

Wave Motion

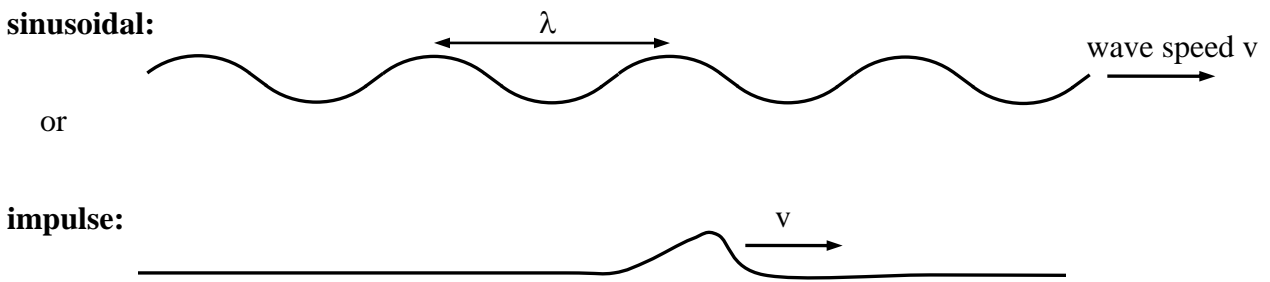
A wave is a self-propagating disturbance in a medium. Waves carry energy, momentum, information, but not matter.

Examples:

- Sound waves (pressure waves) in air (or in any gas or solid or liquid)
- Waves on a stretched string
- Waves on the surface of water
- "The Wave" at the ballpark stadium. The medium is the people.
- Electromagnetic waves (light) – this is the only kind of wave which does not require a medium. EM waves can travel in vacuum. This was quite a surprise to 19th century physicists who had trouble imagining a wave without a medium. The thing that is "waving" in an EM wave is an electromagnetic field which generates itself as it propagates. In a sense, an EM wave "rolls out its own carpet", creating its own medium as it moves forward. More on EM waves next semester.

We can use a wave to send a signal (information) without sending any matter. Imagine a long line of people holding hands. We can send a signal down the line by hand-squeezing (a disturbance in the people medium), and yet no one moves.

Traveling Waves can be categorized as:

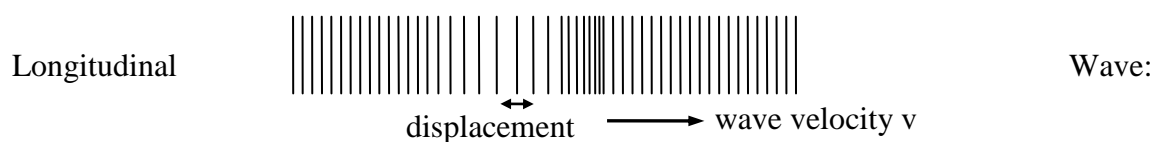


Traveling waves can also be categorized as:

transverse: displacement of the medium is perpendicular (transverse) to the direction of the wave velocity (like a wave on water or a string or drumhead).

or

longitudinal: displacement of the medium is parallel to the direction of the wave velocity (like sound wave or a slinky that has been push-pulled).



Wave Speed

Wave speed is $v = \frac{\text{distance}}{\text{time}} = \frac{1 \text{ wavelength}}{\text{time for } 1 \lambda \text{ to go by}} = \frac{\lambda}{T}$, $T = \text{period}$

$$\boxed{v = \frac{\lambda}{T} = \lambda f} \quad (\text{since frequency } f = 1/T)$$

Almost always, the wave speed v is a constant, *independent* of λ and T .

The wave speed v depends on the properties of the medium, *not* on the properties of the wave.

