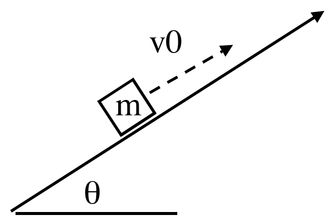
Newton's Laws: Problems and Examples

There is nothing fundamentally new in the first part of this chapter. But, Newton's laws (especially F=ma) are much richer than you may imagine. Almost all of classical physics is explained and understandable from that equation! It's worth doing *lots* of examples. Work through those in the text. Here are some more:

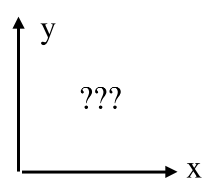
Example: An object is sent up a frictionless air track (angle theta) with initial velocity **v**0. How does the velocity change with time?

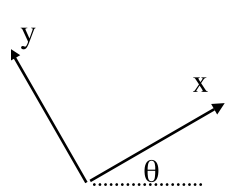


Sol'n: We need to find the acceleration. Then (assuming it's constant, which it will be here), Ch. 2 kinematics tells us **v=v0+a**t, and we'll be done!

(We'll find **a** with **a** = **F**_net/m, of course.)

• Pick a sensible coordinate system. You might think the usual Cartesian coord system is always best, but here it is definitely not. Since the object slides up and down along the track, its motion (and acceleration) will be tilted at angle theta.

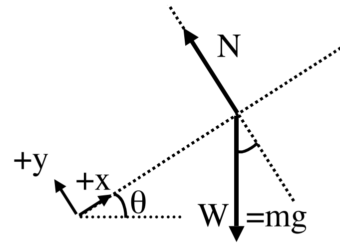




This tilted coordinate system will be much better. (**a**, **v**, and **r** will all be purely in the x direction, i.e. this problem will really become 1-D motion, always easier.)

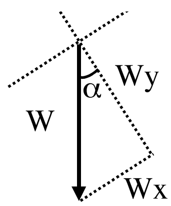
The next step will be to draw a FD (force diagram). You must consider ONLY the physical pushes and pulls on the cart, *after* it's been launched up the track.

I can only think of two forces: gravity and normal (contact) force of the track, which (if its a *frictionless* track) must be perpendicular to the surface.



For N-II, we'll want the components of the forces. Look at the force diagram.

The Normal force **N** is easy, by inspection: N_x=0, and N_y=+N.



The weight **W** is trickier. Here I redraw the weight vector. The important (critical!) thing is to convince yourself that the angle α in this picture is the *same* as the angle θ of the track. (Spend whatever time and sketches you need to convince yourself of this. There will be many problems with similar geometry.) W's components now come from trig:

W_x = -W sin(θ) = $-mg\sin\theta$. (Do you see why W_x is negative, with my axes?)

W_y = -W cos(θ) = $-mg\cos\theta$ (Do you see why W_y is negative, with my axes?)

We're all set up. Write out N-II in the x- and y- coordinates:

F_net, x = m*(a_x) I observed that **a** is totally along the x-axis, so call a_x = a

thus $0-mg\sin\theta$ = m*a Solving this: the m's cancel, giving $a=-g\sin\theta$.

F_net,y = m*a_y. But, **a** has no y-component, so the right side is zero.

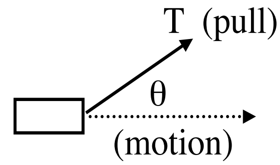
or $N-mg\cos\theta = 0$. This tells us N, if we're interested: $N = mg\cos\theta$.

<u>Comments:</u> The sign of $a=-g\sin\theta$ makes sense, accel is DOWN the track! Notice that N is NOT mg, here. (It doesn't have to be.)

<u>*Checks*:</u> if θ=0 (flat track), we get a=0, which makes physical sense. (And, N=mg, which also makes sense for a flat air track, we saw that in the previous chapter.)

If θ = 90, we get a=-g. Also makes sense - we'd be in freefall.

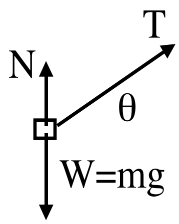
Example: Pull a cart, mass m, sideways along a (frictionless) horizontal track, with a “tilted rope”, pulling with tension T, as shown.



• What is **a**?

• What is the normal force of the track on the cart?

