**Physics 2210, SJP, Spring 2012**

This document contains lecture summaries from the Spring 2012 term.

More details can be found in the **SJP\_daily\_lecture\_notes.docx** file, as well as the daily powerpoint files (look at the comment pane below each clicker question to see details about how the class interacted with each of those)

If future instructors come up with new lecture demo and/or activity ideas, it would be great to summarize those with enough detail that other faculty could regenerate them! Let's keep this file up to date (or, produce an abbreviated version for supplemental ideas)

**Lecture #** (Tues Jan 17) **Intro, Newton's Laws**

**Coverage:** Syllabus and course logistics (~15 minutes). My lecture notes pp 1-3, and most of p8, and diagnostic pretest  
 **Summary:** Review of Physics I, and overview of this course.

- Newton's laws, systems, N2 with an emphasis on a) vector nature b) F(net, external) c) really dp/dt rather than ma, d) clear definition of your "system" e) comes from experiment!) f) limitations (classical regime). Also N-III, with discussion of same topics, especially on limitations/classical regime. (Some segue into where N-III breaks down).

Brief review of torque.

(Cut class off after 60 minutes to do 15 minute pretest)

**Props/demos:** Brought in a bike (for a concept test at end of class, pushing on a lower spoke, which way does the bike move?). Worked well, it's fun, visible, and compelling. No bike in the basement, I had to bring in my own.

**Activities:** 1) Early in the class (at :17 in) I had them write a summary of Phys 1110 on a piece of paper (~1-2 minutes), then "meet their neighbors", and continue working on this. While they worked, I got them to turn them in, and then I went to the board to summarize. This let ME summarize 1110 and compare with what they had, it worked pretty well.

2) End of class (last 15 min) we gave our "diagnostic pretest". 15 minutes was a little short – but ok.

**Concept tests**: I did some demographics (math background, physics background),

- "nature of science" icebreakers (Is F=dp/dt "derivable"? Or, torque = dL/dt?) These generated good discussions. We discussed how force is defined (not ma!), "operational definitions" of measurables, and the experimental origin of physics.

- 1110 review: I had a "centripetal acceleration" question (rock on a string), they did fine, but it was good to discuss the difference between kinematics and dynamics.

- I had a "free body diagram" (pushing on a system of two masses constrained by a massless rope). Good for talking about "what is the system", external forces (it was frictionless, so they had to realize the front object WOULD accelerate), newton's 2nd and 3rd law. Discussed "how do you know the force of the string on the front object is equal to the force of the string on the rear object". (We talked about it via N-III, but also F(net) = ma for a massless object...

- "what happens if you push forward near the bottom of spoke on a bike" question. They were 80% wrong (thought it would go backwards, or not move). We did n’t have time for ANY discussion!

**Lecture #** (Thur Jan 19) **Coordinate systems, Polar coordinates**

**Coverage:** Lecture notes pp 4-11, didn’t quite get to 12.   
 **Summary:** Coordinate systems, physics is independent of choice. Cartesian, and (briefly) cylindrical (I had them find the volume element) and spherical (discussion of convention for theta and phi in physics, different in most math classes!)

Plane polar - unit vectors and Orthonormality, Interpretation of r-hat and phi-hat

Writing these in Cartesian coordinates

Taking the time derivative of \vec r (and interpreting v in this notation)

Taking dv/dt to get a (in plane polar)

Endinwith F=ma in plane polar, interpret the various terms. (Didn’t quite get here)

**Props/demos:** I had a small colorful “right handed coordinate system” (8 inch arrows in a block of wood) which I could walk around with. And, a bungee cord to stretch from the corner of the room up to my “point in space” to show the r vector.

**Activities:** Kinesthetic activity of them pointing in various directions. (Eyes closed, point in jhat direction). Stand point in rhat. A few are confused about direction (in or out). Then, theta hat (many have the direction wrong here, it’s “increasing theta”) People in the right-most row (as I face them, in front of the origin) should all have thetahat pointing essentially STRAIGHT DOWN! Finally, point in phihat (make sure they’re all pointing in plane)

Later – let THEM come up with x and y components of rhat, I set up finding v, then had THEM work on d(vec r)/dt, what it might look like in terms of polar unit vectors.

Use sheet (Velocity\_accel\_polar\_OSU\_) which asked them to *compute* drhat/dt, and dphihat/dt, and then to use that to find v, and finally a (which few get to)

Gave them about 10 minutes (?), it wasn't enough to finish, but we did get everyone started. (Interrupted after few minutes to make sure they were USING the cartesian formulas as an intermediate step) Make sure they understand phi is an arbitrary function of time, it can change, they need to use the chain rule.

**Concept tests**: -pre class was volume element in cylindrical (I have a "cheater/helper" version that I followed up with, so they would get it all correct)

- limits of integration for R, theta, phi in spherical (to discuss conventions)

- plane polar, which unit vectors vary with time?

- write **r =** r rhat one (this is very tricky for them!) I gave it twice - it was mostly all wrong the first pass (silently), and I told them that and let them talk. Second time it was ~60+% correct. We had already defined rhat = **r** / |r| on the board before either version. Still, they find this confusing. (Discussed units, talked about "directions". They were curious why, if **r** is always in the rhat direction, would you ever need/care about phi-hat? (So, I talked about velocity).

- Given rhat, find phi-hat in Cartesian. (They struggled a bit - got the "signs" from the picture, but some sin/cos confusion. I drew a picture on the board and had them figure out the angle phi-hat makes to the \*vertical\*, and that helped, after the fact)

Mention orthogonality.

**Lecture #** (Tues Jan 24) **Newton's law in 2D, and ODEs.**

**Coverage:** My notes pp 12 - 17 (This is Taylor 1.7, and Boas 8.1)  
 **Summary:** Start with Newton’s law on the board – brief discussion of “philosophy” (what is an ODE, find x(t), predict or control motion) Review last time: spherical coords and unit vectors, interpreting "components": First in cartesian (review Fx=max, Fy=may) Then, "orbits" in cartesian. Then, back to polar, discuss radial and phi components of Newton's 2nd law in 2D. Pendulum (including discussion of tension) Derive and solve ODE for phi (linearize, with discussion, and "guessing" method!) Talked about "2 undetermined constants" and boundary (or initial) conditions. Notation for ODEs: define and \*motivate\* "order", "linear", "homogeneous", "separable". Let them work on interpreting the “linear growth” ODE.

**Props/demos:** Mass on string, swung aroud my head, and used as pendulum to talk through the Diff eq (r is fixed, so which terms vanish? What do the equations all mean, how to interpret the terms)

**Activities:** For dN/dt - kN = 0 concept test, had THEM solve it.

**Concept tests**: Many! See CT file for more notes.

For circular orbit (parametric) CT, they knew the answer but were not thinking of it in terms of taking |r| and |v|, they were just "tracing out" the curve, so we discussed that.

The CT about "What is T\_phi" generated lots of discussion, it wasn't as easy as the histogram implies. One student had called it out before we started, but still lots of discussion. We talked about what the notation T\_phi means, the difference between the angle and the phi unit vector... For "what is W\_phi", they were still a little confused, so it's good to have this follow the previous one. Signs were an issue now. We redrew the picture carefully to visualize it better, and discussed the "check that this little angle is phi by taking the limit of phi -> zero" trick.

