Conservation of energy.

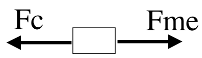
Kinetic friction is a somewhat unusual force. It's always fighting you. It does negative work, no matter which way you go. If *you* do work on an object that feels friction, that work is (practically) lost. You can't easily get it back. We call friction **nonconservative** for this reason. Compare it with, say, gravity. If you do work on an object that feels gravity (e.g., lifting it up), you can get *all* that work back - just let the object fall back down again by itself! Gravity is a **conservative** force.

If I lift an object, where exactly does my work go? Imagine starting and ending at rest, so W(net) = Δ K = 0. Still, I did + work while lifting it. That work (or energy) is stored, in what we call **potential energy**, or "U". An object high up has energy, it has the capacity to do work by virtue of its position. This is *not* kinetic energy, it can be at rest. It's potential energy.

Spring forces are also conservative. If you do work compressing a spring, the work is stored up, as "spring potential energy". Like a cocked dart gun: there is potential energy there, you can get it all back later.

How much energy is stored, quantitatively? Let's consider gravity again.
We know (last chapter) it requires +mgh of energy to lift an object up a distance h. So that's *exactly* how much energy you'll get back later if it falls. In other words, that's how much potential energy has been *stored* by lifting it up. We say the stored gravitational potential energy is U(grav) = mgh, in this case.

Let's be a little more rigorous. Suppose I move an object against some conservative force, call it **F**c.



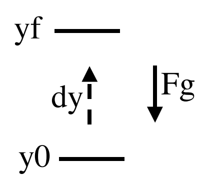
I will have to apply a force **F**_me = - **F**_c to move it around. Then, W(by me) = - W(by Fc) as I move it around. This W(by me) is precisely the work (energy) that's being stored up. That will be the additional potential energy, Δ U, the object has gained because of me pushing it around:

Δ U = W(by me) = - W(by Fc), or (using the definition of W from Ch. 7)

$$\boxed{\Delta U = -\int_{\mathbf{r0}}^{\mathbf{rf}} \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}}}$$

The potential energy here is *associated* with the conservative force **F**c, we might label it e.g. Δ U(spring) or Δ U(grav). For gravity, this integral formula becomes

$$\Delta U(\text{grav}) = -\int_{y0}^{yf} (-mg)\ dy\ = +mg\ \Delta y\,.$$



(The minus sign inside the integral is because gravity is down, opposite to dy) No matter what, the final sign is always correct:

If the object moves down, Δ y<0, and gravitational potential energy has *decreased*.

If the object moves up, $\Delta y > 0$, potential energy has *increased*. (Energy is stored)

• Only Δ U matters, not the value of U itself. We can *define* U to be zero wherever we want, it's like choosing an origin. You can call "zero gravitational potential energy" the floor, or the basement, sea level... All that ever matters is the *change* in U as something moves to different places. I will therefore usually define U(grav) =mgy (which agrees with the *derived* formula $\Delta U(\text{grav}) = mg\ \Delta y$, and sets U(grav)=0 at wherever I happen to chose to call y=0.)

• The path you take moving an object is irrelevant for Δ U, if the force involved is conservative. The change in gravitational potential energy is the same no matter what path you take to get from y0 to yf.

• U can be negative, it's quite meaningful to say an object has "negative potential energy". E.g., if the floor is defined to be zero potential, then a bowling ball in the basement has negative U. You would have to do work ON it to bring it up to zero energy!

• If you had a spring force, instead of gravity, you'd have

$\Delta U(\text{spring}) = -\int_{x0}^{xf} (-kx)\ dx\ = +\frac{1}{2}k(x_f^2 - x_0^2)$, where x is the stretch of the spring.

Once again, the zero is arbitrary, we can e.g. choose $\underline{U(\text{spring}) = +\frac{1}{2}k\,x^2}$,

so a relaxed spring would be said to have no potential energy.

Stretched or compressed, the spring's potential energy is now positive, which makes sense - the spring can do some work.

The central idea of this chapter (and much of modern physics) is **conservation of energy.** Let's begin by considering cases where there are only conservative forces acting (no friction!) We'll come back and talk about friction soon.

The W.E. Theorem says $\Delta K = W_{net}$, but we just defined Δ U= - W(by Fc).

If there are only conservative forces around, W(net) = W(by Fc), and

$\Delta K = W_{net} = W_{by\ Fc} = -\Delta U$. We can rewrite this several different ways:

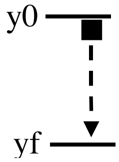
<u>$\Delta K + \Delta U = 0$</u>, or <u>$\Delta(K + U) = 0$</u>, or <u>(K+U) = constant</u>.

