

COURSE SCALE LEARNING GOALS

Math Methods with Classical Mechanics (Phys 2210)

These learning goals were created by a working group of faculty – both those in physics education research and those with other areas of research. This list represents what we want students to be able to *do* at the end of the course (as opposed to what *content* is expected to be covered, as in a syllabus).

1. **Math/physics connection:** Students should be able to translate a physical description of a sophomore-level classical mechanics problem to a mathematical equation necessary to solve it. Students should be able to explain the physical meaning of the formal and/or mathematical formulation of and/or solution to a sophomore-level physics problem. Students should be able to achieve physical insight through the mathematics of a problem.
2. **Visualize the problem:** Students should be able to sketch the physical parameters of a problem including sketching the physical situation and the coordinates (e.g. , equipotential lines, a resonance curve, a pendulum with its angle as the coordinate,) as appropriate for a particular problem.
3. **Expecting and checking solution:** When appropriate for a given problem, students should be able to articulate their expectations for the solution to a problem, such as direction of a force, dependence on coordinate variables, and behavior at large distances or long times. For all problems, students should be able to justify the reasonableness of a solution they have reached, by methods such as checking the symmetry of the solution, looking at limiting or special cases, relating to cases with known solutions, checking units, dimensional analysis, and/or checking the scale/order of magnitude of the answer.
4. **Organized knowledge:** Students should be able to articulate the big ideas from each chapter, section, and/or lecture, thus indicating that they have organized their content knowledge. They should be able to filter this knowledge to access the information that they need to apply to a particular physical problem, and make connections/links between different concepts.
5. **Communication.** Students should be able to justify and explain their thinking and/or approach to a problem or physical situation, in either written or oral form. Students should be able to write up problem solutions that are well-organized, clear, and easy to read.
6. **Build on Earlier Material.** Students should deepen their understanding of Phys 1110 material. I.e., the course should build on earlier material.

7. **Problem-solving techniques:** Students should continue to develop their skills in choosing and applying the problem-solving technique that is appropriate to a particular problem. This indicates that they have learned the essential features of different problem-solving techniques (eg., solving differential equations with constant coefficients, using fourier series methods to solve PDEs with appropriate boundary conditions, etc). They should be able to apply these problem-solving approaches to novel contexts (i.e., to solve problems which do not map directly to those in the book), indicating that they understand the essential features of the technique rather than just the mechanics of its application. Students should move away from using templates. They should be able to justify their approach for solving a particular problem.

...7a. **Vectors and coordinate systems:** Students should be able to compute dot and cross products and solve vector equations without reference to books or external materials, and they should demonstrate comfort with these mathematical tools. Students should recognize whether variables are scalars or vectors, and vector and scalar variables should be clearly distinguishable in students' written work. Students should be able to project a given vector into components in multiple coordinate systems, and to choose the most appropriate coordinate system in order to solve a given problem. Students should be able compute surface and volume integrals in Cartesian, cylindrical, and spherical coordinate systems (i.e., know the expressions for dV in these coordinate systems and how to apply them in a particular situation).

...7b. **Approximations:** Students should be able to recognize when approximations are useful, and use them effectively (eg., recognize when air resistance is a small effect, Students should be able to indicate how many terms of a series solution must be retained to obtain a solution of a given order, and should be able to identify when a Taylor expansion is appropriate and what the variable of expansion is in a given problem.

...7c. **Series expansions:** Students should be able to recognize when a series expansion is appropriate to approximate a solution, and expand a Taylor Series beyond zeroth order.

...7d. **Orthogonality:** Students should recognize that both vectors and functions can be orthogonal and that any function can be built from a complete orthonormal basis. Students should be able to expand functions in an orthonormal basis (e.g. find the coefficients for a Fourier series) and interpret the coefficients physically. Students should be able to determine from the even or odd symmetry of a function which terms in the expansion are zero. Students should be able to define the terms complete and orthonormal in the context of an orthonormal basis.

...7e. **Differential equations:** Given a physical situation, students should be able to write down the required ordinary differential equation, identify the method of solution, and correctly calculate the answer. Students should be able to identify the type of differential equation (homogeneous, linear vs. nonlinear, constant vs. variable coefficients, 1st, 2nd, or higher order, etc.) and choose the correct method to solve that type of ODE. Students should be able to explain how the type of differential equation helps determine which methods of solution will be applicable.

...7f. **Superposition:** Students should recognize that – in a linear system – the solutions may be formed by superposition of components.

8. **Problem-solving strategy:** Students should be able to draw upon an organized set of content knowledge (LG#3), and apply problem-solving techniques (LG#4) to that knowledge in order to organize and carry out long analyses of physical problems. They should be able to connect the pieces of a problem to reach the final solution. They should recognize that wrong turns are valuable in learning the material, be able to recover from their mistakes, and persist in working to the solution even though they don't necessarily see the path to the solution when they begin the problem. Students should be able to articulate what it is that needs to be solved in a particular problem and know when they have solved it.
9. **Intellectual maturity:** Students should accept responsibility for their own learning. They should be aware of what they do and don't understand about physical phenomena and classes of problem. This is evidenced by asking sophisticated, specific questions; being able to articulate where in a problem they experienced difficulty; and take action to move beyond that difficulty.