Many classifying ODE questions, they did fine. Some discussion about whether sin(x) makes the ODE nonlinear. Some question about what the dependent variable was (I didn't specify it in the first question)!

For the dN/dt = kN ODE, I had THEM solve it on their own, took a couple minutes. Clicker question went up after they started, that's how I knew they had finished. Worked nicely. Had a good discussion of the physical interpretation of the ODE, sensemaking of the solution (biology example, though this year someone first came up with Newton’s law of cooling- good idea, though not with my sign choice. s)

**Lecture #** (Thurs Jan 26) **ODEs, motion with drag.**

**Coverage:** Lecture notes pp. 19-21, and the Maine Tutorial "Newton's Laws and velocity-dependent forces". This was Taylor 2.1. Also Boas 8.3)  
 **Summary:** Discuss Unit analysis. Review ODE terminology. Classify y' + P(x)y = Q(x) (order? linear? homogeneous? separable?) Very brief sketch Boas' general solution ("particular" + "homogeneous"), chose not to go into this in much detail this year. Air drag (~20 min) - first in general (as one form of force, looking at F = md^2x/dt^2, and the various types of "F" we can solve), then form of f(v), (constant, linear, and quadratic). Vector nature and notation ( f(v) vhat). Regions of validity, physics of bv and cv^2. Get students to brainstorm what drag might depend on. Viscous drag for the first, b = beta \* D. Why D and not D^2 here. Then quadratic: model as "knocking molecules out of the way", make sense of v^2 (one power to speed up the molecules, and one more power because we "sweep out" more volume per second as we move), and thus sensemake c = (1/2) c\_d rho(air) Area. Physics of c\_d: show typical values. Then, activity.

**Props/demos:** (none)

**Activities:** (Warn students to bring laptops in one week)

1) Started class with "use unit analysis to find the error". (Available within the Concept test files for this chapter) (~5 minutes)

2) Showed the generic 1st order ODE y' + P(x)y = Q(x), asked them to "classify" the ODE (order, linear? homogeneous? separable?) (~5 minutes)

3) Maine Tutorial "TtVDF-RP\_CUModified\_final". 3 pages, ~35-40 minutes for this. Over 2/3 said they were done at the end, timing seemed fine. I interrupted every 10 minutes and had class discussions (~2-3 min) to keep/bring everyone together, this worked out ok. Checkouts on Q1 and Q2 were good. Every group I went to near the end had gotten the sign wrong on the bv term for an upward moving object. This is clearly a "crux"! (But, I didn’t resolve it – told them to think about it, for next time)

**Concept tests**:

Which drag dominates microbiology? Sporting events? (We didn't click on these, but discussed. It was quick but useful)

**Lecture #** ( Tues Jan 31) **Solving motion with drag.**

**Coverage:** Taylor 2.2-2.4, my notes pp 22-26, start 27, including intro to sinh, cosh, tanh.  
 **Summary.** Review Tutorial (Sign issues on **F** = -b**v** -c|v| **v**. This is hard! Takes some time) Derive terminal velocity, and discuss "scaling" with size (including hidden issue that mass also scales with size) Solve ODE for 1D linear case ("separation of variables") (Trick of integrating BOTH sides with proper limits, to avoid constant of integration) Talk about tau = m/b, idea of "natural scales" in ODE's. Discuss sketching, and qualitative features. Discuss integrating v(t) to get x(t) . Add g (signs again), sketch solution, qualitative discussion. Then, quadratic case - what changes? Introduce "hyperbolics", define/sketch/motivate.

**Props/demos:** The plot of "cat injuries vs height fallen" is quite funny. (About Terminal velocity, see clicker question powerpoints)

**Activities:** No Tutorial today - we changed Tutorials (so MMA Tutorial will be next class, and it’s different). I took some of the concept questions out of the MMA NDSolve Tutorial and made them into clicker questions for today.

**Concept tests**: Preclass is about “math-physics” connection, simple Newton’s law question framed in a slightly unusual way.

Next is about signs, take the time to let them discuss their reasoning! I let them first vote silently, it was very mixed, then talk. We heard strong clear arguments for the wrong answer, and the right answer. Nice! The argument is subtle, v\_y is signed, |v\_y| or v\_y^2 is not!

The "terminal velocity of big vs small object" was 50/50. Seems ok after my wrapup, but nice discussion opportunity about scaling. The followup "cat plot" is pretty comic, they laughed a lot.

Three "sketching" CTs, these are definitely worthwhile. Make them sketch FIRST before showing them the outcomes (I animated my slides for this). Many features to discuss. The last points out that tanh is maybe not quite so scary as they think.

**Lecture #** (Thurs Feb 2) **Drag in 2D. Taylor series & Numerical methods.**

**Coverage:** Almost Finishing Taylor Ch 2 (we’ll skip motion in Mag field), Start with Projectile motion, then Taylor/Maclaurin expansion, end with numerical methods.

**Summary:** Start with ODEs for a projectile on board, for general linear+quadratic. (e.g. Fy = my’’ (t) = –mg –bv\_y – c |v| v\_y) Wrote SOLN for x(t) and y(t) on board to start. (x and y equations have been solved!) Elimate t, get y(x) (discuss significance and usefulness ), then ln(1-x/v0x tau), discuss and motivate idea of expansion. I wrote “analytic, approximate, numerical” on board, and spent a few minutes discussing how physics involves all 3 (though it may seem like the focus is on “exact solutions”, that’s rarely how physics or engineering goes in practice). This term, approximation includes Taylor (and later Fourier) expansions. Introduce Taylor series - expanding arbitrary function, the idea of finding f(x) if you know a lot about f(x0). Work out coefficients, both about x=0, and x=x0. Emphasize the idea, of starting at x0 and approximating the function as a power series, simpler than many other functions. Discuss the series for cos(x), let them shout out the terms. Analytic functions, and convergence. Series for cos, sin, e^x are in book- memorize them! What about ln(x)? (They couldn’t answer why not, but when I sketched it they saw it. I pointed out ln(1+x) IS in text! Briefly returned to projectile problem but ran out of time. Motivated the numerical Tutorial – let’s open the hood of NDSolve! General idea of *how* numerical solutions work. (Ask, “how many have programmed”)

**Props/demos:** Students needed to bring laptops today.

**Activities:** When finishing up Taylor series q’s, there’s an activity to do some sketches in their notebooks (it’s described in the powerpoints) Then, Used my MMA notebook, 4MMA\_Taylor\_examples.ma (In the end, go to n=40 to see the whole fn)

**New MMA Tutorial today.** 30 minutes was adequate (barely) Went great, lots of good questions, from simple “Mathematica doesn’t work” to questions about coding, notation (i++, arrays), when do loops increment, what’s the meaning of x = x+v \*dt. (I interrupted twice, once after most were done with page 1 to summarize, again after p.2, and did a 2 minute wrapup at end. The x=x+vdt formula requires thought – it’s not MATH, it’s “updating”, and makes more sense if you think of it as x(new) = x(old) +v dt, that’s just v = dx/dt! Discussed the idea here – we’re “stepping across”, solving a simple 1st order ODE on step at a time.

**Concept tests**: Preclass – proved to be a little too easy.

- String of Taylor series questions. They did fine, quickly moved to just having the shout out answers rather than clicking.

- Convergence/analytic function questions. (cos(3pi) was easy). Emphasize x needn't be <1 for convergence, and that N! beats x^n sooner or later.

