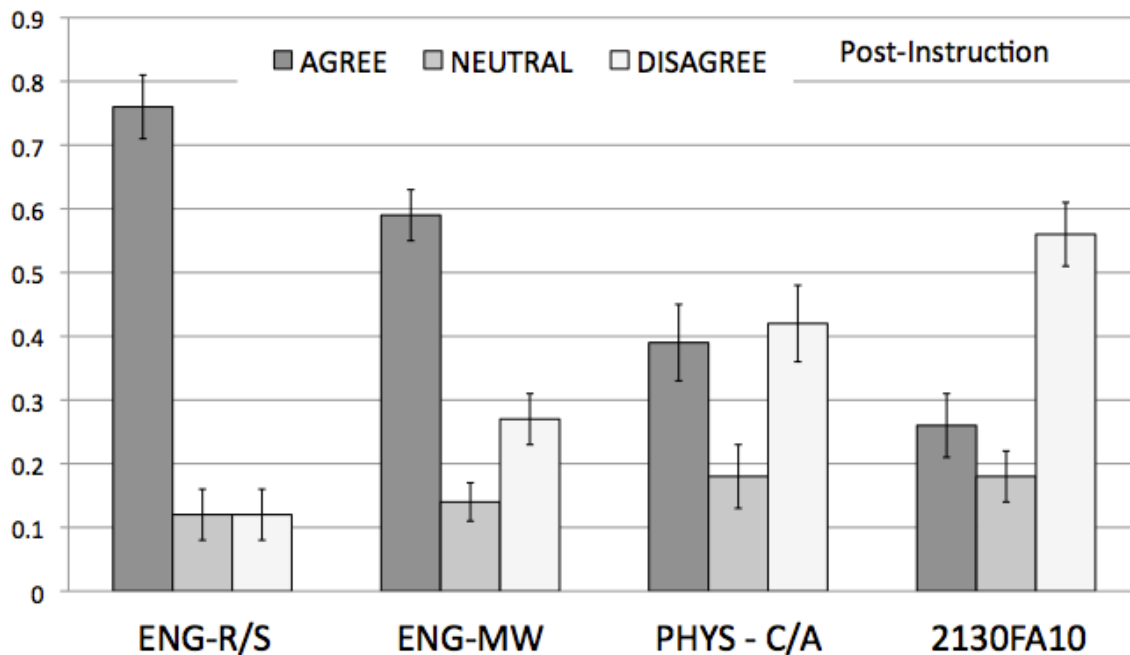


FIG. 6.4. Post-instruction student responses to the statement: *When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time*, from four different modern physics courses, as denoted in Table 3.I. [Error bars represent the standard error on the proportion; $N \sim 50$ -100 for each course.]

We have previously characterized the other two courses as having de-emphasized matters of interpretation in the latter parts of the course, [Chapter 3] and heard from one instructor [PHYS-C/A; Instructor C in Chapter 3] about what had influenced his instructional choices – he felt that giving students a facility with the mathematical tools of quantum mechanics should take precedence over a detailed exploration of its physical interpretation, which might anyways be beyond the sophistication of introductory students. Though different in his overall interpretive approach, it turns out the other instructor [ENG-MW; Instructor B2 in Chapter 3] offered similar reasoning for having made a similar choice, and so it is worthwhile to consider one last time in detail what we consider to be a common motivation for the de-emphasis of interpretive themes in introductory modern physics courses, according to the instructor for ENG-MW:

“This [probabilistic] aspect of quantum mechanics I feel is very important, but I don’t expect undergraduate students to grasp it after two months. So that’s why I can understand why [the statement on atomic electrons] was not answered to my satisfaction, but that was not my primary goal of this course – not at this level. We don’t spend much time on this introduction to quantum mechanics, and there are many aspects of it that are significant enough at this level – it is really great for students to understand how solids work, how does conductivity work, how does a semiconductor work, and these things you can understand after this class. If all of the students would understand how a semiconductor works, that would be a great outcome. I feel that probably at this level – especially with many non-physics majors – I think that’s more important at this point. But still, they have to understand the probabilistic nature of quantum mechanics, and I hope, for instance, that this is done with the hydrogen atom orbitals, not that everyone would understand that, but if the majority gets it that would be nice. These are very hard concepts. At this level, I feel it should still have enough connections to what they already understand, and what they want to know. They want to know how a semiconductor works probably much more than where is an electron in a hydrogen atom. [...] I don’t think the [engineering] students will be more successful in their scientific endeavors, whether it’s a personal interest or career, by giving them lots and lots of information about how to think of the wave function. The really important concept I feel is to see that there is some sort of uncertainty involved, which is new, which is different from classical mechanics. [...] At the undergraduate level, I feel it is important to make the students curious to learn more about it – and so even if they don’t understand everything from this course, if they are curious about it, that’s more important than to know where the electron really is, I think.”

We see the instructor for ENG-MW *would have liked* for his students to disagree with this statement, and yet 75% of them chose to *not disagree*. Recall that this instructor made his own modifications to the first modern physics curriculum, to

include an entire lecture on quantum measurement and interpretation towards the end of the course (but without specific reference to atomic systems).

At the end of the introduction to matter waves, our transformed course and ENG-MW both utilized a lecture slide similar to the one shown in Fig. 6.5 – note that both courses offered similar explicit guidance, albeit decontextualized, on how to think of electrons when not being observed: as delocalized waves. We believe this kind of general guidance is not by itself sufficient to cause most students to reconsider their conceptions of atomic electrons, as evidenced by the distribution of responses from a course that did not apply more specific guidance in the context of atoms. [Fig. 6.4] But specifically telling students to think of atomic electrons as delocalized would also not by itself be sufficient for significantly influencing students' overall perceptions of uncertainty in quantum mechanics.

