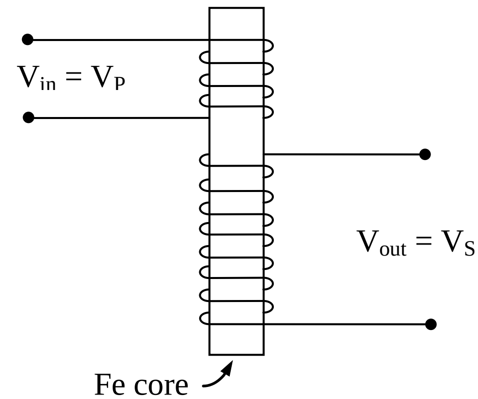
Transformers

The entire electrical power distribution system in the civilized world depends on a simple device called a transformer. A transformer is a device for transforming AC voltage from one value (say 120 VAC) to another value (like 10 VAC or 2000 VAC). A transformer is made of 2 coils of wire, usually wrapped around an iron core. It is a simple device with no moving parts.



Primary coil = input coil, with N_P turns

Secondary coil = output coil, with N_S turns

We will show below that $V_{out} = V_S = \frac{N_S}{N_P} \cdot V_P$ or $\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}}$

This "Transformer Equation" says that, for AC voltage, the voltage ratio is equal to the turns ratio.

NOTE: the transformer only works for AC voltage. If V_{in} is DC, then $V_{out} = 0$.

$\frac{N_S}{N_P} = \frac{V_S}{V_P} > 1 \quad \Rightarrow$ "step-up transformer"

$\frac{N_S}{N_P} = \frac{V_S}{V_P} < 1 \quad \Rightarrow$ "step-down transformer"

(A step-down transformer gives a smaller V, but a larger current I.)

Transformers work because of Faraday's Law:

V_P (AC) $\Rightarrow$ I_P (AC) $\Rightarrow$ B_P (AC) $\Rightarrow$ B_S (AC) + Faraday $\Rightarrow$ $\mathcal{E} = V_S$

Proof of the Transformer Equation:

If we apply Faraday's Law to the primary and secondary coils, we get:

$$\left.\begin{aligned} &(1) \quad V_S = N_S \frac{\Delta\Phi}{\Delta t} \\ &(2) \quad V_P = N_P \frac{\Delta\Phi}{\Delta t} \end{aligned}\right\}$$

same $\Phi = B\,A$ in each turn of primary and secondary because the iron core "guides flux" from P to S.

(1) ÷ (2) ⇒ $\frac{V_S}{V_P} = \frac{N_S}{N_P}$ (End of proof.)

If a transformer is well-designed, only 1 to 5% power in is lost to heating of coils and eddy currents in the iron core.

$\Rightarrow \quad P_{out} \cong P_{in} \quad \Rightarrow \quad I_S V_S = I_P V_P \quad \Rightarrow \quad \frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S}$

A step-down transformer produces a smaller voltage, but a bigger current (same P = I V).

Light bulbs and appliances with motors (vacuum cleaners, blenders) use AC voltage to operate. But devices with electronic circuits (TV's, computers, phones, etc) need DC voltage to function. The "power supply" in computers and TV's converts the AC voltage from the wall socket into DC voltage (usually 10-15 V) that the electronic circuitry needs.

Example of use of transformers: Suppose you want to melt a nail by putting a big current through it. What happens if you try to melt the nail by putting 120 VAC (from your wall socket) across the nail? Answer: you will blow a fuse or trip a breaker. The resistance of a nail is quite small: $R_{nail} \cong 10^{-3}\,\Omega$. The current produced by a 120 V voltage difference across the nail is huge: $I_{nail} = \frac{V}{R_{nail}} = \frac{120\,\text{V}}{10^{-3}\,\Omega} = 120000\text{ A}$. This will never happen since your breaker will trip when the current exceeds 15 A. (Here's an experiment you should <u>never</u> try at home: Bend a nail into a U shape and plug it into your wall socket. Watch the lights go out.)

So how do we melt that nail? Solution: Use a 100-to-1 <u>step-down transformer</u>.

$$\frac{N_S}{N_P} = \frac{1}{100} = \frac{V_S}{V_P}, \qquad V_P = V_{in} = 120\text{ VAC}$$

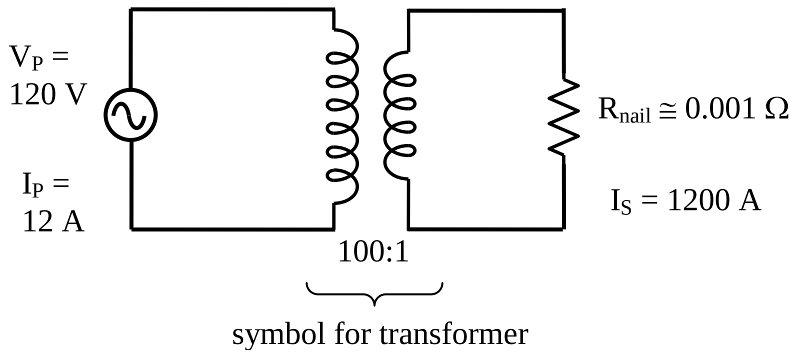
$$V_S = V_{out} = \frac{N_S}{N_P} \cdot V_P = \frac{1}{100} \cdot 120\text{ V} = 1.2\text{ VAC}$$

$$I_S = I_{out} = \frac{V_S}{R_{nail}} = \frac{1.2\,\text{V}}{10^{-3}\,\Omega} = 1200\text{ A} \quad \text{(enough to melt the nail)}$$