- The "string of semicircles" CT is great, opens up good intuitive discussion about what analytic \*means\*.

**Lecture #** (Tues Feb 7) **Conservation of momentum, Center of Mass.**

**Coverage:** Lecture notes pp. 37-41, and 44-46. (Saving 42-43 for next class) This is Taylor 1.5, 3.1, and starting 3.3

**Summary:** Start with e^x, they know then. Then, let THEM work out Sqrt[1+x] about x=1,: Binomial expansion - I gave them time, let them work it out, then did it on board. Some discussion about why you cannot “expand” Sqrt[x] for x near 0.

- CT about y vs x plot - did click (just for “intuition”) worked out how the Taylor expansion is identical to the ideal, out to x^2 terms.

Derive and discuss "double sum" formula/proof (Taylor 1.27) to prove Fnet,ext = dP/dt for systems (i.e. internal forces cancel). Write out the terms to visually represent the double sum (see lecture notes, p39) which is hard/abstract for them. Conservation of momentum. When is it useful? (e.g. explosions, collisions). Car crash - is p conserved starting from "car behind at v, car ahead stopped" to "massive wreck at rest in middle of intersection". (No, Friction = external force. Police use skid distance to trace back to right after the crash). Why is conservation of P ok for SHORT time after the crash (Delta P = F delta t) Fermi/neutrino (talk about nuclear physics, n -> p + e + nubar) (FORGOT to Mention laser cooling.)Do the UW Clicker question about whether F=ma for rotating body – it’s about 50% correct, then do Derivation of CM, motivate definition, Rcm, Ptotal = M(total) vcm, and Fnet, ext= M(total) acm. THEN return to the UW clicker question. Do "Hanging rod" CT, and various other CM CT’s CM calculation in continuous m case , just a brief introduction. Talked about symmetry, breaking up the object into little chunks. Symmetric example (hockey puck) then "half hockey puck". (Qualitative only today)

**Props/demos:** Used my body to talk about CM of a macro object. Grabbed a hammer from demo room, as well as the big flat oddly-shaped object that’s useful to show where the CM is located.

**Activities: None**

**Concept tests**: double sum was straight from text, they did ok but were a little uncomfortable with the notation, seemed happy to see it laid out

Conservation of p in beta decay ended up largely correct, but took them a long time, there was a LOT of discussion about it. (I told 'em cos(45)= .7)

**Newton's law for rigid rod was very split, and I did NOT discuss it right away** - took about 10 minutes of lecture to get back to it (during which I derive F\_net^ext = m a\_cm. On second vote, there was strong shift to correct answer, but many questions, good discussion! What about the rotation, why DOESN't it "use up" some of the force, what about energy, etc. (Discussion of motion of high-divers fits here)

No problems on qualitative CM of sphere, quick and near unanimous.

My little "equilateral" triangle CT was easy for them. The UW clicker question about a hanging rod with uneven mass distribution had a very split vote too, lots of discussion. (But most of the discussion was for the correct answer) No time to discuss (or even really process) the last CT about “origin indepdence).

**Lecture #** (Thur Feb 9) **Relative motion and rockets.**

**Coverage:** More on CM Taylor 3.2 (rockets) and start of 3.4 (angular P). Lecture notes p. 42-43, then pp. 47-54 (I’m skipping angular momentum this year). (no time for torque this pass)  
 **Summary: (Forgot to mention laser cooling from last time?)** Recap CM. (Discuss meaning of notation, "vector under the integral sign", triple integral (or, double?) Why, when we’re looking for “x\_cm” do we still need a triple integral? Should use Cartesian unit vectors to write 3 equations (for x, y, z\_cm), but then can use spherical/cylindrical variables for the triple integral. Do example (half puck). Clicker question: what depends on origin (R, v, a?) (Discuss “kinesthetically”, then formally, write R = R’ + R\_frames and take derivatives. Then discuss “relative velocity" formula (needed for rockets) Clicker sequence – lots of sensemaking of signs, taking limit of v=0 and v=vexh: (DEMOs + videos) Derive the rocket equation (Conserve P, talk about sign of dm!!, get arrows right, talk about direction of v(fuel), discard dm\*dv term,etc) Divide by d/dt to think about THAT version of the ODE, Physics of Delta V (you need to dump lots of mass to get good acceleration). Shuttle images, told them some stats (with discussion/sensemaking)

**Props/demos:** 2 large "physicist skateboard" (carts), ductaped together. (Walked across, to show "addition of velocity") Use with 2 medicine balls to show rockets (and Delta v is bigger for 2nd throw). CO2/fireextinguisher to rocket across room - it's *loud*, WARN students to cover their ears (need ear protectors, + spotter)

Another video to start class (to review “Fnet = m a\_cm”),

TwoPucksDemo\_1min48.mov

Which comes from <http://matterandinteractions.org/Content/Materials/materials.html>

(two pucks)

Later, when doing velocity addition:

video\_vectorAddition\_1minto0\_then\_300.mp4 , extracted from

ww.youtube.com/watch?v=yPHoUbCNPX8&feature=player\_embedded#at=28

(It's quite funny, start at ~minute 1 for ~30 seconds, and then skip right to end)

**Concept tests**: - What depends on origin, R, v, a? Some questions about semantics: does "origin" mean "coord system" (flip it, switch to non-inertial frame, etc).

-String of CTs about computing relative velocity. The sequence was meant to start off \*fast\* and \*easy\*. (They argued that going from r to v equation is just d/dt. No prob interpreting v0) (I got asked about relativity, but (v/c)^2 is about 10^-16 ) -- -- Rockets v's, they fell apart. I think it’s good – the distribution is so even, it’s humorous. See notes in powerpoints, I explicitly rewrote all the symbols in my “v a/b” notation, and let them try again. They went from 25% to 65% correct. +

**Lecture #** (Tues Feb 14) **Rockets (wrapup)** and **Angular momentum,**

**Coverage:** Taylor 3.2 and 3.4 Lecture notes pp 51-56 (Ch 3 wrapup)

**Summary** Start with review of rocket story. Let them struggle with the relation between velocities, and motivate WHY we “replace” v\_fuel in the equation with (vexh-v\_rocket). Get to rocket equation and discussion – show and discuss Shuttle video, talk about shuttle info . (See ppt for talking points)

Briefly, introduce L and torque, derive conservation of L, review cross product properties. Angular momentum: We had a string of concept tests about conservation of angular momentum, and frame dependence. This brought out lots of good discussion and confusions. I got myself confused on the last concept test, but that proved nice in class – see ppt slide for more details.

**Props/demos:** Chocolates (Halloween candy) for the chocolate pretzel activity.

**Activities:** 5line\_integral\_activity.docx (Gave 15 min, 25 would have been better) Very good activity, lots of questions, it wasn't easy for many - but they worked their way through it. Issues of units, of parametrizing, of meaning of line integral.

video\_STS-131\_Space ShuttleDiscovery.mp4

extracted from http://www.youtube.com/watch?v=\_NeCvBCZbC8

(from 30 seconds before launch to just over 2 min after launch, pause video to discuss commentary and numbers on screen) Talk about “Mach” and “G-force”. At

**Concept tests**: Reading questions- they're doing well, and I gave a little pep talk (this is day before exam)

See powerpoint, nice collection of questions today, with some generating long discussions.