Matter Waves (Summary)

- Electrons and other particles have wave properties
(interference)
- When not being observed, electrons are spread out in space
(delocalized waves)
- When being observed, electrons are found in one place
(localized particles)
- Particles are described by wave functions: $|\Psi\rangle = \Psi(x, t)$
(probabilistic, not deterministic)
- Physically, what we measure is $\rho(x, t) = |\Psi(x, t)|^2$
(probability density for finding a particle in a particular place
at a particular time)
- Simultaneous measurements of x & p are constrained by the
Uncertainty Principle: $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

FIG. 6.5. A lecture slide equivalent to one used in each of two modern physics courses for engineers, ENG-MW & ENG-FA10. This slide offers explicit, but decontextualized, guidance on how to think of matter when not being observed.

We may conclude this from our observation that explicit instruction in one context does not necessarily influence student perspectives in other contexts, but also by other considerations. Even if the physical interpretation of atomic wave functions is not a primary learning goal for every instructor, we may safely say that our course shared with ENG-MW and PHYS-C/A a common learning goal that *was* primary: recognizing a difference between the experimental uncertainty of classical mechanics and the fundamental uncertainty of quantum physics. How do these four courses compare with respect to student responses to our last attitudes statement

on the probabilistic nature of quantum mechanics? Realist expectations might lead incoming students to favor agreement with the statement: *The probabilistic nature of quantum mechanics is mostly due to the limitations of our measurement instruments.* We find that the incoming percentage of students from all three of the engineering courses agreeing with this statement is nearly identical, [ENG-R/S: 45%; ENG-MW: 48%; ENG-FA10: 46%] but that incoming attitudes for physics majors were significantly more favorable (with only a quarter of them agreeing, and over half disagreeing before instruction).

It would seem from his explanation of atomic energy quantization that the *Realist/Statistical* instructor would consider the uncertainty in quantum mechanics as being introduced by the *measurement process*, which is not the same as asserting that quantum uncertainty is experimental in origin, or that technology might one day find a way around these fundamental limits on observation. Regardless, there was a mild uptick in students from his course agreeing with this survey statement at the end of the semester. [Fig. 6.6] The instructor for PHYS-C/A had the greatest proportion of favorable responses at post-instruction – despite a de-emphasis on interpretive themes, he was successful in positively influencing student perspectives on uncertainty in quantum mechanics, though we must keep in mind the student population of his course, and the already relatively favorable incoming attitudes of his students.

In fact, the differential impact on student responses from these four modern physics courses is most dramatically illustrated by normalizing (post – pre) shifts in student agreement with this survey statement, according to their rate of agreement at the start of the course.¹ [Fig. 6.7] By this measure, our course had the greatest positive impact on student attitudes regarding the relationship between fundamental uncertainty in quantum mechanics and classical experimental uncertainty.

We conclude this section with some comments on the statements of the instructor for ENG-MW, regarding what we might like for students to take away from our introductory courses. First, if the aim of instruction is not necessarily a universal understanding of concepts, but for students to come away with a continued interest in modern physics, then we would claim that our course was the more successful of the two: student interest in quantum mechanics increased from 70% to 75% for his course (with 10% responding negatively at post-instruction), but the reported interest among students from our course increased from 85% to 98%, which we have argued must be in part attributable to the transformed curriculum itself. Second, we shouldn't presume to know exactly where the interests of our engineering students lie. The results from our curriculum implementation would suggest that students are in fact *just as interested*, if not more so, in questions about the nature of reality, as they are in learning about the theory of semiconductors.

¹ We define *favorable gain* as the negative of this, since we consider a *decrease* in agreement with this statement as favorable. This definition is equivalent to the definition of *normalized gain* = (post – pre)/(1 – pre), except that the target response rate is zero, and not 100%.

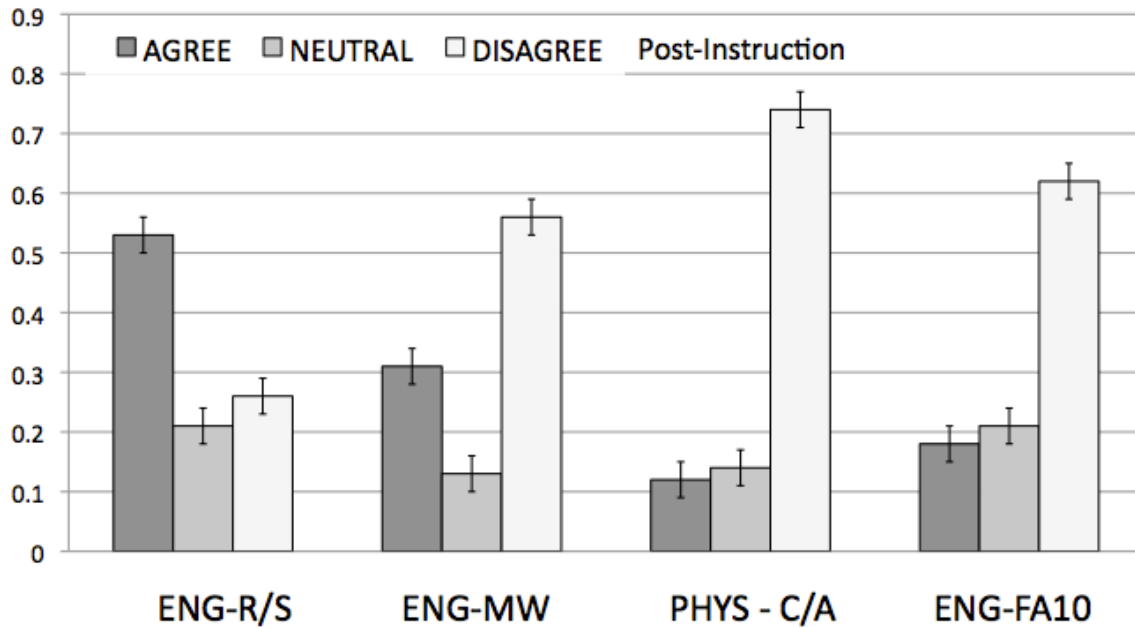


FIG. 6.6. Post-instruction student responses to the statement: *The probabilistic nature of quantum mechanics is mostly due to the limitations of our measurement instruments*, from four different modern physics courses, as denoted in Table 3.I. [Error bars represent the standard error on the proportion; $N \sim 50-100$ for each course.]