Answer: (You might think that normal force on *horizontal* surfaces is always mg: it has been in all problems so far. But it’s not a law of physics. Let’s see!)



Draw a force diagram. There are three forces on the cart, as shown.

(N must be perpendicular to the surface, if it’s frictionless).

N-II in the x-direction says F_net,x = m*a_x

Since we’re told the object moves to the right, let’s call a=a_x.

T cos(theta) +0 +0 = m*a. We’re given all quantities except for “a”, solve for it:

$a = T\cos\theta\ / m$.

Check: to get the maximum acceleration, you want θ=0, which makes sense. Pulling up at any other angle seems somehow wasteful....

To find the normal force, we must write out N-II in the y-direction, F_net,y = 0.

(Acceleration is zero in the y direction, since we said the cart is moving sideways)

$+T\sin\theta + N - m g = 0$. (Verify all three signs simply by looking at the FD)

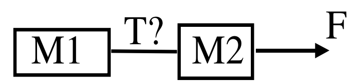
The only unknown is N, so solve for it:

$N = mg - T\sin\theta$**.**

It is less than mg. (Some of the rope tension is pulling *up* on the cart, reducing the normal force the surface needs to apply.)

Example: You pull two objects attached by ropes, with an external force F (given)

The objects have masses M1 and M2 respectively.

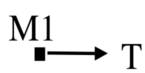


• What's **a** of the system?

• What's the tension T in the rope between them?

Comment: The rope between them is NOT the same rope as the one pulling the system off to the right, so T certainly doesn't have to equal F. (Tension is the same anywhere along a SINGLE rope, but not in different ropes in the same problem.)

But **a** is going to be the same for M1 and M2, because they're hooked together.



This is the FD for object 1. That's *it*. Force F does *not* pull directly on M1. It doesn't *touch* M1. Only the rope with tension T directly pulls on M1! So, for object A, F_net,x = M1*a_x becomes T = M1*a.

(One equation in the two unknowns we're after, "a" and "T")

This is the FD for object B. Notice that object B is touched by *two* ropes, each of which pulls. The rear rope is pulling *backwards*. (Remember, ropes can only pull. They never push!)

The tension "T" pulling backwards has the same magnitude (and symbol) as the tension "T" in the previous diagram, because it's the two ends of the same rope.

N-II for object 2: F_net,x = M2 * a. Looking at the diagram to get F_net, we have

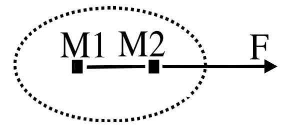
+F - T = M2*a.

Adding our two underlined equations (to get rid of T) gives

F = (M1+M2)*a, or a= F/(M1+M2).

This last answer makes sense if you think about it.

I could have gotten it directly by considering a "composite object", the combined system of 1 and 2 together, like this:



You can apply N-II to *any* object you want. That "object" can be a combination of any number of smaller objects.

Think of the dashed line there as a "paper bag" that surrounds M1 and M2 - how could I know (or care) whether there are two masses attached together, or one mass of M1+M2? Using this force diagram, T is now an "internal force" which doesn't belong in N-II. Only *external* forces contribute to F_net!

So F = (M1+M2)*a is N-II for this system. Just as we had.

Going back, now we know a, we can substitute it in either equation to get T.

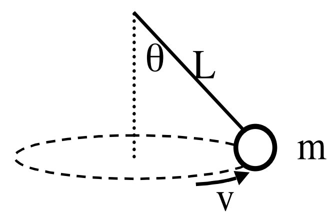
Using T = M1*a, I get

T= M1*F/(M1+M2).

Notice that no matter what M1 and M2 are, T < F. (It's strange, but true.)

F has to pull BOTH masses, accelerating the combined system with "a".

But T really only has to pull the one mass M1, accelerating it with the same "a".

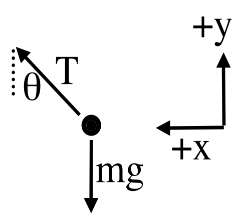


Example: A tetherball hangs on a rope (length L).

It swings in a circular path, in a flat plane, with a constant speed v as shown.

• What's the tension T in the cord?

• Given the angle of the cord, θ, what is the speed v?



Soln: Draw the FD when the ball is in the position shown above. (I chose a coordinate system, too. Can you see why I chose x pointing to the left like that?)

There are two and *only* two physical forces pulling on the ball, shown.