DIdn’t get to discuss the ice skater (pulling arms in speeds them up), student asked how internal forces cannot cause CM motion, but internal energy can. Talked about where the energy came from for the skater - and for your car, where the force is friction, but the energy is from the engine!)

**Lecture #** (Thur Feb 16) **intro to work and energy. (Exam #1 tonight)**

**Coverage:** Taylor 4.1. (Boas 6.8) and notes starting Ch 4, pp. 57-61

**Summary:**

(Briefly Comment on L having richer physics than p (e.g. ice skaters, pulling their arms in, or astronauts turning themselves in empty space) . Introduce work and energy (Feynman lectures - energy is just "a scalar associated with systems that is conserved". Limitation of "energy is ability to do work". Define work, scalar nature and signs, then work-energy theorem formal proof. Define KE and motivate it from the derivation! Various examples, with and without friction. Briefly define conservative forces (for now, independent of path). Line integrals. Work an example, (discuss dr in Cartesian and "parametrizing" the curve.) Do the F = y xhat example. Let THEM sketch it for a few minutes before doing it for them. Walk through the “parabolic path” example (let THEM come up with the equation for y=1-x^2)

Wrapup: summarize the Tutorial activity

**Props/demos:** None

**Activities:** NEW! Used 5line\_integral\_activity\_mod (Gave over 20 min, 25-30would have been better) Very good activity, lots of questions, it wasn't easy for many - but they worked their way through it.

**Concept tests”** "In which situation is net work done" clicker question. I clarified that it's the "point object" approximation, but they SHOULD include friction. **Asked it twice, first before my lecture (with no real discussion), then again after.** Significant improvement, this worked quite well. Some issues about "net work" vs "work done by you", and whether direction changes matter, and still some distress about why/how PE enters into the picture.

**Lecture #** (Tues Feb 21) **Work, PE, and del.**

**Coverage:** Lecture notes pp. 62-64. (This is conservative force, and the relation of force to PE)

**ON Board**: r = (x,y) dr = (dx dy) wordk = integral(F dot dr), and work-energy theorem. Also, “conservative” and “non-conservative” (details filled in later)  
 **Summary:** Discuss exam results (and importance of HW) Review line integrals and work/work-energy story. Remind them mechanics of line integrals. Review the “uniform field/line integral around a loop” case (clicker q) Let class sketch a field **F** = **r**. Then, discuss formal check for “conservative” fields (show them the curl trick as determinant, do it here for them in detail. Also discuss the “paddlewheel” curl trick). Let them pick a path (this is an activity from last class that some didn’t get to). Discuss the various choices, DO the integral for each of the choices for explicit practice on the board. Then move to harder example f = (y, x) (Let them sketch it, it’s HARD for them!) Talk about it. Does it have a curl? (Looks curly, but work it out) Then polar coords, discuss formalism. Discuss the intuition of dr = dr rhat + r dphi phi-hat (a nice picture can really help here). Redo problem ( the curve is r=Sqrt[8], so dr=0) Define conservative forces, and define potential energy as – Work(from reference pt to r) Then, before going into any formalism, do Maine Tutorial on "contour maps", finish with quick wrapup of the “puzzle” questions after the Tutorial, ending with the review clicker question about sign of d(vec r) = dr rhat. (Ended class with question about Conservative forces – need to start there next time)

**Props/demos:** None, except for ask students to sketch the 2 fields (x,y) and (y,x)

**Activities:** 6TtCFP\_Tutorial.doc. :40 - :72. This was enough time for most groups to finish, or at least be on the last page. (54% done, 31% still on last page) (Maybe a dozen students had finished and were working on a "thinker" puzzle we had up on the screen. ) Most students get the "conservation of energy gives same KE for the boulders" part, but struggled just a moment when asked if this made sense/seemed consistent with their intuition that steeper slopes should roll boulders faster! Students struggled a bit with sign and meaning of partial U/partial x. One sharp group was worried about Fnet=-grad U when there's a non-conservative force involved (normal force), but we didn't "go there" with the class as a whole.

**Concept tests**: (Started with "how was the exam" question(

The "is d**L** = +dr rhat or -dr rhat" question was 61%, better than last year - I myself find this one difficult, and have to think really hard about it. In the end, it's well worth discussing. (Consider a "pie-shaped" integral of this same vector function. Argue (without proof here) that the full loop integral should vanish, and in this way simply showthat you do NOT want to introduce that extra minus sign..) that seemed to help some students.

**Lecture #** (Thur Feb 23) **Conservative forces and curl.**

**Coverage:** Lecture notes pp. 64-77 (Taylor 4.2-4.4) It’s a lot!   
 **Summary:** Review conservative forces. Define U(r). (Arbitrary starting point, motivate by conservation of mechanical energy. Discuss the minus sign).   
Example: F = -mg yhat, work out U=mgy.   
Formal argument that Delta E = Delta T + Delta U, and by relating Delta T to +W, and Delta U to -W, find Delta E=0. (Discuss multiple forces, and non-conservative work, with a derivation of Delta E(Mech) = W(nc) (1->2) (and discussion of fact that this doesn't violate conservation of energy, just conservation of *mechanical* energy) )

Next I did a fairly formal derivation: Given U, how do we find F? Work it out: dU = -F dot dr, interpret dU, discuss what partial derivatives mean, discuss what F dot dr looks like, end with F = -grad U. I lost them, this was a waste of lecture! In the future I might minimize the “formal math proof” nature here, and just make more physical sense of du = -grad u dot dr, and du = -F dot dr. (Or rethink how to make this more interactive) Next: Grad f is perp to equipotentials. Invoke Stoke's theorem (without proof or discussion) to get curl f=0 <=> integral around a closed loop =0. Use THIS to prove work is independent of path.

Things I didn’t get to this year: Should discuss grad U at a minimum. (directional derivative,). Also, sSince curl(grad U)=0 (will be on homework), then curl of conservative force vanishes.

**Props/demos:** Road runner cartoon for fun wn.com/Wile\_E\_Coyote (#9) start at :31 seconds: <http://www.youtube.com/watch?v=Jnj8mc04r9E&noredirect=1>

I built a MMA notebook to take them through Curls, this worked quite well. At each step, I had THEM take a minute to draw the field in their notes, then predict the curl.

See 6b\_curl\_SJP\_Mod\_forclass

**Activities:** Near end - students solved the puzzle we had up while they did Tutorial last time: sketch equipotentials for 3 different force fields. (in ppt/clicker files)

Followed up by non-clicker discussion questions: figure out if curl=0 (and why), and what path can you find to give non-zero line integrals (no clicks) (The interesting y=0 case in the middle figure is worth talking about). (Can show the middle graph has no equipotentials by working backwards: if F=f(y)xhat, then U must be -f(y)x, but then there should be a y component of F!)

**Concept tests**: -

Start with the "which of these is conservative" question – this year it was a little hard for them, Getting THEM to articulate why e.g. a velocity INdependent friction is still non-conservative, and why normal force (which is path independent) is non-conservative, seemed productive.