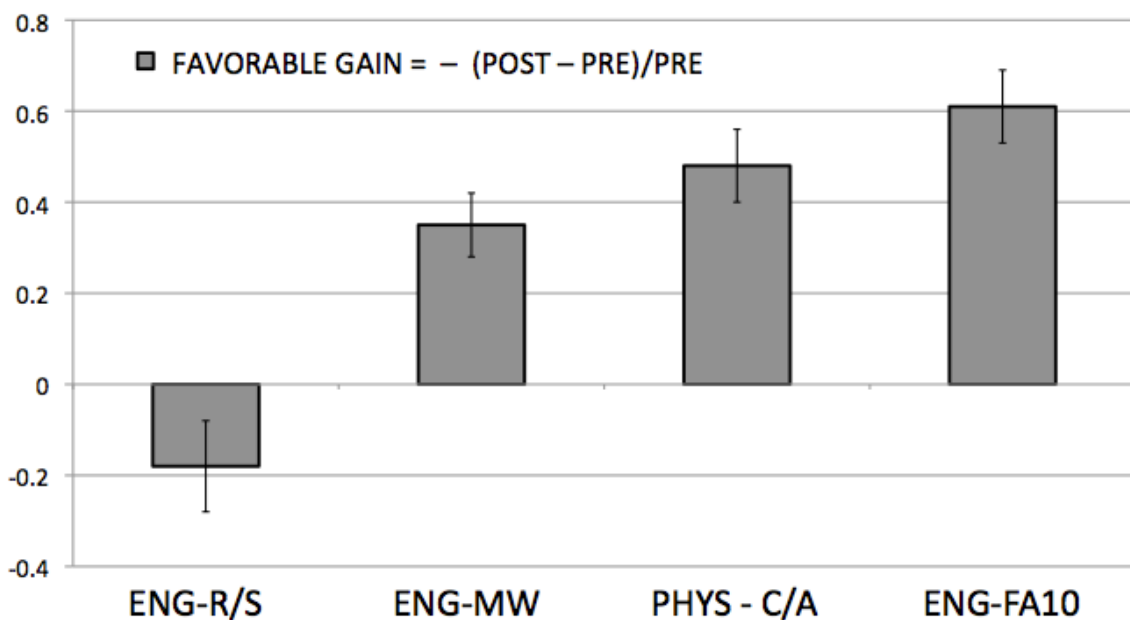


FIG. 6.7. Normalized favorable gain in the (post – pre) rate of student agreement with the statement: *The probabilistic nature of quantum mechanics is mostly due to the limitations of our measurement instruments*, from four different modern physics courses, as denoted in Table 3.I. A positive favorable gain is defined as a *decrease* in agreement with this statement. [$N \sim 50-100$ for each course.]

And finally, we didn't just give our students lots and lots of information about how to think of the wave function – we also gave them lots and lots of information about molecular bonding, conduction banding, semiconductors, transistors and diodes; as well as lasers, scanning tunneling microscopes, and nuclear energy; not to mention applications of nonlocality to quantum cryptography and computing. We had time for this because our course omitted topics from special relativity, which have generally cost other modern physics courses a minimum of three weeks out of a 15-week semester. We wouldn't claim that special relativity is not a relevant and worthy topic for engineering students, but the original decision to omit special relativity was in part a response to an overall consensus among engineering faculty at the University of Colorado, that their students would be better served by a curriculum that emphasized the quantum origins of material structures, and other real-world applications. [1] Every modern physics instructor at CU has had the option of removing special relativity from the engineering curriculum, and its re-emergence following the first round of course transformations is symbolic of a deep sense of tradition surrounding the topic, and stands in recognition of the profound influence its development has had on modern scientific thinking.

Our students had ample opportunity to contemplate the myriad contributions of Einstein's genius to the twentieth-century, but many of them were even more fascinated by the idea that Einstein could have been wrong about *anything!* And his glory was in no way diminished by telling our students this story of his confusion; for as we wove this tale of classical and quantum reality, he became a champion for those who expressed a deep commitment to their intuitions, which had become all the more apparent to them when we made their own beliefs (and not just our own) a topic of discussion. In the end, it is a question for each instructor of the pedagogical costs and benefits when deciding which story from the history of physics to tell our students, but we have made our best argument that the benefits may far outweigh any costs when we make the physical interpretation of quantum mechanics a central theme of our modern physics courses.

III. Curriculum Refinement and Other Future Directions

For the sake of future implementations of this curriculum, efforts should be made to assess where students had the most difficulty, so that suggestions for improvement can be made. Given the volumes of data collected in this dissertation project, we must confine our discussion here to specific examples of potentially fruitful changes, and suggestions for future studies.

III.A. Single-Photon Experiments

We begin by examining student responses to another essay question from the second midterm exam, designed to test student understanding of the single-photon experiments; we focus on this specific topic area for several reasons. First, single-quanta experiments with electrons and photons were the topics most commonly cited by students in their personal reflections as having influenced their perspectives on quantum physics, indicating this to be a key component of this curriculum's successful implementation. Second, the content of this lecture is fairly self-contained, and might easily be adapted by instructors who wish to augment their own courses without adopting the entire curriculum, and is therefore worthy of extra attention. Third, we are unaware of any instructional materials having yet been developed for introductory modern physics students concerning such experiments, and so have had no basis for judging ahead of time whether their implications for the meaning of wave-particle duality would be fully appreciated by our students.