K=Kinetic energy, U=potential energy, K+U = "Total" or "**Mechanical Energy".**

The equations at the top all say *total (mechanical) energy is conserved*, it never changes (if there are no non-conservative forces around)

Energy can change form, e.g. from potential to kinetic and back. It can change from one type of potential to another (e.g. grav. to spring.) But, it never disappears.

<u>Example:</u> Drop a mass m, from a vertical position y0 to a final (lower) position yf.



At the start, (K+U)(initial) = K0 + U0 = $\frac{1}{2} m v_0^2 + mg y_0$

(Assuming here we started at rest, this simplifies to mgy0.)

At the end (K+U)(final) = Kf+Uf = $\frac{1}{2} m v_f^2 + mg y_f$.

Conservation of energy says K+U never changes. Setting initial energy to final E:

$\frac{1}{2} m v_f^2 + mg y_f = 0 + mg y_0$, or dividing through by (1/2)m: vf^2 = 2 g (-Δ y)

Note: this agrees with our old Ch. 2 formula, vf^2 = v0^2 + 2 (-g) (Δ y)

• Conservation of energy does *not* say "K=U". Don't ever write that. Don't ever *think* that! Granted - in some super simple problems, where K0=0 (starting at rest), and Uf=0 (ending at ground level), you have K0+U0 = Kf+Uf, which simplifies to 0+U0 = Kf+0, or U0=Kf. But this is a *very* special case!

ALWAYS, always write it out: E0=Ef, or better yet, K0+U0 = Kf+Uf,

(Think about what each quantity means) *That*'s conservation of mechanical energy.

Example: A spring (constant k) is compressed by an amount x0. It is then pointed vertically, and launches a dart *straight* up in the air. How high does the dart go?

If you think about forces, it's a hard problem. The spring force changes all the time, (F=kx) so acceleration is not constant, and then you can't use any Ch. 2 formulas. But, the only forces in this problem (spring, and gravity) are conservative, so you know you can use conservation of energy.

Initially, we have K0=0 (dart is at rest), U0(grav)=0 (dart is at ground level), but we DO have stored spring energy, U0(spring) = 1/2 k x0^2.

At the top of the path, the dart is momentarily at rest (Kf=0), and now the spring has "unsprung", so it has no more stored energy Uf(spring) = 0. The dart is up in the air, it has Uf(grav) = +mgy. Conservation of energy says

E0 = Ef, or writing out all the contributions to mechanical energy:

K0+U0(spring) + U0(grav) = Kf + Uf(spring) + Uf(grav) ,

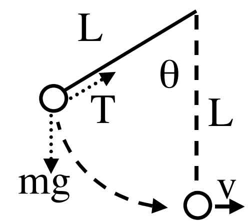
0 + 1/2 k x0^2 + 0 = 0 + 0 + mgyf,

or $y_f = (\frac{1}{2} k x_0^2) / (mg)$

Easy enough...

I could also ask for the speed at any *intermediate* height, it's basically no harder to find this as well. (You would have to add in (1/2 m vf^2) on the right side, and then solve for vf.)

<u>Example:</u> A pendulum swings. Given the initial configuration in the picture, figure out the speed v at the bottom of the swing.



Once again, using forces is nasty. The relative angle of "T" (tension) changes as we swing: a is *not* constant.

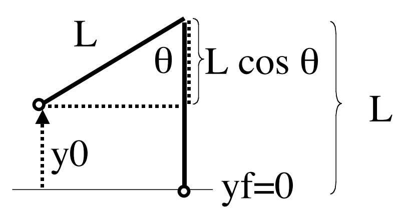
Can we use conservation of energy? We have a new force, the tension of the string, to worry about. If it does work on the system, then we could be in trouble. (If an outside force does work, you won't conserve energy. Like a "nonconservative" force, there's no potential associated with it. We'll discuss this at the end of the chapter). But here we're o.k.: Look at the picture and convince yourself "T" is ALWAYS perpendicular to velocity, in other words, "T" does zero work! (Remember, if **F** is perpendicular to displacement, no work is done).

The only force doing work is gravity, and that's conservative. So we can use

E0 = Ef

K0 + U0 = Kf+Uf.

We start at rest, so K0=0



The height y0 is obtained from this little geometry diagram: y0 = L-Lcosθ.