- Find U given E0 ihat. Easy enough, good to talk about signs (not hard for them)

- PE of pendulum -make students work it out first instead of guessing

- Which way is grad(f)? (It was split on direction when they did it alone)

**Lecture #** (Tues Feb 28) **Stability conditions, U in 1-dimension.**

**Coverage:** My notes pp. 78-84, Taylor 4.6 and 4.7

ON Board: U(r) = -Integral(ro to r) F(r’) d r’. (E.g. PE from gravity is mgz)

And F(r) = -gradU(r)   
 **Summary:** Review potential,curl and conservative forces. Energy in 1D: simpler, but still useful. Any F(x) is conservative in 1D (proof). Fx=-dU/dx is easy to visualize - spring example, (signs, stability, "rolling downhill") Stability conditions. HCl molecule ex: (strong repulsion at small x, U=0 at infinity, binding, etc). Forbidden regions, turning points, graphical representation of T = "difference of E-U". Let them work out pully example (activity below). Discuss choice of U=0, multiple solutions, stability. Did not get to the m=M special case. ( "instability").

**Props/demos:** Angular acceleration demonstrator (the pulley demo/stability example in lecture notes) (Took just a couple of minutes, follows Activity. Did not have time to ask them why it doesn't behave exactly as our problem (largely because, mass on rim is not pointlike, and not at r=R) Show stable equilib, and unstable equilib.

**Activities:** 1st CT we used whiteboards to come up with formula for vector function (and, taking curl) When we got to the "pully with fixed M on rim, and hanging m on other side" example, I had groups doing it all on whiteboards. Took ~5 minutes.

**Concept tests**: - New starting questions about gradient and zero of PE. Review question asked about Curl. Good discussion, see notes in clicker file (had them come up with as many different ways as they good to see that curl was nonzero)

- Walked through "Review" PPT slide, which isn't a clicker question, but allows for many rhetorical/class call out questions. They were a little quiet on this, though.

-Where is F=0? This was easy, but it led to nice discussion about stability, and significance of the zero (none!) and visualization of motion from U(x)

- The "Fig 4.12 of Taylor" one – spend some time eliciting “what’s the physics, why is this so generic?” I gave them several minutes to discuss the motion BEFORE giving the answer choices, this was very productive. Good long discussion about whether it will return (since, after all, the force is leftward after you start heading past b), and why not, and how to "read off" T from the graph. Next slide - talked about the significance of negative energy, defined turning points, forbidden regions (mentioned QM)

- "pulley example", instead had them do it on whiteboards. Deal with signs, and zeros. They seemed to have fewer issues this year – I didn’t hear as much discussion about the "1" in "1-cos(phi)" (it's just there to make U(0)=0)

**Lecture #** (Thur Mar 1) **Newtonian Gravity.**

**Coverage:** This is not in Taylor. My lecture notes, "Gravity 1-4" (This is mostly from Marion and Thornton CH 5 on gravity). On board, just F = -grad(U)

**Summary: (Review from last time:** Work "Taylor expansion of U(x) about x0" in detail, explained why so much physics is the Harmonic Oscillator.) History/physics culture (Newton, the fundamental forces, the weakness of gravity, its ongoing role in exp'tl and theoretical physics.) Define g-field (as versus force), Summarize the “3 ways to find gfield” (direct summation -> integration, using a potential, (next time!) or Gauss’ law (wait till after the clicker question where they come up with this on their own). Set up the triple integral (and then, the sextuple integral) needed to compute force with extended masses. Compare to formula for R(CM) (the symbol "vector r" in the integral has slightly different interpretation, depending on where your origin is) Discuss Symmetry. E.g. flat disc, consider "little man" observer (laws of physics need to be invariant under transformations, so symmetry can eliminate certain possibilities). E.g. no "g\_x" or "g\_y" above the center of the disc, PROVEN by rotating little man by 180 degrees. Cylindrical coordinates (If you change your z axis to point along the line, cylindrical is a win, because g points in the rhat direction, and depends only on r) Finite disk problem: general solution, then simplify to symmetry line. Work it out, with their help. (Didn’t quite finish)

When they “derive” Gauss - Discuss how you USE gauss law to get g (always true, but only useful when you can pull g out of integral). Analogy: integral (0 to 10) of f(x) dx = 17. Can you "extract" f(x)? (No. But, what if you know f(x) is constant?)

**Props/demos:** (Didn't bring a "little man", but should have!)

**Activities:** white boards on which they sketched v(x) for the roller coaster review, and later come up with Gauss’ law for gravity, on their own.

**Concept tests**: "Roller coast" review question. Straightforward.

-Newton's gravity. Worth discussing sign, and rhat = rvec/r (and why that's handy)

- Pentagonal symmetry. had them vote silently. It was 80% correct already, I was a little surprised but not completely. Their ability to articulate why is probably not great, but we used this as a warmup rather than talking it thru.

- "Little man" above wire. This raised good discussion, students are resistant to making the argument by invariance/contradiction, rather than by "matching patches". I motivated it (e.g. when they get to magnetism! Or, a NEW force of nature, we don’t have to know what it is to make arguments about what it can NOT do!)

- "Little man", dependence rather than direction. Good discussion, the argument is a little different, need to be very explicit (arguing by "matching patches" can a good job of telling you about direction, but not as well about functional depenendence)

Returned to the pentagon and had them articulate their reasoning - this went very smoothly, a normally quiet student in the back did a great job here.

-2 calculational questions in the middle of the "gravity from a disc" derivation. These break up the calculus nicely. No big problems, helps to discuss/remind the notation. **Lecture #** (Tues Mar 6) **Gravity and Gauss' law.**

**Coverage:**. Lecture notes gravity 4-13. (This is mostly from Marion and Thornton CH 5 on gravity). On board: Gauss, g from m at origin, and g from –grad(Phi)

**Summary:** Finish the example problem. (Mention sign Issue: Sqrt[z^2]=+z if z>0. ) Discuss \*checks\* of units, sign, and limits. (Note that z->0 limit is surprising, it's NOT zero, it's a constant. Don’t cover in detail, but student picked up on it and pointed out the discontinuity) Discuss PE(grav), signs, sketch U(r). Review integral formula for gravity, "warning" about what r means (draw a picture, ask if rhat/r^2 referred to position vector of M2 from origin. It does not!) Return to formula from last time (g above center of disc), Taylor expand (what is the "small parameter" if z is large?) Discuss difference between saying "it goes to zero" and "it goes to zero like 1/z^2". Constants out front give -GM/z^2! As z->0, it goes to a constant (like near earth) and is opposite sign "under" the disc . (Could sketch g(z) for all z, there's a discontinuity at z=0. ) Gauss' law activity (below) Gave them about 20 minutes, they were mostly (80%) done with pages 1 and 2, but not with pp 3-4. End of class (~10 minutes) was a rushed first pass at using Gauss’ law (clicker question sequence)

**Props/demos:** (none)

**Activities:** 6d\_gravitation\_tutorial.pdf We spent 20 minutes on this. (21% done with p. 1 only. 22% done with p. 2 only. 16% done with p. 3 40% done with all 4 pages) They took a surprisingly lot of time on p.1! Many groups didn’t quite see where there density formula came from, and many were content with the volume element in spherical but could not derive it or explain it. Could spend a little more time on this – I felt rushed because the Homework on Thursday has a bunch of Gauss’ law questions, it’s too bad.