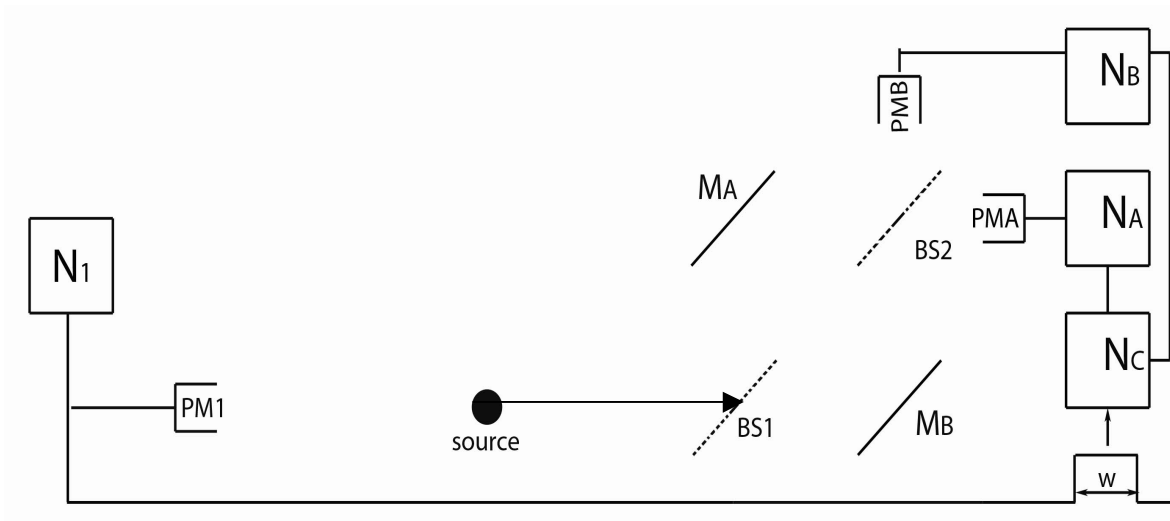
For this midterm, students were required to answer the first essay question on interpretations of the double-slit experiment, but were given the option of answering just one of the remaining two essay questions; if students chose to answer both of the remaining two problems, they received credit for the higher of the two scores. Naturally, we will have no insight into the difficulties faced by students who opted out of answering this question, but 75% of the 103 students who took the exam did respond, which should represent a fair sampling of overall student understanding of this topic. Generally speaking, students performed well on this question: the average total score was 6.75 out of 8 points, and 85% of responses received a total score of 6 or better. We shall first give the problem statement below, and then consider individual responses of our four students (A–D) from Chapter 5. Their individual answers will help to illustrate the coding scheme that emerged in our analysis of aggregate student responses, but also the quality of responses from students with whom we are already somewhat familiar.

The beginning of each of the first two parts asks students to identify for which experimental setup, X or Y, (see below) they would expect photons to exhibit particle-like behavior, and which for wave-like behavior. Calling these two experiments X and Y (instead of 1 and 2, as in the lecture slides; see Chapter 5), and reversing their order of presentation seemed to have no impact on student responses, since all students but one were correct in their identification for each case. We felt a key step in assessing student understanding of the implications of these experiments would be to determine whether they could describe in what sense the photon is behaving like a particle or wave in each setup. We were also interested in finding out which kinds of epistemological tools would be favored by students in justifying why each type of behavior could be expected in a given situation. The final part of the this essay question concerns a delayed-choice experiment that is the reverse of the situation described during lecture: here, the second beam splitter is in place at the time a photon encounters the first beam splitter. If the second beam splitter were to be quickly removed before the photon had passed through the apparatus, there would be no opportunity for the photon to

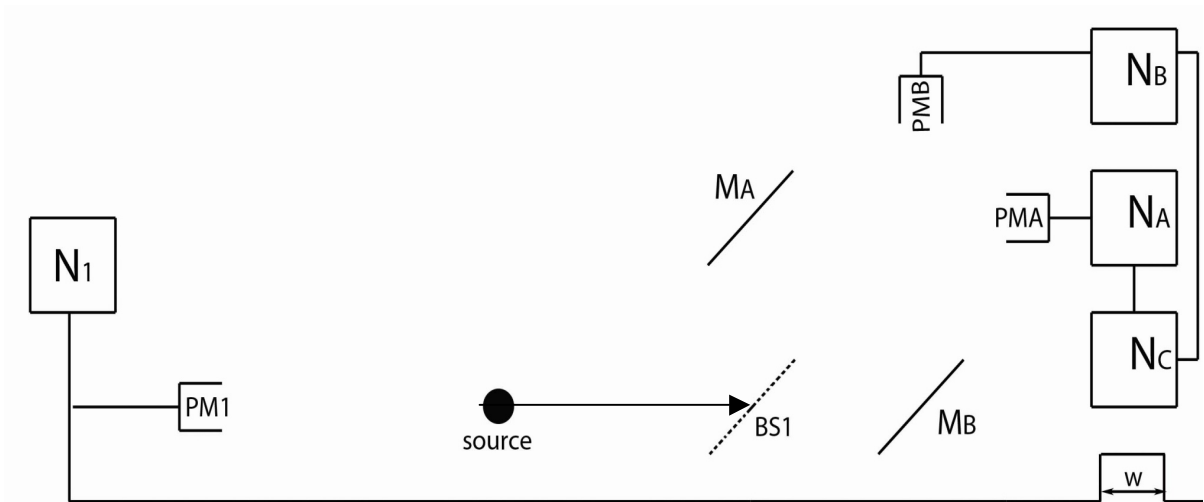
interfere with itself, meaning there is an equal likelihood for it to be detected in either photomultiplier.

E3. (OPTION TWO - 3 PARTS, 8 POINTS TOTAL) For the diagrams below depicting Experiments X & Y, M = Mirror, BS=Beam Splitter, PM = Photomultiplier, N = Counter. In each experiment a single-photon source sends photons to the right through the apparatus one at a time.

EXPERIMENT X



EXPERIMENT Y



E3.A (3 Points) For which experimental setup (X or Y) would you expect photons to exhibit particle-like behavior? Describe in what sense the photon is behaving like a particle during this experiment. What features of the experimental setup allow you to draw this conclusion without actually conducting the experiment?