(Some people are tempted to add a left pointing arrow, because they're thinking ahead and know the acceleration is in that direction. But, a FD should only contain real, physical pulls on the object! Such a force is not there, and does not belong.)

• Do NOT label "L" in the force diagram. It's a length, *not* a force. Although the "L" line and the "T" line are the same orientation, don't confuse them. L is a length, and should *not* appear in a force diagram!

N-II in the y direction: First of all, "swinging in a flat plane" means that the ball is not rising or falling - it has NO motion in the y direction. So, a_y = 0.

From the FD, F_net, y = m*a_y = 0 tells us

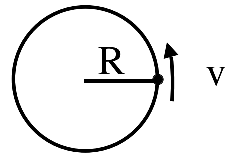
+T $\cos\theta$ - mg = 0 (Look carefully. Convince yourself of the signs, and also the cos)

So T = mg/$\cos\theta$. Part of the problem is solved already.

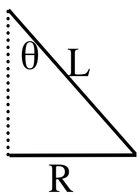
Now look at N-II in the x-direction. F_net, x = m*a_x. Do we know a_x?

Yes! Look *down* on this motion.

We have uniform circular motion! So we know a_x = v^2/R, pointing towards the center of the circle of motion.



Here R is the radius of the circle the ball is rotating around. That's not L, it's the radius of the "flat plane" circular motion.



To find L, go back to the original geometry, and you can see

$R = L \sin\theta$.

N-II in the x-direction gives $+T \sin\theta = +m\, v^2/R = +m\, v^2/(L \sin\theta)$

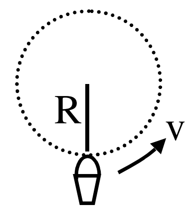
Signs are important. I defined +x to be leftwards, both **T**_x and **a**_x are leftwards.

This equation gives $v = \sqrt{\dfrac{TL\sin^2\theta}{m}} = \sqrt{\dfrac{gL\sin^2\theta}{\cos\theta}}$.

(I plugged in $T = mg/\cos\theta$ in the last step)

<u>Check:</u> When $\theta=0$, the formulas give T=mg, and v=0. This makes sense. The ball is just hanging there. If you want it tilted at a higher angle, you have to spin it faster, and the tension in the rope increases.

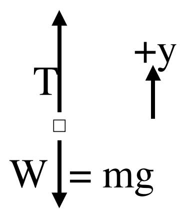
The case $\theta=90$ is kind of weird. It says T and v both go to infinity. But this is formally correct. You *cannot* spin the tetherball at 90 degrees! The rope would be horizontal, and there's no possible way to compensate for the downward force of gravity. It's impossible! What physical force would prevent the ball from falling down, at least a little? You need SOME upward component of tension to compensate gravity.



Example: A bucket swings in a vertical loop around your head, *at constant speed, v*. (In practice it's difficult to do this at constant speed. But it's possible, think of a ferris wheel - and makes the analysis simpler.)

Question: What's the tension T in the rope at the bottom (and top) of the swing?

Incorrect analysis: We know $a = v^2/R$ for uniform circular motion, so since $F=ma$, shouldn't the tension in the rope just be mv^2/R? NO, tension is not the only force in the problem, you need F_{net} for N-II. As always, we *need* a force diagram.



This is the FD when the bucket is at the bottom.

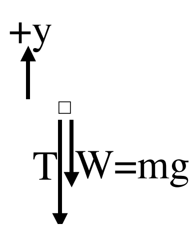
I picked a coordinate system (shown) with up = "+".

The rope can only *pull*. Weight is always down.

N-II (y-direction): $F_{net,y} = ma_y$. Now it's correct to argue $a_y = v^2/R$, up.

So $+T - mg = m v^2/R$, which means $T = m(g + v^2/R)$.

Tension is larger than mg. You pull hard when you swing a bucket and it's at the bottom - the faster you swing it, the harder you have to pull.



This is the FD when the bucket is at the *top*.

Same coordinate system, up = "+". Important: it's still true that ropes can only *pull*. That's why "T" points down now!

N-II (y-direction): $F_{net,y} = m*a_y$....

Now $a_y = -v^2/R$ (centripetal acceleration is down, towards the center of the circle, when you're at the top.)

So $-T - mg = -m v^2/R$. Solving for T give $T = m(v^2/R - g)$

That minus sign in front of g makes the story quite different at the top.