**Concept tests**: See comments in clicker question file!

**Lecture #** (Thur Mar 8)  **Gravitational PE, and intro to Complex numbers**

**Coverage:** Wrap up Gravity (about 30 minutes), notes p 14-17, start Oscillations, complex #'s (Taylor Ch 2.6, Ch 5.1 and part of 52, pp 1-165), notes pp. 1-4, parts of Boas Ch 2) On board: Gauss, g from m at origin, and g from –grad(Phi)

**Summary:**  Start with review of Gauss, Solve g above sheet (What surface to choose, why does a sphere FAIL, how did you "know" g is constant, etc) Then, g around spherically symmetric distribution, work out in detail why g = GM(enc)/r^2.. Discuss nonuniform case (first in r, then in theta or phi) Galaxy rotation curve and dark matter. ("Machos" and "Wimps”) See CT for how I presented this, it was kind of fun. Force inside a hollow sphere. (Why \*don't\* get the (possibly intuitive) result that near the edge it might point towards the wall?) Discontinuity of F at the shell. Graph of U(r), continuity of U ("prove it"), F = -grad U. Spent some time discussing GRACE satellite and gravity anomaly, showing/discussion geophysics of plots (and movie).

(Dang, Still have NOT discussed gradient in spherical coordinates (why it's NOT dU/dtheta (theta-hat).) Formulas in flyleaf, and fundamental idea, dU = grad U dot dr, but dr is not just dr rhat)

Introduce oscillations, "mass on a spring" as a model for many systems. Taylor expand "Generic U(r)" to find U = 1/2 c x^2, and F=-kx, near equilibria Set up ODE, (2nd order linear, natural frequency) Informal definition of "independence" of solutions. Demonstrate cos(omega t) works, as does exp[i omega t]. Segue to complex numbers: Euler's theorem. Derive it by Taylor expansion, (Taylor expand sin and cos, sense-make missing terms. )

**Props/demos:** the “spherical chalkboard”, dark matter slide and gravity anomaly slides (in ppt), movie file for Grace data Mass on spring, Used whiteboards to sketch F(r) and U(r). See greenland\_movie\_whar.mpeg

**Activities: White boards with sketching activities – see ppt.** The sketch of U(r) for a thin spherical mass shell is very interesting, good discussion and questions.

**Concept tests**: With stellar rotation curves, asked them, given the data, what would THEY conclude as astrophysicists? (Would they throw out Newton's law?)

**Lecture #** (Tues Mar 13) **SHM**

**Coverage:** Taylor 5.2, lecture notes pp. Osc 5-12

On board: ODE for mass on spring (and more generically), with solution, and omega.

**Summary:** (Discuss div and curl in spherical cords at start of class, since I kept forgetting to mention this before!) Start with review of SHM, and generality of the result. Complex #s ("ordered pair in complex plane". PHYSICS of complex numbers (real measurements are always real, but the math is often simplified). Introduce "polar coords", and connect back to complex plane representation. Connect to oscillations (cos(omega t) = real part) Showed how you can combine exponentials to get cos (mention how this proves useful when we need to integrate cos[ax] cos[bx] dx.) ) Review 2 forms of general soln of SHM. Connect coefficients of exp[+/- i w t], to coeff’s of cos and sin[wt]. Which are real, which complex? (Discuss how, if you have 2 complex coefficients, how is that still only 2 undetermined constants) z''=-w^2 z has complex solutions, and Re(z) is ALSO a solution (x(t) and v(t) must be real.) Introduce A cos(wt-delta) notation, related A and delta to B1 and B2 through cos(a-b) = cos cos + sin sin. Discuss reasons for picking these different general solutions (e.g. if you have v(0)=0, or x(0)=0, or if you know the amplitude) Introduce “energy” method, U = ½ kx^2, remind them of the formula and the idea of v(x) rather than v(t).

**Props/demos:** Mass on spring, with several masses

**Activities:** 1) 7aa\_complex.pptx Use Whiteboards, let them draw points in complex plane (See ppt file, also) This went quickly (couple of minutes) but was worthwhile, and not everyone got it as fast as I expected, this comes just after the first clicker question on this topic.

1) PPT 7a\_SHM.pptx (also in CT ppt file), whiteboards. . (Given some data about motion, construct x(t) in the form x0 + A cos(wt+phi). Took over 5 minutes, and they weren't fully correct. (See concept test file) The idea of introducing an offset to x was not hard this time around, the phase part was hard.

**Concept tests**: Students didn't immediately catch on to how Euler "shows" you how to plot a point in the complex plane, I had to be very explicit about that after the first clicker question about it. (Then mentioned my freshman demo of oscillating object and its shadow which exhibits SHM)

1) Preclass about dependence of period on k. Pretty Quick, but not unanimous – did a revote which helped. Breeze through dependence on mass (scaling), and on amplitude. No problems here. (Good discussion from class about WHY amplitude doesn’t increase period)

3) Sign of phase difference when one cos curve leads another. Lots of discussion, lot of confusion, students kept asking questions about this. One student was convinced I had it backwards. Possibly some issue about the explicit minus sign in the formula I use (but I was consistent with Taylor's notation)

**Lecture #** (Thur Mar 15) **Phase Space, 2D SHM,**

**Coverage:** Taylor 5.3 & 5.4 (Boas 8.5) , my notes p 12-14

**Summary:** Review phase space, do activity. 2D oscillators: ODE and general solution: A cos(w t-d), general sol'n. Physical demo (mention "coupled oscillators") Introduce x-y plot (not phase space, it's physical!). Discuss convention of letting one delta=0. Define Isotropic vs anisotropic.

Next - Damped 1D oscillator. ODE, define beta. Discuss/define linear operators, trick of auxiliary equation. Mention "double root" solution. Wronskian: work out e^{+/- i w t} case (independent, except if w=0!)

**Props/demos:** Big physical demo - mass on 4 springs (can show 2D motion). Let them debate what delta is when you start "at rest, x>0", then x-y plot if you release from rest at nonzero x,y.

Lissajous laser pointer toy.

Lissajous Mathematica sim (see below) .

Simple mass on spring to demonstrate aspects of damped SHM.

**Activities: -** 2) 7TtPhase-SHM\_ADM.doc (intermediate mech Tutorial). Gave 30 minutes, they were largely close to done, but not quite. Good activity, this is "introduction to phase space", plotting xdot vs x for SHM. Many groups chose to match "x" in ellipse with "xdot", and "y" with "x", which is not so much wrong as confusing (and inconsistent with my xdot vs x plot scheme) Many struggled with "interpreting" a and b, but could come up with it with some prodding. Many were not looking at the numbers on the back side (e.g. omega=1.5, or energy is 4x). Didn't see anyone discussing the concept test I had up during the Tutorial.

Ppt animated slide with review of last Tutorial and 2 leading q's about phase space led to long discussions and good student responses.

- Ppt slide with guiding questions from Tutorial (about orthogonality of ellipse and axes) got many student responses, good math/physics connections.

- 8\_Lissajous mathematica notebook was used to spur conversation, and demonstrate various answers to questions. Let them predict changes as change phase, amplitude, w. (Also run as "animation", fun! Discuss how this models an oscilloscope trace) (Show little Lissajous laser pointer here, too)

**Concept tests**: (See ppt file for more details).

Preclass: "k on one side, 2k on the other" question. Good level of interest/activity, many seemed to have the procedure down, but many got it wrong.