Student A: In setup Y, the photons exhibit particle like behavior because the photon can only have one path to get to a particular photomultiplier. I know this because beamsplitter one will either allow the photon through or reflect it. If it reflects it it will go to PMA, if it is let through it will go to PMB. It can't take Path A to get to PMB thus there is one path to take, it acts as a particle.

Student B: Particle-like behavior expected in setup Y. Photon's path is predictable depending on the detector in which it was detected. It either gets reflected or transmitted at BS1, thus if detected at PMA, it must have been reflected and if detected at PMB, it must have been transmitted. We also know that $\alpha = P_C / (P_A P_B) = 0$ if there is only one photon in the apparatus during the time constant. This implies that $P_C = 0$ and no wave like behavior, acts like a particle. There is only one BS, so it will act like a particle (we know this even before conducting exp.)

Student C: Experiment Y should show photons acting like a particle. This is due to the fact that which path the photon takes can be determined by which photomultiplier is triggered. If the photon struck mirror B, PMB will fire, if the photon struck mirror A, PMA will fire. If there was truly only a single photon in the source only one of the photomultipliers will fire, and each would fire with a 50/50 chance.

Student D: Experiment Y (Aspect's 1st Experiment)
The photon may take one of 2 paths, but not both, and thus travels along a defined path consistent with the behavior of a particle. The way the experiment is set up, a photon may only take one of:
source – beamsplitter – mirror A – photomultiplier A
source – beamsplitter – mirror B – photomultiplier B
If a photon is to be detected in PM1, its pair must have exited the source in exactly the opposite direction, and by geometry can only take one of the two paths listed above.

Of the three parts to this essay question, this one presented the least problems for students, and 95% of them received full credit for their responses. Students were fairly uniform in the types of argumentation and reasoning they employed, and a simple coding scheme was almost immediately apparent. Many students offered multiple justifications for their answers, and so we ranked each type of argument according to its prominence in the student's response, or by which appeared first if they seemed to carry equal weight; we report here statistics only on students' primary responses.

In describing the behavior of a photon in Experiment Y, 58% of students said that, as a particle, it is only taking one path or other on its way from source to detector (Students A & D); and 40% said its particle nature is demonstrated by

being detected in either one PMT or the other, but not both (Students B & C). It seems significant that the majority of students associated particle behavior with definite trajectories (taking a single path), while fewer students associated particles with localized detections. This focus is also reflected in their identification of which features of the setup would allow them to predict particle-like behavior: 66% cited the fact that only a single path existed between source and each detector; 14% claimed the ability to determine which path a given photon had taken was sufficient for predicting this specific behavior. [16% focused on the literal difference between the two experiments – the absence of a second beam splitter.] So, a relatively small number of students relied on the new and more abstract epistemological tool developed in lecture, the availability which-path *information* as a determiner of behavior (as opposed to the existence of a single path). Not only did fewer students associate particles with localized detections, only 10% of all students made mention of measuring the anticorrelation parameter, or referred to counting rates and coincidence detections, even though these had been significant aspects of our presentation. This suggests that students are not entirely comfortable with the statistical nature of the argument for interpreting particle-like behavior in this experiment, which likely has implications for why students had greater difficulties with the flip side to this question:

E3.B (3 Points) For which experimental setup (X or Y) would you expect photons to exhibit wave-like behavior? Describe in what sense the photon is behaving like a wave during this experiment. What features of the experimental setup allow you to draw this conclusion without actually conducting the experiment?

Student A: In setup X, the photons exhibit wave like behavior because the photon can take either Path A or Path B and still get to PMA or PMB, we don't know which path it took, thus since it is unpredictable, it acts like a wave. Since it can take either path and still get to either photomultiplier, I know it can be represented as a wave.

Student B: Wave-like behavior expected in Setup X. Photon behaves like a wave because there is interference if we change the path length (move BS2). Thus it seems to interfere with itself. In this experiment, we can't know which path the photon takes due to the existence of BS2 (it could be detected by either PM, and have taken either path). We can also change BS2's location such that all the photons are detected in PMA or PMB. Throughout the experiment, it seems that the photon somehow "knows" that there are both paths. The BS2 lets us conclude this before starting the experiment (that it can behave like a wave).

Student C: Experiment X should show photons acting like waves. The path the photon took is undeterminable. Mirror B could have been hit with a photon and either PMB or PMA could fire. This implies a wave is being propagated through both possible paths. The wave then describes an equal probability of triggering each photomultiplier provided each path is the same length. Interference can happen if the paths are different length and cause only one photomultiplier to trigger.

Student D: Experiment X (Aspect's 2nd Experiment)

The exact path taken by the photon is rendered indeterminate by the second beamsplitter; we can't know which path the photon actually took to PMA or PMB. If we vary the path length of A or B, and observe interference as a result in the detectors, a logical explanation is that the wave that represents the photon split at beamsplitter 1, and then (due to the difference in phase created by the changed path length) interfered with itself to produce the observed results. The presence of the 2nd beamsplitter essentially randomizes whether a photon travelling along path A or B ends up in PMA or PMB (50% chance of either for fixed path length), thus rendering the path of the photon indeterminate, which allows for the above conclusions to be drawn.

Only 51% of students received full credit for their responses to this part of the question, but a total of 90% were given a score of 2/3 or better. 43% said that photons manifest their wave behavior in the form of interference (Students B, C & D), and 35% claimed that wave-like photons take both paths in this experiment. This is not precisely what Student A said – he mentions that photons are *capable* of taking both paths, but not that photons *are* taking both paths. In fact, his responses to this part of the question and the last suggest that he associates wave-like behavior with indeterminacy – photons are still presumed to take only one of two paths – it is our knowledge of which that is indefinite.