If v is large, you still have to apply lots of tension, which makes physical sense.

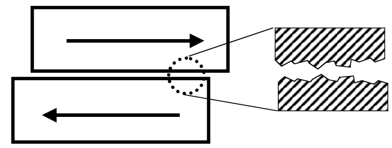
As v gets smaller, there comes a point when T=0. (That's the *critical* speed, when v_crit^2/R = g, or v_crit = Sqrt[R*g].)

At that slow speed, the rope goes slack at the very top, but the bucket will (barely) continue to move around in a circle. Gravity supplies the required centripetal force.

What happens if you swing the bucket with a speed v<v_crit? The equation says you need a negative T to keep the bucket moving in a circle. But ropes cannot provide negative T, this is nonsense! Something is wrong with our equations. There's a physical explanation - at such a slow speed, the bucket won't travel in a circle any more. Think about super slow v's - the bucket will simply FALL (in some parabolic projectile trajectory) instead of continuing around in a circle. (If v=0, it would fall straight down, certainly *not* uniform circular motion, which we implicitly assumed in our analysis.)

If this was a roller coaster instead of a bucket, you could rig it up so that the roller coaster was "attached" to the track, which means some mechanical connection that would *hold the coaster car up*. That would correspond to our required "negative" |T|. But in the problem as stated, the mathematically nonsensical negative T solution means that our starting assumption (uniform circular motion) has broken down.

FRICTION: Friction is obviously an important force in the real world. I'm afraid we don't have a complete fundamental understanding of it, even today (compared to, say, gravity or electricity.) The microscopic understanding of friction involves complex chemical bond formation, tearing, and breaking.



Different materials exhibit very different frictional forces.

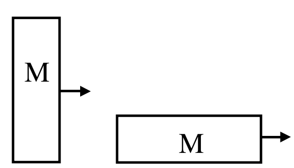
There's no simple, precise formula like "F=mg" for friction. Still, many materials do exhibit approximately simple frictional behavior -we'll discuss some crude empirical rules for friction to at least get a preliminary quantitative feel for it. (If you work as a mechanical engineer, you'll need to learn more about real complexities of friction.)

Kinetic friction (also called sliding friction) is denoted F_fr,k or just $\mathbf{F}_k$.

Kinetic friction *always opposes motion*.

This requires care in force diagrams, because you can't draw the arrow for **F**_fr,k until you first know which way the object is moving.

F_fr,k has some surprising properties for many materials. It does *not* generally depend on the speed of sliding. It also does *not* generally depend on the surface areas in contact. That one is quite a surprise. It says a block on a table feels the same sliding friction force, whether it's laying flat, or up on its small end.



You might have thought that when it's flat, more area => more friction, but there's a compensating factor - the same weight is spread out over more area, so microscopically the surfaces aren't bonding as tightly.

The (approximate) formula we'll use for kinetic friction comes from experiment.

$\boxed{F_{fr,\,k} = \mu_k N}$,

where N is the normal force between the object and the surface its sliding over.

μ_k (or "mu_k") is the **coefficient of kinetic friction**. It's a unitless, positive number, generally less than 1. (You must measure it, it depends on materials.)

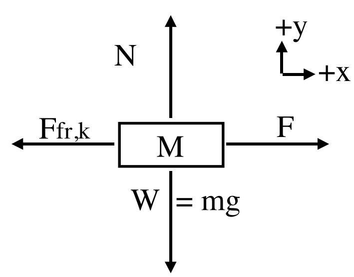
E.g. rubber on dry concrete has mu_k = .8 or so.

That equation is *not* a vector equation! It tells you only about the magnitudes. The direction of **F**_fr will be perpendicular to **N** (i.e. parallel to the sliding surface, opposite the motion)

<u>Example:</u> A block slides to the right along a table, with constant speed v.

You apply an external force F to make this happen.

What is the coefficient of kinetic friction?



(Notice in my FD I do not draw **v**, but I needed to know it was to the right, in order that I can correctly draw **F**_fr to the left.)

N-II (y-direction): +N - mg = 0 (there's no acceleration in the vertical direction)

N-II (x-direction): +F - F_fr,k = M*a_x = 0 (constant speed means no acceleration!)