Is CCW phase diagram physically possible? Led to nice student discussion and comments. Which path has larger k. Good response about physics of "stiff spring"

#3 - isotropic 2D oscillator with changed initial conditions. No problems, decent discussion, student reasoning was clear, they seemed to be getting this. (I used the 2D "prop" here. Discussed (w/o clicking) the followup about frequency unchanged)

#4 - Lissajous figure, frequency ratio. Good discussion, Follow with MMA, ask them to predict, then CT (MMA gets a very different shape, a parabola, until we fix up the phases!) Discuss (w/0 clicking) the followup question about the relative amplitudes

Rest are ct's without clicking - signs in damped ODE, solving several auxiliary equations (I regret not clicking on the one which is in fact SHM)

**Lecture #** (Tues Mar 20) **Damped SHM, critical damping**

**Coverage:** Taylor 5.4, lecture notes pp. Osc 15-23   
 **Summary:** Intro to Damped ODE. Start from SHM ODE, add damping conceptually, have them think about signs and physics. Work out general solution method (two roots, ancillary equation, also think about it as “operator method”, where D is d/dt, but you treat it algebraically). Have them connect physics (drag) to math (ODE), e.g. thinking about whether the period changes, and where the maximum of velocity occurs. Get "3 cases" on the board (with terminology). and physics of all 3 cases. Limit of beta->0. Plot underdamped case (define w1) Discuss e^(-beta t) envelope. Overdamping: had them deduce both roots<0 (and physics agrees) Sketch x(t) for underdamped

**Props/demos:** Mass on spring, with a cardboard card (I used an LA flyer) to add damping. (Also, showed Lissajous toy from last class)

**Activities:** Tutorial: 9TtPhase-DHM-REPmod.doc (~20 minutes for this was not enough, last year was 35. This year, 30 would have done it)

This was challenging, people worked hard, good participation and questions.

**Concept tests**: See ppt for more notes. Lots of discussion on todays questions!

- How does |Fnet| compare when you add damping? (No problems here, but resistance to drawing Free body diagram)

- How does period change when you add damping? (Lots of debate, split vote when it was silent, not a lot of change when they could talk!) See ppt for notes of student ideas/comments, it was quite interesting. This led us to a mathematical solution, see above.

- Where does vmax occur? Again, very big split and good discussion (I had a hint which I didn’t have time to pop up – last year it improved the answer pattern somewhat) Loud discussion.

- Last question was up during the whole half hour of the Tutorial. They did fine.

**Lecture #** (Thurs Mar22) **Damped HM, and exam review. (Notes pp. 21-23 again)** (**Note: midterm Exam #2 tonight).**

**Summary:** . Review, sketch/discuss v(t) too. MMA activity - have them call out predictions of what would happen as b (beta) increases, and v0 increases. Critical damping to overdamping. Overdamping refers to beta, so it's damping SPEED (not position, per se. Some students were confused on the preflight about why overdamping doesn't "damp faster".) What happens when v0 is not zero? (MMA notebook lets you investigate that, including negative v0 case). Wrap up: the "approach to zero" for the critical oscillator. Look at (C1+C2t) exp^[-beta t] form of critical damping, and discussed whether this could ever cross zero (yes, once) and what physically makes that happen (v towards origin). Showed it on MMA.

End with 30+ minutes of exam review (clicker questions)

**Props/demos:** Mathematica notebook: 9damped\_SHM.nb

MMA notebook allows you to "zoom in" and talk about "ringing", and velocity, and energy

**Concept Tests:**

Answered q about physics of critical damping (they came up w. their own physical ex's) One student argued cars should be slightly overdamped, he might be right…

* I showed a nonsensical phase space diagram to remind them of such diagrams
* See PPT file for more notes.

(SPRING BREAK next week!)

**Lecture #** (Tues Apr 3) **Driven oscillators**

**Coverage:** Taylor 5.5, notes pp. 24-26

On board: the homogeneous ODE>

**Summary:** Review damped oscillator (10 min). Also go over exam. (10 min) Intro to driven oscillators - some motivation of the physics, and many examples where this matters (see ppt slide) Classify the ODE. Find particular and Homogeneous solutions and add. Try to guess the particular solution. Example of constant drive, then exponential driver. (Didn’t really get to: What if you have sum of two drivers Beauty of linear equations! Can get to this later) Guess/derive solution for Bexp[c t] driving force, then let c become i w, so we can have sinusoidal drivers (discuss taking Re or Im parts) Rework the equation for the response with this case. Wrapup - review formula, get to x(partic) = A E^[I w t], and plugging in gives A(-w^2 + 2 beta I w + w0^2) = f0. This is it after long times! Briefly talk about writing the response as A e^(-delta) E^[i w t]. Then 30 minutes for Tutorial.

**Props/demos:** Mass on spring to talk about driving. (Wiggle it fast, slow, and on resonance)

**Activities:** Tutorial 10TtFHM. Seems like a productive tutorial. They were not done at the end, needed more than 25 minutes I gave them, though 10% did finish. It's a good Tutorial. I liked to ask them right at the start what "Steady state" really means (how long do you have to wait to get to it?). There's also some subtlety on Q1 about energy conservation - (I claim energy is conserved from period to period, but NOT within a period if you're off resonance) I spent much of the time working with people on page 1. Some questions about what "leading" or "lagging" means, and how to sketch cos curves with a phase. Needs more time - we added 25 minutes (!) next class period.

**Concept tests**: (See ppt). Started with exam question to show M is not rho\*V (this is not trivial for them ) then simple review of x, v, and a graphs (easy question, but decent discussion)

Then an ODE where they were asked for a particular solution – they did ok, and I felt students "got it" when the answer was revealed/discussed, that you CAN just guess, and it can be simple! So I like this one.

Later, an ODE requiring full solution with BC's. It's tricky – they had whiteboards and I gave them a couple minutes to get started. I think that’s very productive. This one is easier just to "reverse engineer", so after some time I suggested they try that - looking at the 3 choices and picking the right one. That seems like it worked well, discussion was better (and more correct) than last year.

**Lecture #** (Thurs Apr 5). **Driven damped HM and Resonance**

**Coverage:** Taylor 5.6,lecture notes Osc 27-37

**Summary:** Start with review and discussionof general solution to driven system (1st clicker question provides a nice center for this) Talk about physical interpretation of amplitude A, derive the complex coefficient of exp[I wt] Finish Tutorial from last time (with a "filler" for those who finished already) (25 min) Review/summary : resonance, max power input, F and v are thus in phase, F and x are thus 90 degree out of phase) Formalism: R(t) = f0Exp[i w t], particular soln x=D Exp[i wt], with D=f0/(w0^2-w^2+i 2 beta w) Hard to interpret, nicer if write as D = Aexp[-i delta]. If A=f/a+ib, then |A| = f/Sqrt[a^2+b^2]

Did NOT get to working out Delta, ran out of time!

**Props/demos:** (none)

**Activities:** 11\_TutorialTfFHM\_followup.docx (+ need spares of Tutorial 10TtFHM which is what most people are finishing.)

Started by letting them finish the Tutorial from last time. We gave them 25 minutes, and still everyone was not done. It seemed very worthwhile, good conversations around the room.