Examples:

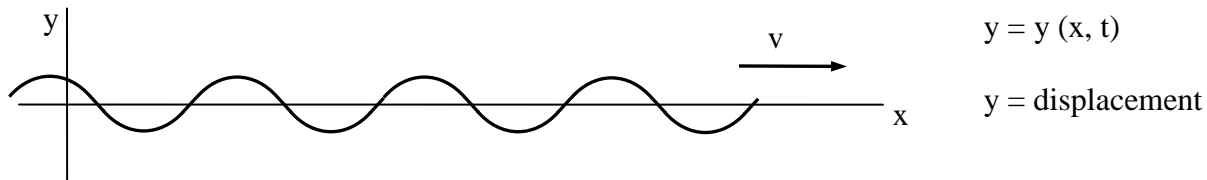
medium = string, properties = tension, mass per length

medium = air, properties = temperature, mass per molecule, etc

$v = \lambda f = \text{constant} \Rightarrow \lambda$ increases as f decreases, λ decreases as f increases

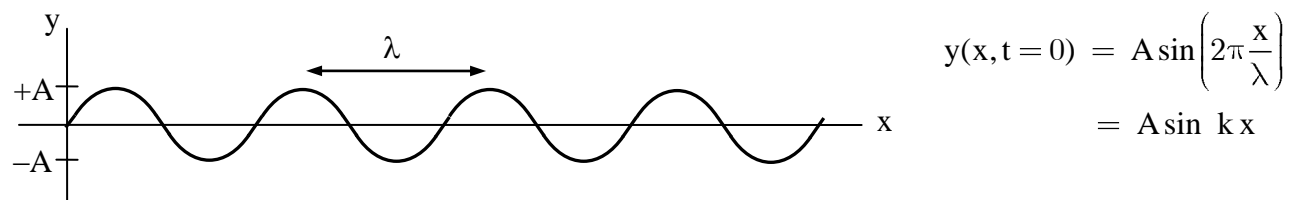
Mathematical description of traveling sinusoidal waves

Sinusoidal waves have a wavelength λ and a frequency $f = 1/T$. (Impulse waves have neither.)



$y = \text{displacement from the equilibrium (no wave) position}$

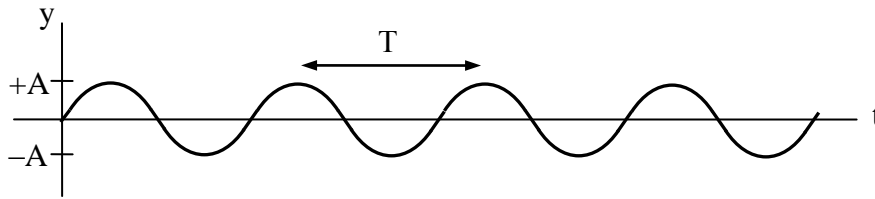
Snapshot in time: freeze time at, say, $t = 0$



A is the *amplitude* of the wave. The displacement oscillates between $+A$ and $-A$.

$$k = \frac{2\pi}{\lambda} = \text{wave number}$$

Now freeze position, watch wave go by at position $x = 0$:

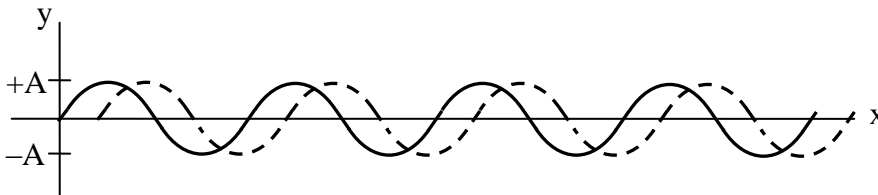


$$y(x=0, t) = A \sin\left(2\pi \frac{t}{T}\right) = A \sin \omega t$$

$$\omega = \frac{2\pi}{T} = \text{angular frequency}$$

Wave traveling to the right: $y(x, t) = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] = A \sin kx - \omega t$

Notice: When position x changes by distance λ or time t changes by period T , the sine function goes through one complete cycle. When x increases by one λ AND t increases by one T , then the sine function stays at the same *phase*; we are then riding along with the wave.



The *argument* of the sin function $kx - \omega t$ is called the *phase*. A point on the traveling wave (traveling along with the wave) corresponds to a particular value of the phase $kx - \omega t$. As t increases, x must increase in order to keep $kx - \omega t$ a constant value, hence a point of constant phase corresponds to a point moving to the right (increasing x).