The 1st Eq tells me N=mg. Plugging that into the next eqn (using $F_{fr,\,k} = \mu_k N$) gives

$\mu_k = F/(mg)$ (Do you see why?) This is a simple exp'tal way to *measure* mu_k.

<u>Comment:</u> If you apply MORE force, i.e. if $F > \mu_k mg$, then the object will accelerate, and the acceleration is given by N-II: $F - \mu_k mg = ma_x$

STATIC FRICTION: When an object is just sitting (not sliding), there can still be friction. Think of a box of books on the floor. You can push the box, and it won't budge. There's a force (by you), but **a**=0, so N-II says there MUST be an opposing force! It's the **static friction**. There is no formula for static friction, because (like Normal forces) it *adjusts* to the situation! If you push that box harder, static friction increases, to keep it sitting there!! This is true up to a maximum, at which point you finally overcome static friction. There is an approximate formula for this *maximum*:

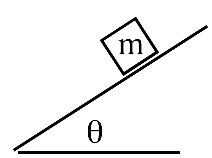
$\boxed{F_{fr,\,s}(\max) = \mu_s N}$.

Some people will then write $\boxed{F_{fr,\,s} \le \mu_s N}$. (But be careful! Do NOT assume an equal sign here. In general, you will be wrong if you do.)

μ_s (or "mu_s") is the **coefficient of static friction**. It is always *larger* than μ_k .

(It's harder to get that box of books *started* sliding than it is to keep it going)

Example: A block sits at rest on an incline, sticking because of static friction. If you tilt the angle of the incline higher, there is a critical θ where the object suddenly starts to slide.

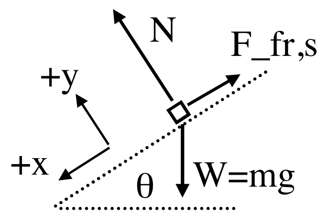


a) What is this critical angle?

b) After it just starts to slide, what will the acceleration be?

(This is a somewhat nasty little problem!)

As always, we must begin with a careful FD. Like our tilted hill problem earlier, I will pick a tilted coord. system. The trickiest part is to think physically about which *direction* the static friction must point. (It holds the object UP!)



Sticking means that a_x = a_y = 0. (It's not moving.)

We'll need the x and y components of "W=mg": go back to our last tilted ramp example (p. 6-2), and remind yourself how to get them. (Note: I picked +x *down* the ramp, this time.)

N-II in the x direction: $-F_{fr,\,s} + mg\sin\theta \;=\; ma_x = 0$

N-II in the y direction: $N - mg\cos\theta \;=\; ma_y = 0$

Look at the picture and convince yourself all these signs are correct.

Solving, I get two results: <u>N = mg cos</u>θ , and <u>F_fr,s = m g sin</u>θ

Please note that F_fr, s is NOT $\mu_s N$, I haven't used this. F_fr, s adjusts itself!

The steeper the hill, the bigger F_fr,s gets to hold it up. However, the bigger θ, the 2nd formula says the bigger F_fr,s will have to be.

This means there will be a MAX angle, where F_fr,s (max) = $\mu_s N$.

$\mu_s N = F_{fr,s\,max} \;=\; +mg\sin\theta_{max}$. Now plug in our result above: $N = mg\cos\theta$

$\mu_s\, mg\cos\theta_{max} = +mg\sin\theta_{max}$. Cancel the common factor mg:

$\mu_s = \tan\theta_{max}$, or

$\theta_{max} = \tan^{-1}\mu_s$

A small amount of static friction (small mu_s) means a small θ_{max}, which makes sense... (A pretty typical mu_s might be around 1 for many materials, which gives a theta around 45 degrees...)

b) What if $\theta = \theta_{max}$, so the block JUST barely starts to slide?

(Maybe you give it the slightest, infinitesimal little "tick" to get it going)

What happens next? What's **a**?

You might think **a=**0, and it slides with constant speed, if we're right at $\theta = \theta_{max}$.

But no - as soon as you start to slide, F_fr changes from static to kinetic. Remember, kinetic friction is always smaller, so there's be suddenly a jump in force - suddenly less force "holding it up", and it *will* accelerate.

The force diagram is the exact same (only now it's F_fr,k uphill)

The y-equations are the same, so N=mg cos(theta) still.