Most significant was the finding that 21% of students mistakenly believed, in Experiment X, that photons would be detected in both PMT's simultaneously; 5% explicitly stated that measuring the anticorrelation parameter as greater than one (coincidental detection) would be evidence of the photon's wave behavior in this case. In fact, for the data run presented in class demonstrating interference through path length modulation, the anticorrelation parameter was calculated to be 0.18 (less than unity, as it should be). We believe this confusion may be likely attributed to two factors. First, we only implied individual photon detections in our comparison of counting rates, but did not explicitly point out that the anticorrelation parameter had been found here to also be less than one. The specific wording of Slide 12 from this lecture [Fig. 6.8] could also be confusing for students. We want them to associate wave-like behavior in this experiment with what each photon does at the beam splitter, yet this slide could lead them to believe that wave behavior should be universally associated with coincidental detections. This misunderstanding could be directly addressed by placing greater emphasis on the connection between wave behavior and self-interference, or indefinite trajectories; and by placing greater emphasis on the continued particle-like *detection* of photons, focusing student attention instead on the behavior of the photons at each beam splitter.

Anti-Correlation Parameter

$$\alpha \equiv \frac{P_C}{P_A P_B}$$

- If N_A and N_B are being triggered randomly and independently, then $\alpha = 1$.
 - $P_C = P_A \times P_B$ which is consistent with:
 - Many photons present at once
 - EM waves triggering N_A & N_B at random.
- If photons act like particles, then $\alpha \geq 0$.
 - $P_C = 0$ when particles are detected by PMA or by PMB, but not both simultaneously.
- If photons act like waves, then $\alpha \geq 1$.
 - $P_C > P_A \times P_B$ means PMA and PMB are firing together more often than by themselves (“clustered”).

FIG. 6.8. Slide 12 from Lecture 20 (Single-Photon Experiments, see Chapter 5). In the first experiment, wave behavior is associated with coincidental detection; it is associated with indefinite trajectories and self-interference in the second.

We have further indication that students are uncomfortable with how wave interference is manifested in this experiment, which is different from directly observing a fringe pattern. Of all the students who mentioned interference as evidence of wave behavior, only half specifically said that it would be observed by making changes to the relative path lengths; the other half only commented that interference would be observed. Moreover, only 26% correctly spoke of interference in terms of modulated detection rates in the two photomultipliers, and 5% incorrectly believed that fixing the mirrors would cause every photon to take just one of the paths (as opposed to being detected in just one of the PMT’s). Whereas only 16% of students had cited the absence of the second beam splitter as being the key feature of Experiment Y, a full 38% of students focused on its presence in Experiment X as being key to determining what kind of behavior would be observed. 36% employed an epistemological tool developed in class: no *which-path information* would be available; and 23% said the availability of two paths for the photon was key to predicting wave-like behavior in this experiment.

These results, and those from the first part of the question, suggest that students attach greater significance to the question of which path a photon takes (strong associations with particle behavior), and focus less on its behavior at the beam splitter (weak associations with wave behavior). The argument for wave behavior presented in class centered on the behavior of the photon at the beam splitter, and so perhaps this emphasis was not properly communicated to students; but we may also consider exploiting the strength of student preference for which-path arguments by giving them greater prominence in our argumentation. After all,

we had been trying to develop the concept of *which-path information* as an epistemological tool, which might be aided by placing less emphasis on the response of photons to beam splitters, where students are less likely to have had any exposure to in previous classes. We had discussed them earlier in the context of the Michelson-Morley experiment, but perhaps there was insufficient connection made between the coherent 50/50 splitting of a classical EM wave, and a 50/50 probability for transmission or reflection of a photon.

Responses to the third part of this question show that the subtlety of the delayed-choice experiments was not entirely lost on students, but also provide additional evidence of student difficulties with probabilistic descriptions of measurement outcomes:

E3.C (2 Points) Suppose we are conducting Experiment X (the second beam splitter (BS2) is present) when a photon enters the apparatus and encounters the first beam splitter (BS1). Afterwards, while the photon is still travelling through the apparatus (but before it encounters a detector), we suddenly remove the second beam splitter (switch to Experiment Y). Can we determine the probability for the photon to be detected in PMA? If not, why not? If so, what would be that probability? Explain your reasoning.

Student A: No, we could not because we don't know which path the photon took, it could have taken path A in which it would be detected by photomultiplier A or it could have taken path B and not been detected by PMA. Since it has not been detected yet we can't determine the probability it's already on a definite path.

Student B: This is the delayed-choice experiment. We can indeed predict the path that the photon took if BS2 is not present depending on the detector in which it was detected. Thus, the probability of being detected in PMA would be 50/50 (0.5). It would act just as if we ran experiment Y and behave like a particle. Put the beam splitter back and it acts like a wave again. There is no "tricking" the photon!

Student C: First assume that experiment X is set up so that interference occurs and only PMA is firing. If the photon is still traveling through the apparatus, and BS2 is then suddenly removed, the photon will switch to acting like a particle. The photon will no longer only fire in PMA due to interference, but will instead show particle-like behavior and trigger either PMB or PMA with a 50/50 probability. BS1 results in either path from BS1 being 50/50 probable. Because when BS2 is removed, the path the photon took is now better known and particle like behavior is observed. In other words, once BS2 is removed PMB firing means MB was hit by a photon, and PMA firing means MA was hit by a photon.

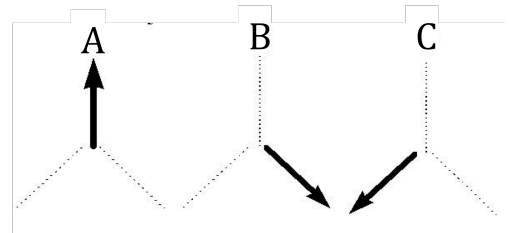
Student D: Yes, the probability will be 0.5 – same result as Experiment X with equal path lengths, but with a definite path for any given photon. A photon may exhibit either wave-like or particle-like properties, but not both in the same instant. Removing the 2nd beamsplitter "forces" the photon to exhibit particle-like behavior by making its path definite retroactively – example of a "delayed choice" experiment.

