**Definition of potential energy PE.**  The change in PE is the amount of work done on a system by an external force, when the KE does not change and no heat flows (no friction).

PE = Wext , when KE = 0 and no friction or heat

Units of PE = units of work = joules. KE, PE, and work all have the same units, the unit of energy = joule.

Potential energy is a kind of *stored energy* associated with position or configuration. PE comes in several varieties. We can use our definition of PE to derive a formula for the **gravitational PE**. If an arrangement of masses has gravitational PE, there exists a *potential* to change that PE into KE.

Change in PE of a mass of mass m when it changes height by y = 

Only *changes* in PEgrav and *changes* in y have physical significance. We are free to set the zero of height and the zero of PE wherever convenient. If we agree that that PE = 0 when y = 0, then we can write 

m

+y

mg

Fext

y

**Derivation of PEgrav = m g y.**  Suppose I slowly and steadily lift a m = 1 kg object a distance y = yf – yi = 1 m above a table. What is the *change* in the gravitational potential energy? According to the definition of PE,PE = Wexternal = Fext y. Since v = constant and acceleration a = 0, then Fnet  = 0, and the external force of my hand has magnitude Fext = mg. So



Gravitational PE is associated with the system of (mass m + earth + gravitational attraction between mass m and earth). The PE is not "in the mass" or "in the earth"; it is in the mass-earth system.

When a spring is stretched or compressed, the spring contains **elastic potential energy**:



This formula comes from our definition of PE: PE = Wext If you slowly compress (or stretch) a spring by an amount x, the force you exert varies from zero (when x = 0) to Fmax = k⋅x . The *average* external force you exert is (1/2)k⋅x , and the work you do is Wext = force × distance = Faverage × distance = (1/2)k⋅x ⋅x = (1/2)kx2.

Another definition: **total mechanical energy of a system** = Emechanical  ≡ KE + PE

Think of work as "energy input". If positive work is done by an external force on the system, the system energy increases. If negative work is done by an external force, then the system energy decreases. If no work is done on the system by the surroundings (and no heat is added), then the energy of the system is constant.

**work done on system = energy change of system**

**System**

**Surroundings**

+work = +Emech

E­mech = KE+PE

– work = – Emech

**dissipation** = conversion of mechanical energy into thermal energy (because of frictional forces)

IF NO DISSIPATION (meaning no friction – more on friction later), then….

1) IF a system is *isolated* from outside forces, then one can prove that

Emechanical  ≡ KE + PE = constant (isolated system, no dissipation)

KE can change into PE, and PE can change into KE, but the total (KE + PE) is constant. This is a statement of the conservation of energy.

2) If the system is *not isolated* from outside forces and there is no heat transferred, then the work done on a system by the external force is equal to the change in total mechanical energy (KE+PE) of the system. When *positive* work is done by the external force on the system, the energy of the system increases. (Energy is transferred into the system from the surroundings.) When *negative* work is done by the external force, then the mechanical energy of the system decreases (energy is transferred from the system to the outside).

Wexternal force  = Emechanical  = KE + PE (no heat transfer, no dissipation)

**Example of KE + PE = constant for isolated system.** A book of mass m falls from rest a distance h, but has not yet hit the floor. Here, the system = (book + earth + gravity). What is the change in PE? What is the change in KE? What is the change in (KE + PE)?

+y

yf, vf

yi , vi = 0

m

h

PE = mg y = –mgh (from definition of PE)

vf2 = vi2 + 2a(yf – yi) = 0 – 2g(–h) = +2gh (from constant *a* formula)

KE = KEf – KEi = KEf = (1/2)mvf2 = (1/2)m(2gh) = +mgh

Emechanical  = KE + PE = +mgh – mgh = 0

As the book falls, PE is converted into KE, but the total KE+PE remains constant. Once the book hits the floor, the KE will get converted into thermal energy (book, ground, air will heat up) – an example of the process we call dissipation.

Commentary: For the particular case of a falling book, we have shown that (KE + PE) = constant for an isolated system with no internal friction (no dissipation). This turns out to be true always.

