Calc-based Physics 1

Introduction:

This course is a fast-paced, demanding, and difficult. In my opinion, we try to cover too much in too short a time, but somehow, every year, most freshmen science students taking this course do just fine. But everyone who does well in this course has to work hard. You can vastly improve your learning experience by following this …

Recipe for Success.

1. Take this course seriously, starting on the first day. That means scheduling study time every day, starting today. Organizing your time is an essential part of learning.

2. Read the chapter and/or the online notes before the lecture. Read carefully, do not skim. When you study, you must devote your full attention to the task.

3. Read with a pen and notebook. Take notes while you read, and work out example problems.

4. Attend every lecture, and participate in lecture: confer with your peers, ask questions, pay full attention, don't surf the web, don't text. Read how multi-tasking damages your brain.

5. Don’t attempt the homework without studying the chapter first. But start your homework assignments EARLY. If you wait till the last day, and you get stuck, you won’t have time to get help.

6. Don’t get isolated: Get in a study group. Go to the Physics HelpRoom.

7. Don’t fall behind; you’ll never catch up.

8. Be an aggressive learner, not a passive listener. You must constantly ask yourself: Does this make sense? Do I understand it? How would I explain it to someone else? If it doesn't make sense to you, do something about it.

9. Just reading the text, attending lecture, and doing the homework is ***not enough***. You have to ***understand*** the material. Here is the Test of Understanding: If you can explain the material, in words, to someone else, without referring to the text, then you understand. This course is not about memorizing; it's about understanding.

Necessary background for this course.

This course requires that you have a good command of algebra and trigonometry. Calculus 1 is the co-requisite for this course, meaning that you should either be taking a first course in calculus now or have already taken it recently.

We also assume that you are familiar with the metric system (meters, kilograms, seconds…) and know the various metric prefixes: millimeter means 1/1000 of a meter, kilometer means 1000 meters, "mega" means million, etc. Also, you should be comfortable with exponential notation: 3.2×10^{8} = 3.2E8 = 320,000,000 ; 4.5×10^{-6} = 4.5E–6 = 0.000 0045 ; etc.

We do not assume that you have taken a course in physics before, but any previous exposure to physics will help.

Necessary attitude toward this course

Many students mistakenly believe that learning means memorizing. In this course, you will not be required to memorize anything. Memorizing is a component of learning, but in this course, it is a relatively minor component. On all the exams, you will be allowed bring a formula sheet on which you can write whatever you want. So you never have to worry that you might forget some formula, because you can just put it on your formula sheet.

On the exams, we will never test whether you have memorized some formula; instead we will test things like: do you understand what the formulas means, when does it apply, and how to use it to solve a problem. Questions on the exams will be similar to, but never the same as, questions you have seen previously in the course. So memorizing answers to questions won't help you on exams. Instead you need to understand and remember the *strategies* for getting the answers. The exams are hard, because *understanding strategies* requires mastery of the material. Memorizing answers only requires an acquaintance with the material. Monkeys can remember; only humans can understand. Before each exam, we will give you access to a practice exam so you can test yourself and know what to expect. Although I have downplayed the importance of memorization, you cannot really master the material unless you have worked so long and so hard that some memorization is inevitable. If you have studied so little that you cannot remember the few fundamental laws and definitions in each chapter, then that is a warning signal that you are studying ineffectively or too little.

In order for you to do well in this course, you need to take the attitude that you will study the material until it *makes sense* to you, until it seems logical. If it makes sense, then you understand it, and if you understand it, then you can explain it to someone else. Being able to explain something to someone else is the acid-test of understanding. How many times have you heard someone say: “I understand it. I just can’t put it into words.” ? Anyone who thinks this is fooling themselves. Our brains process information using words, so if you cannot explain something using words, then you do not understand it. Equations in physics are a short-hand notation for ideas that can be expressed in words. The surest way to test that you understand something is to try to explain it to another human being. The act of communicating with others is an essential part of learning, and that is why we emphasis that you discuss physics with your peers in this course.

Just enough about derivatives

You don't have to know a lot about derivatives in this course. But what you do have to know, you have to know extremely well. Here is a review of what you will need to know about calculus for this course.

The derivative of a function f(x) is another function f '(x) = df / dx, defined by

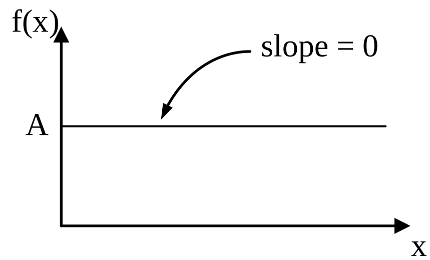
$\frac{df}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$. The derivative of f(x) is the slope of the tangent line to the curve f(x) vs. x.