Could also have a wave traveling to the left: $y(x, t) = A \sin\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right]$

We could have used cosine instead of sine for the form of the wave. The only difference between sin and cos here is where we put the zero of time.

Wave speed is $v = \frac{\text{distance}}{\text{time}} = \frac{1 \text{ wavelength}}{\text{time for } 1 \lambda \text{ to go by}} = \frac{\lambda}{T}$

$$v = \frac{\lambda}{T} = \lambda f \quad (\text{since frequency } f = 1/T)$$

Another way to see that our formula for the wave $y = y(x, t)$ corresponds to a wave moving right with speed $v = \lambda/T$ is to rewrite the formula like so:

$$y(x,t) = A \sin \left[2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \right] = A \sin \left[\frac{2\pi}{\lambda} \left(x - \frac{\lambda}{T} t \right) \right] = A \sin \left[\frac{2\pi}{\lambda} x - vt \right]$$

A point traveling with the wave is a point with $(x - vt) = \text{constant}$ or $x = vt + \text{const}$. This is the equation for a point moving right with speed v (graph of x vs t has slope $\Delta x/\Delta t = v$).

Claim: Any traveling wave $y = y(x,t)$, traveling with a rigid shape ("dispersionless") has the form

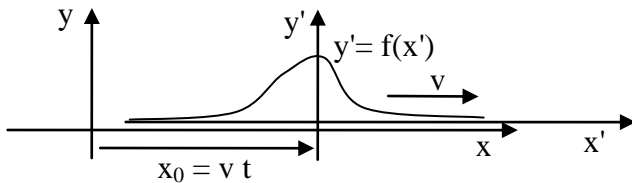
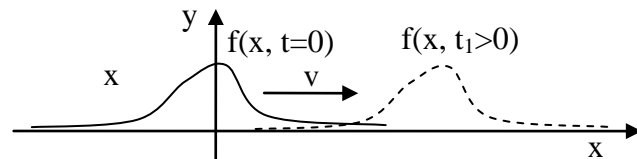
$$y(x,t) = f(x - vt) \quad (\text{wave traveling to right with speed } v) \text{ or}$$

$$y(x,t) = f(x + vt) \quad (\text{wave traveling to left with speed } v)$$

Proof: Consider a wave initially with some shape $y = f(x)$ (at $t = 0$)

that is moving to the right with speed v .

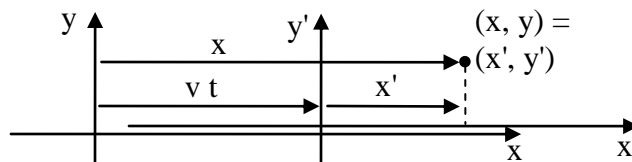
And consider a *moving* coordinate system (x', y') , that is moving along with the wave



Coordinate transform:

$$x = x' + vt, \quad x' = x - vt$$

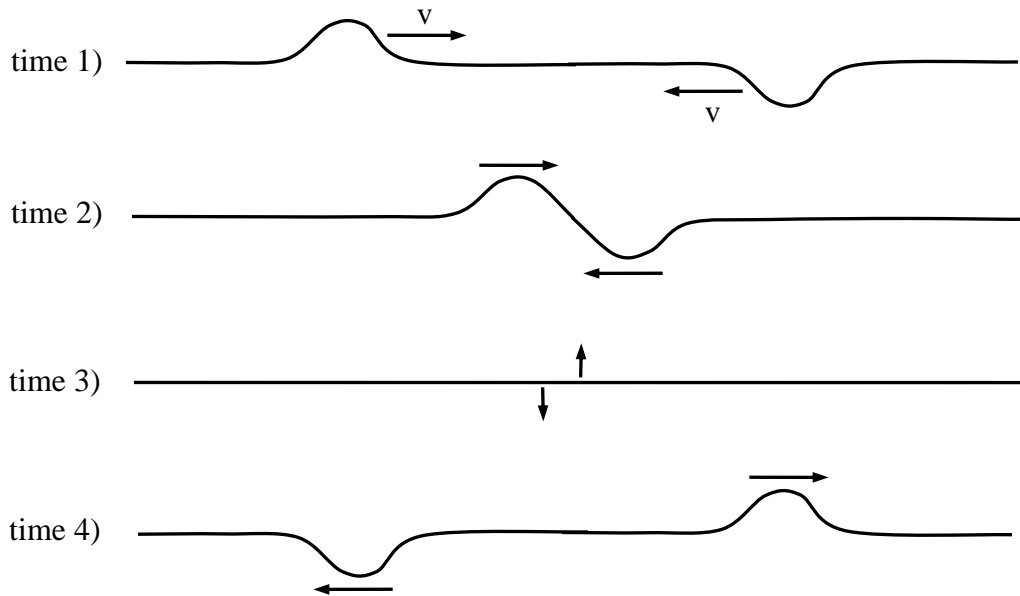
$$y = y'$$



In the moving $x'y'$ coordinate system, the moving wave is stationary: $y' = f(x')$. Transforming to the xy coordinate system, the wave $y' = f(x')$ becomes $y = f(x - vt)$. Done.

Interference of waves

Superposition Principle: If two or more waves are present in the same place, at the same time, the total wave is the *sum* of the individual waves: $y_{\text{tot}}(x,t) = y_1(x,t) + y_2(x,t)$. You get constructive or destructive interference depending on whether y_1 and y_2 add (both have same sign) or cancel (opposite signs).

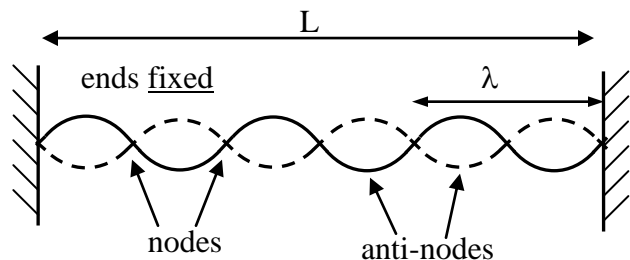


Waves carry energy. For a string wave, the energy is both KE (string moves as wave passes) and PE (originally straight string must stretch a little as wave passes – elastic PE similar to $(1/2)kx^2$).

At time 3 above, where is the energy? Answer: in the KE. At time 3, the string is moving, while instantaneously flat.

Standing Waves

Two sinusoidal traveling waves of the same λ (and therefore the same $f = v/\lambda$) and the same amplitude, traveling in opposite directions, overlapping in the same region of space, make a *standing wave*.



If the ends of a string of length L are fixed, as in a stringed musical instrument, then standing waves are only possible at certain *resonant* frequencies given by the condition:

$$L = n \cdot \frac{\lambda}{2}, \quad n = \text{integer}, \quad L = \text{fixed total length}$$

Sound

Sound is a pressure wave in air. It is a longitudinal wave.

$$\text{pressure} = \frac{\text{force}}{\text{area}}, \quad p = \frac{F}{A}$$

Air consists mostly of oxygen and nitrogen molecules. At room temperature, the molecules have thermal energy and are moving around rapidly (speed ≈ 400 m/s), colliding with each other, and

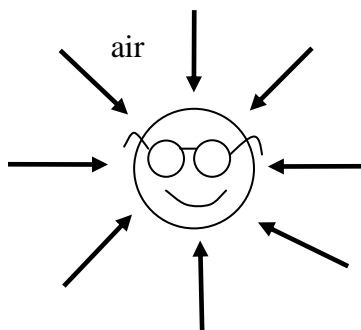
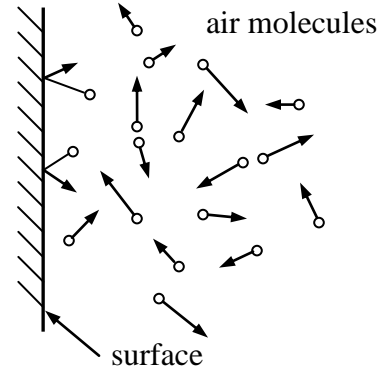
with every exposed surface. The pounding of the air molecules on a surface, like the pitter-pat of rain on the roof, add up to a large force per area.