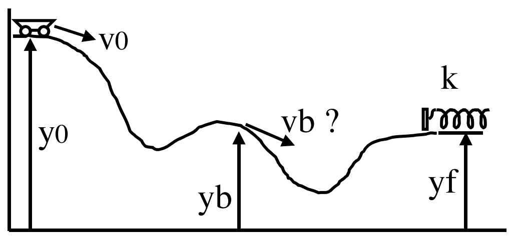
We end at the bottom, yf=0. Putting it all together, I have

0 + mg(L-Lcosθ) = 1/2m vf^2 + 0, which is easy to solve for vf.

Indeed, I could find the speed *anywhere* along the path, e.g. at any other intermediate height y', because 0 + mg(L-Lcosθ) = 1/2m v'^2 + mgy'.

(A problem that's hard to do with Newton II is often as easy as geometry, and writing E0=Ef, when energy is conserved.)

<u>Example</u>: A roller coast starts at the top of a hill, height y0, with some given v0. Can you find its speed at some later position, say height yb? Also, in the end it runs into a large spring (constant k) which stops it. How far is the spring compressed.?



N-II is a nightmare now. Think how complicated the Normal force is!

But (if friction is negligible), we can use conservation of energy.

K0+U0 = Kf+Uf

$$\frac{1}{2}m v_0^2 + mg\,y_0 = \frac{1}{2}m v_b^2 + mg\,y_b$$

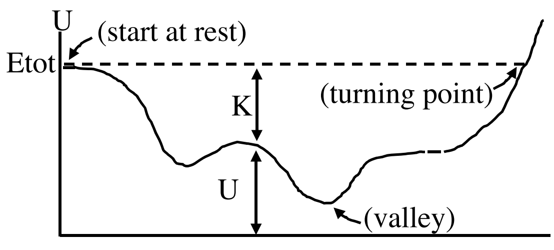
If you want vb just solve for it: $v_b = \sqrt{2g(y_0 - y_b) + v_0^2}$. Done!

How about the compression of the spring? Energy is STILL conserved!

After the spring has compressed by the unknown "x", the car is stopped, i.e. Kf = 0.

$$\frac{1}{2}m v_0^2 + mg\,y_0 = 0 + mg\,y_f + \frac{1}{2}k x^2$$ Just solve for x...

The roller coaster example leads to a way to graphically visualize conservation of energy. Since U(grav)=mgy, a graph of y vs x (like the picture above) is the SAME shape as a graph of U(grav) vs x. We call this a graph of U(x).



This time, assume we start at rest. So

Etot = E0 = =0 + U0 = mgy0.

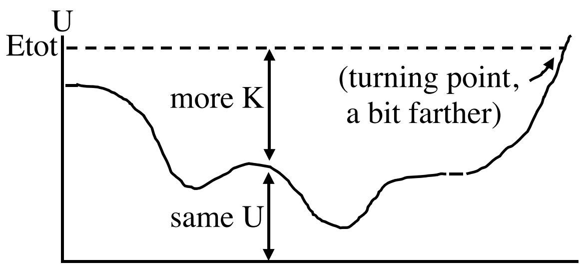
Etot is shown by the dashed line.

(It's constant, never changes!)

At any intermediate point, U+K=Etot. That's shown in the picture. U is the height of the graph, so K is the distance from the graph up to the dashed line...

At a "turning point", the graph of U(x) reaches up to Etot. Since U+K=Etot, that means that K=0 there. The object is stopped, and turns around (hence the name)

If you added a little energy at the beginning (e.g., if the cart started with some initial nonzero K), the graph of U(x) is unchanged, you just lift up the Etot line a little.



<u>What about if there's friction?</u>

The W.E. Theorem says $\Delta K = W_{net} = W_{by\ Fc} + W_{nonconserv} = -\Delta U + W_{nonconserv}$.

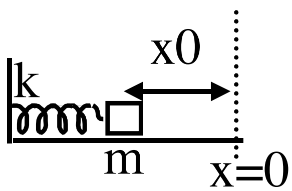
That means $\Delta K + \Delta U = W_{nonconserv}$.

If you have nonconservative forces (like friction), this means total mechanical energy is NOT conserved. (Friction => the right side will be negative, the change in Etot will be negative, total energy will be decreasing.)

You can use this formula to solve lots of problems that have friction in them, as long as you can calculate (or estimate) the right side, i.e. W(non-conservative).

Similarly, if there is any external or "outside" force (like e.g. your hand) which is not conservative (there's no potential energy, U, associated with the force), then you'll have $\Delta K + \Delta U = W_{nonconserv} + W_{external}$. If you do work on a system, W(external), the mechanical energy of the system will change. (Makes sense.)