How much current will this draw from the wall socket? Recall that $P_{in} = P_{out}$ or $I_S V_S = I_P V_P$.

$$I_P = I_{in} = I_S \cdot \frac{V_S}{V_P} = I_S \cdot \frac{N_S}{N_P} = (1200\text{ A})\left(\frac{1}{100}\right) = 12\text{ A} \quad \text{(Not enough to blow the fuse.)}$$

Circuit diagram:

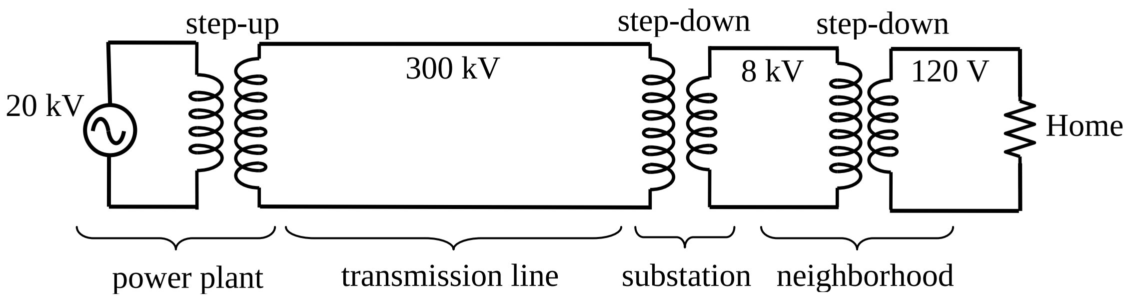


Power dissipated in nail = $I_S^2 R = (1200)^2 (10^{-3}) = 1440\text{ W} \Rightarrow$ will melt nail.

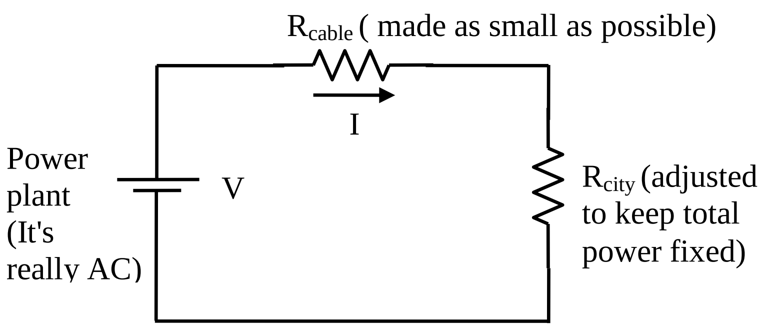
Transformers and Power Distribution

Economical power distribution is only possible because of transformers. Electrical power is transmitted from the power plant to the city by big aluminum cables (power lines). Some energy is inevitably wasted because the power lines have a resistance, and so they get hot: $P_{lost} = I^2 R_{cable}$. In order to minimize this waste, the power must be transmitted from the plant to the city at very high voltage (typically 300 kV). A high voltage allows a small current, at a given power (since P = IV). And a small current means small I^2R losses in the cable.

When the high-voltage power is delivered to the city, step-down transformers are used to transform the very dangerous high voltage down to the not-so-dangerous 120 V before it enters your home. The voltage is stepped down in stages as it is distributed throughout the city.



A very simplified model of power distribution (transformers not shown):



Some typical numbers:

Power output of plant = P_{out} = 100 MW to 1 GW = 10^7 to 10^8 W (<u>fixed</u> by demands of the city)

$P_{out} = I\,V$, $P_{lost} = I^2 R_{cable}$ Using $I = P_{out} / V$, we get $P_{lost} = \frac{P_{out}^{\;2}}{V^2} \cdot R_{cable}$

Fraction of power wasted = $\frac{P_{lost}}{P_{out}} = \frac{P_{out}}{V^2} \cdot R_{cable}$

If $R_{cable} \approx 10\Omega$ and $P_{out} = 10^8$ W, then

If V = 50,000 V : $\frac{P_{lost}}{P_{out}} = \frac{10^8}{{5\times10^4}^{\;2}} \cdot 10 = 0.4$ (40% lost!)

If V = 200,000 V : $\frac{P_{lost}}{P_{out}} = \frac{10^8}{{2\times10^5}^{\;2}} \cdot 10 = 0.025$ (2.5% lost)

Boosting the voltage at which the power is transmitted makes the losses acceptably small.