At sea level, atmospheric pressure is

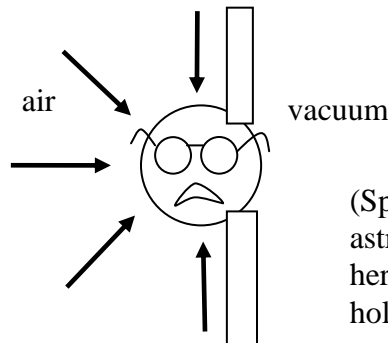
$$1 \text{ atm} = 14.7 \text{ psi} = 1.01 \times 10^5 \text{ N / m}^2$$

(psi = pound per square inch)

This is a big pressure! We are not ordinarily aware of this big pressure because the air pushes on us equally from all sides (even from our *insides* due to the air in our lungs). The big forces on us from all sides cancel out and there is not *net* force on us.



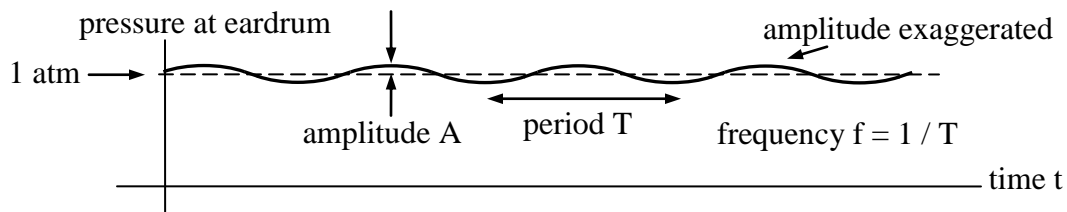
forces balanced



forces unbalanced

(Space Shuttle astronaut heroically plugs hole with head,)

Ordinary sounds are caused by relatively small changes in the air pressure at our ear drum. A pressure change of a few parts per million is enough to wiggle the ear drum and make the listener perceive a loud sound (the ear is sensitive!). When we here a tone of constant frequency $f = 1 / T$, the pressure vs. time at the eardrum looks like this:



The amplitude A of the pressure wave controls the loudness of the sound. Big $A \Rightarrow$ big pressure oscillation \Rightarrow big wag of eardrum \Rightarrow loud sound.

The frequency is the "pitch" of the tone. Hi $f \Rightarrow$ hi notes (soprano) Lo $f \Rightarrow$ lo notes (basso). More on frequency and music below.

Frequency range of human hearing: $f = 20 \text{ Hz} \rightarrow 20 \text{ kHz}$. [frequency f is usually measured in hertz (Hz), cycles per second] The upper end degrades as we age. At age 65, most humans cannot hear above 10 kHz. Dogs can hear up to 35 kHz (ultrasonic).

As with most waves, the speed of sound in air depends on the properties of the medium = air (temperature, mass of molecules, etc) NOT on the properties of the sound wave.

$v_{\text{sound}} = 343 \text{ m/s}$ (this varies by about $\pm 10 \text{ m/s}$, depending on the temperature of the air)

Sounds travels about 1 mile in 5 seconds.. If you see a flash of lighting and then start counting seconds (1-1 thousand, 2-1 thousand, 3-1 thousand...) until you hear the thunder, you can estimate how many miles away the lighting is. The speed of light is very, very fast (light speed $c = 3 \times 10^8 \text{ m/s}$) so the light arrives almost instantly, while the sound wave takes a while to catch up.

Example: What is the wavelength range of human hearing?

$$v = \lambda f \Rightarrow \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{20 \text{ s}^{-1}} \rightarrow \frac{343 \text{ m/s}}{20000 \text{ s}^{-1}} = 17 \text{ m} \rightarrow 0.017 \text{ m (17 cm)}$$

Music

The frequency is the "pitch" of the tone.