**Example of Wext = KE + PE.** I steadily lift a book a distance *h*, starting and ending at rest. System = book+earth, and my hand is exerting an external force on the system. What is PE? What is Wext?

KE = 0 at beginning and end, so KE = 0.

+y

vi = 0

m

vf = 0

h

Fgrav = mg

Fhand

Forces on book:

PE = mg y = +mgh

Lift book with v = constant ⇒ Fnet = 0 ⇒

Fhand = Fgrav = mg

Wext = Whand = F|| d = +mgh

(work by hand positive because Fhand in same direction as displacement.)

Notice that Wext = KE + PE in this case.

My hand did positive work (+mgh), so system's mechanical energy increased (by +mgh). Energy flowed out of me through complex bio-chemical processes (I'm burning up the eggs I ate that morning) and went into the gravitational potential energy of the book+earth system.

Commentary: We have just shown that Wext = KE + PE) in two particular cases where there is no heat transfer and no dissipation (no friction). Using Newton's laws, it is possible to prove that Wext = KE + PE) is always true (when no dissipation or heat transfer occurs).

**Example of Conservation of Energy (no friction).** A pendulum consists of a mass *m* attached to a massless string of length *L*. The pendulum is released from rest a height *h* above its lowest point. What is the speed of the pendulum mass when it is at height *h*/2 from the lowest point? Assume no dissipation (no friction).

h

h/2

v = ?

L

vo = 0

In all Conservation of Energy problems, begin by writing **(initial energy) = (final energy)** :

E i  = E f ⇒ KEi + PEi  = KEf + PEf

⇒ 0 + mgh = (1/2) mv2 + mg(h/2)

(cancel m's and multiply through by 2) ⇒

2gh = v2 + gh ⇒ v2 = gh⇒ 

Notice: Using Conservation of Energy, we didn't need to know anything about the details of the forces involved and we didn't need to use **F**net = m**a**. The Conservation of Energy strategy allows us to relate conditions at the beginning to conditions at the end; we don't need to know anything about the details of what goes on in between.

h

vf = ?

yo = 0

**Another example of Conservation of Energy (no friction)**: A spring-loaded gun fires a dart at an angle  from the horizontal. The dart gun has a spring with spring constant *k* that compresses a distance *x*. Assume no air resistance. What is the speed of the dart when it is at a height *h* above the initial position?

E i  = E f ⇒ KEi + PEgrav,i + PEelas,i = KEf + PEgrav,f + PEelas,f­

0 + 0 + (1/2)kx2 = (1/2)mv2 + mgh + 0



Notice that the angle never entered into the solution.

**KE+PE=total energy graphs**

Suppose a roller coaster of mass m rolls along a track shaped like so:

h

x

m

track

The shape of the track is a graph of height h vs. horizontal position x. Since the gravitational potential energy of the coaster is PE = mgh, where mg is a constant, a graph of PE vs. x looks the same as the graph of h vs. x, but the vertical axis measuring energy (joules) rather than height (meters). Assuming no friction, the total mechanical energy Etot = KE+PE of the roller coaster remains constant as it rolls along the track. We can represent this constant energy with a horizontal line on our graph of energy vs. x. From this "energy graph", we can read the KE and the PE of the coaster at any point.

PE

x

PE

KE

Etot = KE+PE

turning point

KE max,

PE min here

**What if there is friction and thermal energy involved?**

Up till now, we have assumed that there is no sliding friction and no heat transferred in any of these problems. (Having static friction in a problem causes no difficulties, because static friction does not generate thermal energy.) How do we handle sliding friction and heat generated?

If a system is isolated from external forces so that no work is done, and if no heat is transferred, and if there is no sliding friction so that no thermal energy is generated (that's a lot of "if's"), then we can assert that

KE + PE = constant (isolated system, no sliding friction)

If, however, there is sliding friction, then some of the mechanical energy (KE+PE) can be transformed into thermal energy (Etherm). In this case, we can say

KE + PE + Etherm = constant (isolated system)

It turns out that the amount of thermal energy generated is the negative of the work done by friction:

Etherm = – Wfric­

The work done by sliding friction is always negative, since sliding friction always exerts a force in the direction opposite the motion. Consequently, –Wfric is a positive quantity.