TOPIC SCALE LEARNING GOALS

2210 Topic-level learning goals. (There is some copy-paste from course-scale learning goals when appropriate).

Topics covered in 2210:

Math	Physics contexts
Vectors, curvilinear coordinate systems. Quick review of vector addition, dot and cross products. Spherical and cylindrical coordinate systems, simple derivatives.	Kinematics. Position, velocity, and acceleration.
ODEs. Guess and check, linear ODEs, constant coefficient ODEs.	Newton's Laws. Reference frames, $F=ma$, 1D motion, 3D motion.
Line integrals. Gradient operator. Taylor expansion.	Conservation Laws. Kinetic and potential energy, small oscillations, momentum and angular momentum.
Complex numbers. ODEs. Fourier series. Fourier transforms (cover transforms quickly).	Simple harmonic oscillator. Damped and driven oscillators, resonance.
Fourier series applications. PDEs. Separation of variables (in Cartesian and polar coordinates).	Heat equation. Poisson equation.
Surface and volume integrals. Gauss' theorem. Legendre polynomials. Laplace equation. Selected Vector Calc.	Gravitation.
Delta functions	Gravitation

Goals:

Vectors, curvilinear coordinate systems and kinematics:

-[Students should be able to compute dot and cross products and solve vector equations without reference to books or external materials, and they should demonstrate comfort with these mathematical tools. Students should recognize whether variables are scalars or vectors, and vector and scalar variables should be clearly distinguishable in students' written work. Students should be able to project a given vector into components in multiple coordinate systems. Students should be able compute surface and volume integrals in Cartesian, cylindrical, and spherical coordinate systems (i.e., know the expressions for dV in these coordinate systems and how to apply them in a particular situation).]*

-Students should recognize that \dot{x} is the same as dx/dt , and identify these mathematical terms with the physical idea of rate of change of position with time. Students should similarly recognize that \dot{v} is the same as dv/dt , and identify these mathematical terms with the physical idea of rate of change of velocity with time.

* This paragraph copied from course-scale learning goals.

Students should recognize that $\dot{x} = v$ and $\dot{v} = a$. Students should be able to explain the physical meaning of position, velocity, and acceleration and describe how they are related to each other.

- Students should be able to break a vector equation in to three equations – one for each component.

- Students should be able to solve problems in plane polar coordinates. They should be able to draw r -hat, ϕ -hat, and θ -hat for a given point.

ODEs and Newton's Laws:

- [Given a physical situation, students should be able to use Newton's laws to write down the required ordinary differential equation, identify the method of solution, and correctly calculate the answer. Students should be able to identify the type of differential equation (homogeneous, linear vs. nonlinear, constant vs. variable coefficients, 1st, 2nd, or higher order, etc) and choose the correct method to solve that type of ODE. Students should be able to explain how the type of differential equation helps determine which methods of solution will be applicable.]*

- Students should be able to use initial conditions as part of their solutions to ODEs.

- Students should be able to identify (and draw) an appropriate coordinate system before beginning to write down an ODE from a physical situation.

- Students should be able to make physical sense of the mathematical solution to a differential equation, which includes testing limiting cases and sketching the function.

- Students should be able to determine if an ODE is separable, be able to separate the equation if possible, and solve separable ODEs.

- Students should be able to solve first order linear ODEs with constant coefficients.

- Students should have solutions to several common ODEs at their fingertips (i.e. be able to give the solution to the ODE without calculation or reference to external materials). These include the ODEs whose solutions are sin/cos, exponentials, and linear function.

- Students should be able to decide if a given differential equation can be solved analytically. If it cannot, they should be able to use Mathematica to find a solution, and recognize if Mathematica has returned a bogus result.

Motion with air drag: (a good concrete example of the above learning goals)

- Students should be able to translate a given physical situation for an object moving with air resistance to a correct differential equation, including the correct sign for each term of the equation.
- Students should be able to predict the direction of drag when given the velocity of a moving object.
- Students should be able to describe the concept of terminal velocity and be able to calculate it for a given object and form of the drag force (i.e. quadratic, linear, or both terms).
- Students should be able to explain what quadratic and linear drag are and that they are limiting cases of the full equation for drag which is a combination of both.
- Students should be able to list the variables that air resistance depends on (e.g. velocity of the object, viscosity of the liquid, shape of object, etc.) and should be able to predict whether the air resistance increases or decreases when these variables are changed.
- Students should be able to sketch the qualitative path of an object given the differential equation involving air drag and the initial conditions.

Taylor Expansion:

- [Students should be able to recognize when approximations are useful, and use them effectively (eg., recognize when air resistance is a small effect, Students should be able to indicate how many terms of a series solution must be retained to obtain a physically significant answer, and should be able to identify when Taylor expansion is appropriate and what the variable of expansion is in a given problem.]*
- Students should be able to take the Taylor series expansion around zero for common functions (cos, sin, exp, $1/(1+x)$, $\sqrt{1+x}$, $\ln(1+x)$) and express the solution both as a list of terms and using summation notation.
- Students should recognize that Taylor series are often used when a variable is $\ll 1$ and be able to choose an appropriate variable (or combination of variables) to expand in for a given situation.
- Students should be able to explain when a Taylor expansion is exact, a good approximation, or not a good approximation (e.g. near the point of expansion the Taylor expansion will be a good approximation with only a few terms, but farther away more terms are needed for a good approximation of the actual value. To be exact at any point, one may need an infinite number of terms.)