The x-equation changes slightly, because you *can* use F_fr,k = μ_k N, and gives

$$
\begin{aligned}
-F_{fr,\,k} + mg\sin\theta &= ma_x, \\
-\mu_k N + mg\sin\theta &= ma_x, \\
-\mu_k(mg\cos\theta) + mg\sin\theta &= ma_x
\end{aligned}
$$

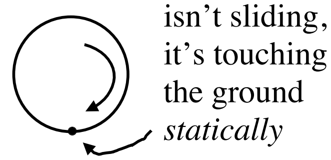
Finally, solving for a_x, which is the desired acceleration, I get

$$
\begin{aligned}
a_x &= -\mu_k g\cos\theta + g\sin\theta \\
&= g\cos\theta(\tan\theta - \mu_k)
\end{aligned}
$$

Yuck. It's a formula... has the right units. Can we make *any* sense of it? I notice one thing : since $\mu_k < \mu_s = \tan\theta_{max}$, then we know the ($\tan\theta$ - mu_k) in that last formula will always be positive for any angle θ LARGER than or equal to the critical angle, so a_x >0: at least the sign makes sense.... Otherwise, it's not the most illuminating formula in the world. But, it should be correct.

Rolling friction and tires: When a car *skids*, the rubber is scraping along the road, and you have kinetic friction, F_fr,k = μ_k N.

But what if the tire rolls, as it normally does? Then the contact point of rubber with road is NOT scraping along - it is instantaneously *stationary*.



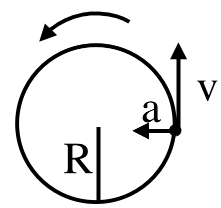
Think about the tire tread marks of a car in snow.

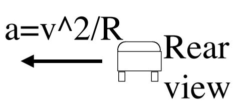
If it is not skidding, the tread marks are clear - the tire contact point must not be scraping or sliding at all. So the force of friction for a normally rolling tire is F_fr,s, static friction!

Example: A car drives around a flat, circular race track (radius R) with constant speed v.

How fast can it go without skidding off the track?

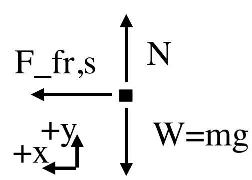
The sketch to the right shows a "birds-eye view", from above.





The sketch to the left is a rear view of the car from ground level.

Finally, a force diagram (from that same rear view):



Why did I draw **F**_fr,s pointing left? I was thinking ahead!

N-II says F_net=ma, and I *know* the car is accelerating to the left (at this moment), so there *must* be a force to the left. But what could provide such a physical force? Only static friction. It *must* be to the left. Suppose there was no friction - the car would not accelerate left - it would continue in a straight line (which means, in the birds-eye view, it skids off the race track. That's what happens on an icy track without any friction)

N-II in the y-direction: N-mg = 0, or N=mg.

N-II in the x-direction: F_fr,s = m*a_x = m v^2/R.

If v is small, you won't need much friction. But as v increases, this formula says the ground WILL supply more and more static friction. Static friction *adjusts* itself, in this case to prevent the car from skidding. However, there is a maximum amount of F_fr,s, which means a maximum speed. Let's find it:

$mv^2(\mathrm{max}) / R = F_{fr,s}(\mathrm{max}) = \mu_s N$.

Letting N=mg, the m's cancel, and $v(\mathrm{max}) = \sqrt{R\mu_s g}$

The units are correct (convince yourself, remember mu_s has no units.)

The bigger R is, the faster you can go - makes sense.

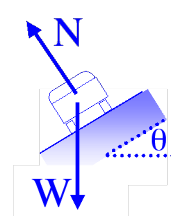
The more static friction you have, the faster you can go - makes sense.

If mu_s=0.8 (typical for rubber on dry concrete), and R=100 m (a reasonable value)

you get $v(\mathrm{max}) = \sqrt{100\ \mathrm{m}\ \ (0.8)\ 9.8\ \mathrm{m/s^{\wedge}2}} = 28\ \mathrm{m/s}$, about 65 mi/hr.

If you try to go any faster, the car *will* skid.

That's why some race tracks are *banked,* so that **N** will have a "centripetal" component, and you won't need as much friction to provide the required centripetal force. If you're going just the right speed, you won't not need *any* friction, when the inward component of N (i.e. $N^*\sin\theta_{bank}$) is just equal to mv^2/R.



(See if you can work that out for yourself... What angle do you need, to go 65 mi/hr around that 100 m radius track if it's an icy day?)