<u>Example:</u> An object at the end of a spring, initially compressed by x0, is released from rest. It sits on a rough surface, the coefficient of kinetic friction is μ_k. When the object has reached the "relaxed" position, x=0, how fast is it going?



Again, the force is not constant, so we couldn't use any Ch. 2 "constant a" Eqns. N-II won't be easy. There IS friction, so we *cannot* assume conservation of energy. But it's not hard to figure out W(nonconserv) here:

W(friction) = |F(friction)|*|Δ x|*cos(180) = - μ_k N x0.

(N is the normal force: in this flat case, it's going to be mg. Do you see why? |Δ x| is x0, the distance traveled. Do you see why the - sign is there?)

$\Delta K + \Delta U = W_{nonconserv}$ tells us

(Kf-K0) + (Uf - U0) = W(friction) = - μ_k mg x0.

Now, $K_f - K_0 = \frac{1}{2} m v^2 - 0$ (it started from rest), and

$U_f - U_0 = \frac{1}{2} k 0^2 - \frac{1}{2} k(-x_0)^2$ so we have

$\frac{1}{2} m v^2 - \frac{1}{2} k x_0^2 = -\mu_k \, mg \, x_0$, or

$v = \sqrt{\frac{k}{m} x_0^2 - 2 \mu_k g x_0}$

That's it. We used the principle of energy conservation to solve this problem, although technically energy was lost due to friction. But we were able to calculate the lost energy easily enough.

Relation of force and potential:

If you apply a (conservative) force over a *small* distance, the amount of work done is easy to compute, W(by Fc)= $\mathbf{F_c} \cdot \Delta\mathbf{r}$, or in 1-D, $F_c\, \Delta x$.

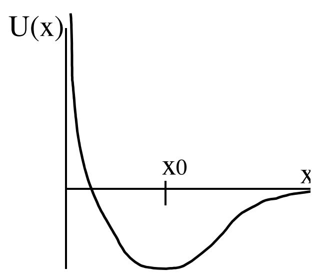
Now recall, we defined $\Delta U = -W(\text{by } F_c) = -F_c\, \Delta x$.

Dividing both sides by Δx, and taking the limit of very small Δx, we get

$$F_c = -dU/dx.$$

If I give you the potential energy as a function of position, U(x), you can thus immediately compute the associated force by taking the derivative.

Example: Diatomic molecules typically have a potential energy as a function of separation that looks something like this:



The potential energy is large and positive when x is small - so, if you squeeze it, the energy is large (think of a spring, with lots of stored energy when squeezed.) If x is large (atoms far apart), energy is basically zero.

U is *negative* in the middle, that means the molecule is happy there! (Everything likes to go to the lowest possible energy, like balls rolling down to the basement.)

The formula says F= -dU/dx. For small x, look at the graph: dU/dx (slope) is -, the force is therefore + (repulsive). (This *shows* the molecule acts like a spring.)

For large x, dU/dx is +, the force is therefore -, the atoms are *attracted* together.

If x=x0, the slope is zero: no force. This is "equilibrium", the position where the molecule likes to be.

F is a basically a derivative of U, U is basically an integral of F. Makes some sense.

Energy is always conserved. It doesn't look like it when there's friction in the story, but it's really true. If there's friction, mechanical energy is indeed "lost", but the table and object heat up. Mechanical energy has been transformed into heat energy. If you had a formula for heat energy (and we will, later in the book) you could INCLUDE heat energy as part of "total energy", add it to the K+U, and then you'd restore conservation of total energy.

Mechanical energy, K+U, is not conserved if there's friction, but *total* energy, K+U+thermal+chemical+nuclear+rest mass energy+...   *is* conserved.

You have to keep track of all *forms* of energy, and then energy is exactly conserved.

That last term, "rest mass energy", is given by E=mc^2. Einstein discovered it, it's another form of energy which is very real, and needs to be included if you want to use conservation of energy completely accurately.

There is no approximation involved with conservation of energy, this is one of the greatest and most accurate laws of physics. Unlike Newton's laws (which were modified by relativity and quantum physics), conservation of total energy is still believed to be absolute and fundamental. The universe has a certain amount of energy which will *never* change. The *form* of that energy can change, from kinetic to potential to heat, back and forth, but the total remains fixed. (Some forms are more *useful* to us than others!)

When something is conserved, it's a wonderful tool to solve problems and understand physics, because if you know the total to start, you know it forever: it makes solving for (certain) quantities quick and simple arithmetic.