In chapter 2, we will consider functions of time: x = x(t) and v = v(t). Position x and velocity v are related by $v = \frac{dx}{dt}$. Velocity v and acceleration *a* are related by $a = \frac{dv}{dt}$.

There are 4 important theorems about derivatives that we will need again and again in this course.

Theorem 1. The derivative of a constant is zero.

Proof 1: Function f(x) = A , where A is a constant. The plot f(x) vs. x is a straight line with zero slope. Remember that the derivative is the slope of the tangent line to the curve. The slope is zero, so the derivative is zero.



Proof 2: Start with the definition of derivative:

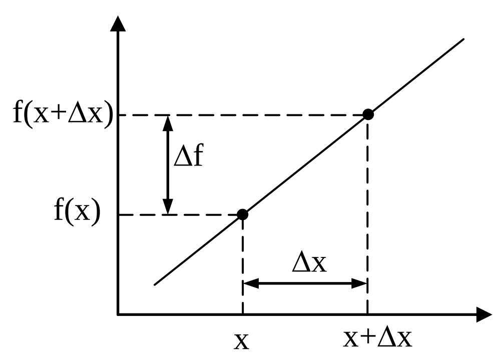
$$\frac{df}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{A - A}{\Delta x} = \lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0 .$$

Theorem 2: If $f(x) = A\,x^n$, where A and n are constants, then $\frac{df}{dx} = A\,n\,x^{n-1}$.

Let's prove this for the special case n = 2: $f(x) = A\,x^2 \Rightarrow \frac{df}{dx} = 2\,A\,x$

Again, we start with the definition of derivative. (In any proof, you have to start with something you know to be true. Definitions are always true, by definition.)

$$\frac{df}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} ,$$



$$f(x) = A x^2, \quad f(x+\Delta x) = A(x+\Delta x)^2 ,$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{A(x+\Delta x)^2 - A x^2}{\Delta x}$$

$$= \frac{A(x^2 + 2x\Delta x + \Delta x^2) - A x^2}{\Delta x} = 2Ax + A\Delta x$$

Taking the limit $\Delta x \to 0$, we have $\dfrac{df}{dx} = 2Ax$. Done.

> Theorem 3: The derivative of a sum is the sum of the derivatives:
>
> $$f(x) = g(x) + h(x) \quad \Rightarrow \quad \frac{df(x)}{dx} = \frac{dg(x)}{dx} + \frac{dh(x)}{dx}$$

Proof:

$$\frac{df(x)}{dx} = \frac{d[g(x)+h(x)]}{dx} \equiv \lim \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim \frac{\left[g(x+\Delta x) + h(x+\Delta x)\right] - \left[g(x) + h(x)\right]}{\Delta x}$$

$$= \lim \left\{ \frac{\left[g(x+\Delta x) - g(x)\right]}{\Delta x} + \frac{\left[h(x+\Delta x) - h(x)\right]}{\Delta x} \right\} = \frac{dg}{dx} + \frac{dh}{dx}$$

> Theorem 4: The derivative of a constant times a function is the constant times the derivative of the function $g(x) = Af(x)$, where A = constant $\Rightarrow \dfrac{dg}{dx} = \dfrac{d\left(A f(x)\right)}{dx} = A\dfrac{df(x)}{dx}$

Proof (next page):

$$\frac{d(Af)}{dx} = \lim_{\Delta x \to 0} \frac{Af(x+\Delta x) - Af(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{A\left[f(x+\Delta x) - f(x)\right]}{\Delta x}$$

$$= A \cdot \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = A \frac{df}{dx}$$

Exercise. Starting with the constant acceleration formula $x(t) = x_o + v_o t + (1/2)at^2$, use your knowledge of calculus to prove that $v = v_o + at$. (Hint: take the derivative dx / dt and use the theorems above.)

Example: A rocket in space has position as a function of time given by $x = x_0 + At^3$, where A is a constant. What is the velocity and acceleration of the rocket?

Hey! You should try to work this out yourself, before looking at the solution below.

Solution: $x(t) = x_0 + At^3$

Take the first derivative to get velocity

$$v = \frac{dx}{dt} = \frac{d\left(x_0 + At^3\right)}{dt} = \frac{dx_0}{dt} + \frac{d\left(At^3\right)}{dt} = 0 + 3At^2$$

(Notice how we needed the theorems .)

Now take another derivative (the second derivative) to get the acceleration.

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d\left(3At^2\right)}{dt} = 3 \cdot 2At = 6At.$$

Notice that this is a case of non-constant acceleration, so none of our constant acceleration formulas applies here.

Comment about notation: The acceleration is the second derivative of position w.r.t time:

$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$. We usually write the 2nd derivative like this: $a = \frac{d^2x}{dt^2}$.

The third derivative $\frac{d}{dt}\left(\frac{d}{dt}\left(\frac{dx}{dt}\right)\right)$ is written $\frac{d^3x}{dt^3}$.