Gradient Operator:

- Students should be able to calculate the gradient of a function in Cartesian coordinates without reference to external sources such as a textbook. Students should be able to

* This paragraph copied from course-scale learning goals.

compute the gradient of a function in cylindrical and spherical coordinates with the use of a reference.

- Students should be able to explain the physical meaning of the gradient, predict relative direction and magnitude for several points given equipotential lines, and relate the gradient to the 1-d idea of slope.

Conservative forces:

- Students should be able to calculate the curl of a function in Cartesian coordinates without reference to external sources such as a textbook. Students should be able to compute the curl of a function in cylindrical and spherical coordinates with the use of a reference.

- Students should be able to determine from an equation or a drawn vector field whether the underlying force is conservative.

- Students should be able to explain both conceptually and mathematically how force (\mathbf{F}) and potential (U) are related and when this relation is applicable.

- Students should be able to determine the relative magnitude and a direction of a force at several points on a set of drawn equipotential lines.

- Students should be able to determine the direction of a force at a point based on a plot of U vs. position (in 1-d). Students should be able to recognize equilibrium points in the plot and should be able to determine if these points are stable given the function $U(x)$.

Line integrals:

- Students should be able to take the line integral of $\mathbf{F} \cdot d\mathbf{r}$ and should be able to explain what this sum means physically (i.e. the sum of dot products along a line).

- Students should be able to predict the magnitude (zero, non-zero) of line integrals for a given path in a drawn vector field.

Simple Harmonic Oscillator:

- Students should be able to solve second order linear ODEs with constant coefficients.

- Students should be able to write down Hooke's Law and explain what each variable means. Students should be able to explain the physical meaning of this equation.

- Students should be able to explain, using the idea of a Taylor series, why Hooke's law is a good approximation for the force near any stable equilibrium. Students should be able to give a physical example of a system that obeys Hooke's law that is not a spring.
- Students should be able to write the differential equation for harmonic motion with and without damping and driving forces, and should be able to explain the physical meaning of each term.
- Students should be able to draw a phase diagram for a physically or mathematically given oscillation and vice-versa. Students should be able to explain the physical meaning of the crossing points on the x and \dot{x} axes.
- Students should be able to explain, both mathematically and physically, how underdamped, overdamped, and critically damped motion come from a single differential equation.
- Students should be able to determine from a given physical situation if the motion will be over-, under-, or critically-damped.
- Students should be able to predict whether w increase, decrease, or stays the same for damped compared to simple harmonic motion.
- Students should be able to use real-world examples to explain how critically damped motion can be useful.
- Students should be able to explain the concept of resonance both conceptually and mathematically.
- Students should be able to sketch the motion of simple harmonic oscillators, with and without damping and driving forces on the same set of axis, and should have qualitatively correct the relative amplitudes and frequencies.
- Students should be able to use Fourier Series to solve for the motion of a harmonic oscillator driven with a given arbitrary periodic forcing.

Complex Numbers:

- Students should be able to write a complex number in the following representations: $x+iy$, (x,y) , $re^{i\theta}$, and on a set of two axis. Students should be able to fluently move between these different representations and to choose the most appropriate representation to solve a given problem.
- Students should be able to break a complex number equation in to two equation – one for imaginary and one for real.

Fourier Series:

-[Students should be able to expand functions in an orthonormal basis (e.g. find the coefficients for a Fourier series) and interpret the coefficients physically. Students should be able to determine from the even or odd symmetry of a function which terms in the expansion are zero. Students should be able to define the terms complete and orthonormal in the context of an orthonormal basis.]

Fourier Transforms:

-Students should be able to match graphs of functions to graphs of their Fourier transforms.

PDEs/Separation of Variables:

- Students should be able to explain the difference between ODEs and PDEs and should be able to give examples of physical situations that lead to each.

- Students should be able to derive the relevant separated ODEs in Cartesian coordinates from Laplace's equation.

- Students should be able to use boundary conditions to solve Laplace's equation using separation of variable in 2-D Cartesian and plane polar coordinates. For plane polar coordinates, students may refer to a book for the relevant separated ODEs.

Gravitation:

-Students should be able to apply Gauss's law in the context of a gravitation problem

- Students should be able to explain what criteria must be satisfied for Gauss's Law to be useful, and should be able to predict mass distributions whose gravitational field can and cannot be determined using Gauss's law.

-Students should be able to find the total mass for a $\rho(\mathbf{r})$ using volume integration.

-Students should be able to translate the physical situation in to an appropriate integral to calculate the gravitational force at a particular point away from some simple mass distributions (a half-infinite line of charge, a ring of charge while on the axis of the ring, etc.)