We note that each of the four students suggested that removing the second beam splitter forces a photon into taking a definite path (not that self-interference would no longer be possible), and only Student C's response makes explicit mention of the lack of interference. Again, student associations seem to be strongest between particles and definite paths. 81% of students said that the probability for detection in PMA could be known, but only 3/4 of those students explicitly stated that probability as being 50%. Student A seems to be close to drawing this conclusion, but there appears to be a disconnect between a completely indeterminate outcome and a 50/50 likelihood for either occurrence. Regardless of whether they felt the probability could be known, almost 40% of students did not state that the probability for detection in PMA is 0.5; this suggests that students require more practice with the use of probabilities, beyond the single lecture we devoted to classical probability and probability distributions.

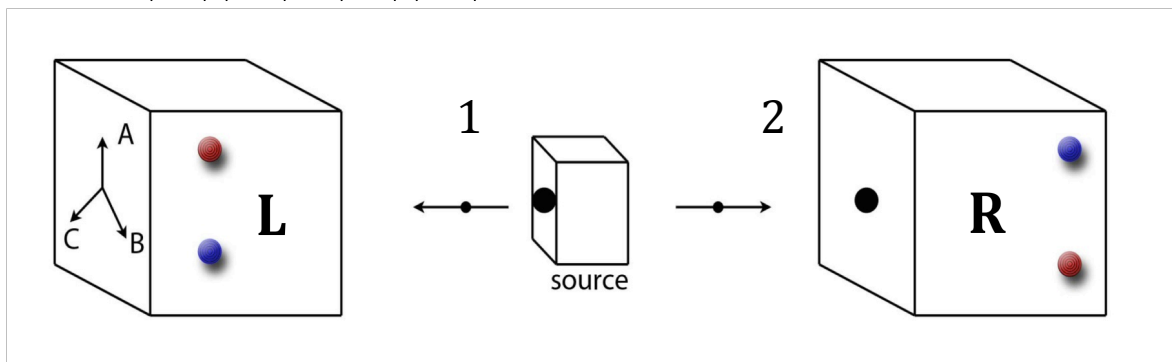
II.B. Entanglement and Correlated Measurements

The need for future studies into student difficulties with our transformed curriculum is illustrated by responses to a multiple-choice exam question concerning distant, anticorrelated measurements performed on entangled atom pairs:

6. Suppose we have two “Local Reality Machines” (Stern-Gerlach analyzers capable of being oriented along three different axes: A, B, & C, each oriented at 120° to each other, as shown) set up to detect atom-pairs emitted in an entangled state:



$$|\Psi_{12}\rangle = |\uparrow_1\rangle|\downarrow_2\rangle + |\downarrow_1\rangle|\uparrow_2\rangle$$



The leftward travelling atom (1) reaches the left analyzer (L) before the rightward traveling atom (2) reaches the right analyzer (R). The left analyzer is set on A and measures atom 1 to be “up” along the vertically oriented A-axis (it exited from the plus-channel). A short time later, atom 2 enters the right-side analyzer. If the right analyzer is set on B (120° from the vertical axis), what is the probability for atom 2 to exit from the plus-channel of the right analyzer?

TABLE 6.II Distribution of student responses to multiple-choice question #6 from the second midterm exam – correct response highlighted in bold.

A) 0	B) 1/4	C) 1/2	D) 3/4	E) 1	RIGHT	WRONG
14%	34%	12%	42%	0	42%	58%

The majority of the class got this question wrong (58%, see Table 6.II), but we have little insight into the reasons for this, since each option is a significant distractor, with many potential sources of confusion. We note that no student selected option E (1), and so we may at least conclude that students did not believe that a measurement of “up” in the first detection requires an “up” measurement for the second. Option C (1/2) was chosen by 12% of students, which may indicate they did not recognize how the entangled state of the atom pair, and therefore the outcome of the first measurement, establishes a definite state of “down” for the second particle along Axis-A; this response would be correct if there were no influence of the first measurement on the second. Option B (1/4) was the most popular incorrect response, which comes from using $(120^\circ/2)$ as the relevant angle in calculating the probability for an “up” measurement along Axis-B, when it is actually $(60^\circ/2)$ – it varies according to the cosine squared of the half-angle between incoming state and axis of analyzer orientation. Students who correctly identified this angle may have forgotten to divide by two; or they may have correctly applied the formula, but thought the second atom would also be measured as “up” along Axis-A; or they may have simply been distracted by the prominence of the 120° angle in the problem statement. Option A (0) was also a popular response (14%), which may imply these students felt that an “up” measurement for the first particle precluded an “up” measurement for the second particle along *any* axis. Adapting this specific question into a short-answer problem, where students would be required to provide their reasoning, would be a first step toward understanding some of the difficulties students have with entanglement and distant correlated measurements.

II.C. Atomic Models and Probability

One of the questions adopted from the QMCS [1] for our post-instruction content survey was designed to elicit common student misconceptions regarding the outcome of a position measurement for an atomic electron in the ground state of hydrogen:

30. The electron in a hydrogen atom is in its ground state. You measure the distance of the electron from the nucleus. What will be the result of this measurement?

- A. You will measure the distance to be the Bohr radius.
- B. You could measure any distance between zero and infinity with equal probability.
- C. You are most likely to measure the distance to be the Bohr radius, but there is a range of other distances that you could possibly measure.
- D. There is a mostly equal probability of finding the electron at any distance within a range from a little bit less than the Bohr radius to a little bit more than the Bohr radius.

TABLE 6.III Distribution of student responses to multiple-choice question #30 from the post-instruction content survey – correct response highlighted in bold.