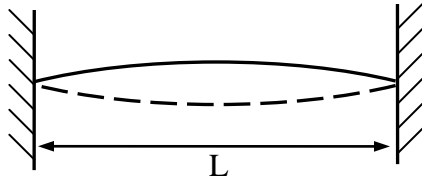
"middle C" = 262 Hz, D = 294 Hz, E = 330 Hz, etc. Each "octave" on the musical scale is a factor of 2 change in frequency, for instance, "C above middle C" = $2 \times 262 = 524 \text{ Hz}$. (It's called an "octave", only because, in the Western musical scale, there are 8 notes in an octave.)

A single frequency sound wave (a pure tone) is an irritating sound to most people. A pleasant musical note is usually a *mixture* of frequencies = lowest or *fundamental* frequency + integer multiples of the fundamental or *overtones*

$$\begin{aligned} f_1 &= \text{fundamental} = 1^{\text{st}} \text{ harmonic} \\ f_2 &= 2 \cdot f_1 = 1^{\text{st}} \text{ overtone} = 2^{\text{nd}} \text{ harmonic} \\ f_3 &= 3 \cdot f_1 = 2^{\text{nd}} \text{ overtone} = 3^{\text{rd}} \text{ harmonic} \\ f_n &= n \cdot f_1 = (n-1) \text{ overtone} = n^{\text{th}} \text{ harmonic} \end{aligned}$$

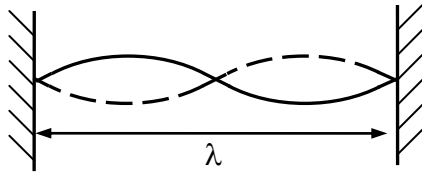
How do musical instruments manage to produce this nice mixture: $f_1, 2f_1, 3f_1$, etc ??

If a string of length L has its ends fixed, as in a stringed instrument, then only standing waves at certain *resonant* frequencies are possible:



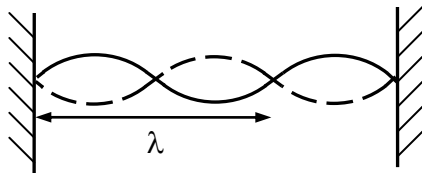
$$L = \frac{\lambda}{2}, \quad \lambda = 2L$$

$$f = \frac{v}{\lambda} = \frac{v}{2L} = f_1 = \text{fundamental or 1st harmonic}$$



$$\lambda = L$$

$$f_2 = \frac{v}{\lambda} = \frac{v}{L} = 2f_1 = 2^{\text{nd}} \text{ harmonic}$$



$$L = 3\left(\frac{\lambda}{2}\right), \quad \lambda = \frac{2}{3}L$$

$$f = \frac{v}{\lambda} = 3 \cdot \frac{v}{2L} = 3f_1 = 3^{\text{rd}} \text{ harmonic}$$

For the n^{th} harmonic ($n = 1, 2, 3, \dots$), $L = n \cdot \frac{\lambda}{2}$, $\lambda = \frac{2L}{n}$, $f_n = \frac{v}{\lambda} = n \frac{v}{2L} = n f_1$.

The frequency of the n^{th} harmonic is $n \times$ the frequency of the fundamental. (In all this, the wave speed v is fixed by the tension and mass of the string. It turns out that the speed of the wave is given by $v = \sqrt{\frac{F_T}{m/L}}$, where F_T is the string tension and m/L is the mass per length of the string.)

When a guitar string is plucked, *all* of these resonant frequencies, or *normal modes*, are excited all at once. The listener hears all frequencies f_1 , $f_2 = 2f_1$, $f_3 = 3f_1$, etc. It is this mixture, which gives the instrument its pleasing quality. The fundamental, or lowest frequency, established the "note". The fundamental $f_1 = v/(2L)$ can be adjusted (tuned) either by changing the tension in the string (changing v) or by holding string at a fret (changes L).