A	B	C	D	E	%Correct	%Incorrect
30%	2%	49%	18%	0	49%	51%

An analysis of student responses to a midterm exam question on atomic models showed that only ~10% of students exclusively employed a planetary model in their descriptions of hydrogen, yet 30% of students incorrectly answered on the post-instruction survey that the electron would definitely be found at the Bohr radius, and 18% thought it was equally likely be found somewhere in that vicinity. This apparent disconnect may be explained by further difficulties students have in using probabilities to describe the outcome of quantum measurements, but it may also indicate realist commitments that were not revealed by the attitudes survey statement on atomic electrons. [See above, Section II.B.] Option D may have been popular among students that favor the de Broglie atomic model over a planetary description, but we must only speculate without the opportunity to further question students on the reasons for their responses, which is impossible in an end-of-term, multiple-choice format.

IV. Concluding Remarks

Perhaps the most important take-home message from these studies is that students will develop their own attitudes (right or wrong, sophisticated or not) regarding the physical interpretation of quantum mechanics when we, as instructors, do not explicitly attend to the realist beliefs that are so common among our introductory modern physics students. We have frequently heard that a primary goal when introducing students to quantum mechanics is for them to recognize a fundamental difference between classical and quantum uncertainty. The notorious difficulty of this has led many instructors to view this learning goal as superficially possible, but largely unachievable in a meaningful way for most introductory students. We believe our studies demonstrate otherwise. By addressing the physical interpretation of quantum phenomena across a variety of contexts, but also by making questions of classical and quantum reality a central theme of our course, we were able to positively influence student thinking across a variety of measures, both attitudinal and in content-specific topic areas.

We have developed a framework for understanding student interpretations of quantum mechanics, which show how their overall perspectives may be influenced by their specific attitudes toward several individual themes central to the question of probabilistic measurement outcomes. Is the wave function physically real, or a mathematical tool? Is the reduction of quantum superpositions to definite states an ad hoc rule established to make theory agree with observation, or does it represent some kind of physical transition not described by any equation? Is an electron, being a form of matter, strictly localized at all times? We have identified student attitudes regarding these questions as playing a key role when formulating their thoughts on quantum phenomena, and have seen how the myriad ways in which these attitudes may combine can lead to a variety of overall interpretive stances. If we wish to have significant influence on student perspectives, and if we are to take seriously the lessons learned from education research on the impact of *hidden curricula*, then we must choose to explicitly address these beliefs in our introductory courses.

We also believe that a *static* view of student and expert ontologies, however useful in addressing student difficulties in classical physics, is too limited to account for the contextually sensitive and highly dynamic thought processes of our students when it comes to ontological attributions. We have seen students blend attributes from the classically distinct categories of particles and waves; they may switch between views according to their cognitive needs of the moment; and they often distinguish between their intuitive perspectives, and what they have learned from authority. At the very least, we may conclude that ontological flexibility does not come easily to most students, and that the contextual sensitivity of their responses is most consistent with students engaging in a piecemeal altering of their perspectives, rather than some wholesale shift (or replacement) in ontologies. Most importantly, many of our students demonstrated exactly the kind of ontological flexibility that is required for a proper understanding of quantum mechanics. We believe this learning goal is more easily achieved by placing greater emphasis on the meaning of wave-particle duality, and by providing experimental evidence that

favors dualistic descriptions, but also by explicitly addressing in class the commonly held beliefs of students revealed by our studies. Among the many learning goals for our transformed curriculum was for students to be consciously aware of their own (often intuitive and tacit) beliefs, but also for them to acquire the necessary language and conceptual inventory to identify and articulate those beliefs. This was accomplished in part by presenting them with specific terminology relevant to perceptions of reality and locality, but also by making the beliefs of students (and not just the beliefs of scientists) a topic of discussion in our course.

It would be too simplistic to say that our aim was for students to consistently *not agree* with realist interpretations of quantum phenomena. After all, there are a variety of situations in quantum mechanics where the physical interpretation of the wave function has no relevance or bearing on the outcome of a calculation. It is not that a particle view of matter is entirely illegitimate in quantum mechanics; it is simply that its consistent application in all contexts is not adequate in accounting for all of what we observe in nature. We suggest that a significant amount of the confusion introductory students feel when learning about quantum mechanics results from the paradoxical conclusions that come as a consequence of realist expectations and ontological inflexibility.

Nor would we wish to connote too much negativity with the fact that students are relying on their intuition as a form of sense making. It is true we are telling them that their everyday thinking can be misleading in quantum physics, but that is not a sufficient argument for the wholesale abandonment of productive epistemological tools. Indeed, our approach to teaching quantum interpretations frequently required an appeal to student intuitions about the classical behavior of particles (they are transmitted or reflected; they are localized upon detection), and similarly with waves. A more important goal is for students to achieve more internal consistency in their thinking, which may be cultivated by developing epistemological tools that aid in deciding which type of behavior should be expected in which type of situation. Considering the observed strong associations students make between particles and definite paths, it seems that framing such tools in terms of *which-path information* [two paths = interference; one path = no interference] may be particularly useful for students.

Of the many potential studies that might be conducted as an improvement on those presented here, we believe that focusing on the thinking associated with *Agnostic* students would be particularly beneficial. We have never considered an agnostic perspective to be unsophisticated; in fact, our *Agnostic* category [as defined in Chapter 4] was meant to include both students *and* experts who acknowledge the potential legitimacy of competing perspectives, without taking a definitive stance. Agnosticism, by this definition, would therefore involve an acknowledgement of evidence that favors more than just a single interpretation, which is clearly different from students who exclusively assert the legitimacy of their realist intuitions *in spite of* evidence to the contrary. At the same time, an agnostic stance may be indicative of the perception that nothing can truly be known or understood in science, since many of the assumptions we make about the world turn out to be demonstrably false, and so much in quantum physics cannot be directly observed. Either way, an agnostic stance among students may be interpreted as an intermediary stage in the

transition away from realism, but might also signal unfavorable perceptions on the *nature of science*. Negative perceptions about what can and can't be known in science might be an unintended consequence of our curriculum transformations, and require further detailed consideration.

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