**Concept tests**: See ppt for more comments.

Limit of A^2 as w-> infinity (was a little too easy, but nice math<-> physics connectins)   
Limit as beta-> 0. (subtle trick, otherwise no problem)   
What is shape of A curve? (Again, this was pretty easy for them. I think the new images may help here)   
What happens to shape if you fix w and vary beta? (Hard, tricky, got some good discussion here)

**Lecture #** (Tues Apr 10) **Ending Driven Oscillators, intro to Fourier series**

**Coverage:**. Taylor 5.7 and 5.8, My notes Osc 28 (we hadn’t done that), 34-37 (wrapup), and start on my notes Four 1- 3

**On Board:** Our familiar ODE with f0 e^i w t on right, then general form of solution, and particular solution ce^i w t. (Leading to the equation for c)

**Summary:** Wrap up driven oscillations: Summary: general ODE with cos (or e^i w t) on right, solution as homogeneous (damped) + particular. Origin of constants (A\_trans and delta\_t arise from init conditions, A and delta arise from omega and beta) Numerically estimate wr for a radio oscillator. Some stuff we didn’t get to last class: Multiply by conjugate upstairs and down, rewrite as A(cos delta - i sin delta) to figure out tan (delta). Get the class to "call it out" as you write it. Relate back to the Tutorial (where w=w0, resonance, gives pi/2, where tan() blow up) and talk about the other familiar limits (w=0 and infinity) Reminder/discussion of the transients. exp[-beta t] has a timescale tau= 1/beta for fading away. RLC circuits (Kirchhoff's law) and the physical example (radio) where fixing w and varying w0 makes sense. Then, fix w0 and vary w - e.g. AMO physics, the atom has a natural w0, and you vary the laser frequency to hit resonance. Derivation of peak of A^2 when you hold w0 fixed. w(res) = Sqrt[w0^2-2 beta^2]. Did NOT Show trick (w0^2-w^2)=(w0-w)\*(w+w0) apprx (w0-w)\*2w0 to get Lorentzian shape near resonance, but did discuss FWHM =+/- beta. Discuss, define Q = w0/2beta. (Bigger beta, wider resonance) Mention Q(old clock)=100, Q(quartz) maybe 10000 (?) Q(cold atoms) = 1E15! Derive/mention Q tells you "tau/T".

Intro to *Fourier series*: history, motivate (in context of problem we’ve been solving, now we can have ANY periodic f(t) on right side, not just pure frequency), and talk about formula. Example function (periodic square pulses, like tapping a kid on a swing) Commonalities and differences, with Taylor expansion. When/how is Fourier method convenient? Many physics examples (see ppt slide), the idea of "encoding" functions in a (maybe small) set of coefficients. Make connection to previous unit - each fourier term in driver function generates a solution, and linearity => add it all up. Discuss Even and odd functions. Formula (without proof yet!) for coefficients. Analogous to those for Taylor coefficients, they give numbers, definited integrals, you can "always" compute them. End with FourierPhet sim, talk them through it, sketch out a few functions, and finally to listen.

**Props/demos:** - youtube breaking of wine glass video from Mythbusters. (Short version: http://www.youtube.com/watch?v=I4jdGf3RzCs)

Mass on spring (low damping, Q about 100) clicker question. Then watch it - e.g. if you wiggle it fast, which terms dominate in that expression? At first, it's the transient! How do you know? And, after a long time?

- Phet sim (with sound cord plugged in) for Fourier series. When get to generating tones, it's interesting to add many harmonics, then "flip" one of them on and off. The ear can clearly pick it out, both when adding AND removing it.

**Activities:** For radio question - led them through the numbers, gave them lots of time on their own to work on it. Importance of "simple arithmetic" without a calculator. When get to 100 MHz, ask "what does this number signify?" (Someone may think of FM radio) . Pointed out these Ex #'s are cheap, simple C and L values, this could be a high school crystal radio kit.

**Concept tests**:. Interpret formula for delta. (seemed fine, good discussion)

Review of Q (used the physical demo, asked them to estimate Q). Most got it, but I think there was a lot of confusion about how to be quantitative, how to really compute/estimate it.

Even/odd function questions -asked a couple (did them silent at first). They all *thought* they knew this, but it was not perfect scores, and was definitely worth the review. Mostly they got it from their own conversations.

**Lecture #** (Thurs Apr 12) **Fourier series.**

**Coverage:** Nearly Finishing Taylor Ch 5, this is lecture notes Fourier-4-6, and 11a.

On board: Fourier formula, omega=2Pi/T, and the formulas for a\_n’s and b\_n’s.

**Summary:** Review/reminder of even/odd, and fourier series. Spent ~30 minutes on the Tutorial (see below) Parallels between expanding a vector in basis vectors, and expanding a function in basis functions. What is an "inner product" for functions, how do you interpret coefficients, why do you "normalize" the integral (to make the basis functions orthonormal), essence of "Fourier trick" (dotting a vector with a unit vector tells you the coefficient!) Didn’t quite finish this sequence!

**Props/demos:** none

**Activities:** 12\_Tutorial-Fourier. Spent 25 minutes on this. This went well, seemed very worthwhile. Good questions, lots of issues came up that they could easily resolve (sketching, finding period, even/odd, average values, etc)

Walked through powerpoints for today. (See comments beneath those slides for more details. Tried to get them to generate as much of the "fourier sum as vector" analogy as possible. This is abstract and hard, but they seemed to keep with me)

**Concept tests**: See slides - we had a preview to start.

Student asked me about stereo equalizer during class what the knobs do (I described it as decreasing the value of the "bn's" for a range of frequencies.)

Very active class, lots of questions. These slides are super abstract for them, students are trying hard to make sense of this analogy of “Fourier series” and “expansion in an orthonormal basis”.

Several "orthogonality of sin(nwt) and sin(mwt)" questions. Many good questions – This time no one asked about how to deal with non-periodic drivers (which I would have talked through a little bit, talked about needing all frequencies, and wrote out the fourier transform formula. This will come up soon.)

Tutorial followup question was not perfect (but they may have rushed to answer without considering it at the very end, and many did not get to the end)

Finally a pair of questions about summing over Kronecker delta. (Again, they did fine, with good productive discussions).

**Lecture #** (Tues Apr 17) **Finish Fourier**, **Dirac Delta.**

**Coverage:** Lecture notes Fourier 7-18,

On board: Fourier formula, omega=2Pi/T, and the formulas for a\_n’s and b\_n’s.

**Summary:** Walk through 13fourier\_series mathematica code, interactively. Discussion of Gibbs phenomenon. Demo: C512 tuning fork, ran Raven Lite program, discussion. Finish up the “Fourier trick” derivation of our a\_n and b\_n formulas. Quick crude discussion of non-periodic functions, sketching out p 15 of my notes (without math). Introduce Dirac Delta. Motivation: model a non-periodic driver as short, square "taps". Model "unit area" functions, sketch, take limit of narrow width. Not a function, but so useful to physicists. (Ex: "charge density of an electron" in 1D) Definition (0 everywhere, infinite at x=0, unit integral) Ex: what is delta(x-2)? What's integral of f(x) delta(x-a?). (Did not get to Deriv of delta(kt)=1/|k| delta(t), (Have THEM shout out shorthand for 1/k when k>0, -1/k when k<0)

**Props/demos:** C512 aluminum Tuning fork and corresponding resonance box, and Raven Lite software. Practice with this in advance, it's tricky, need to think about how to "zoom" in both axes.

With Raven: Looked at the waveform of tuning fork. (or, in my case, my voice) Predict, then zoom in and count out period, it’s indeed .002 seconds! Then, looked at "fourier" graph, interpret, we saw the 512 line and the 1024 line very clearly. (With the resonance box, you can see LOTS of lines, the box has many resonances.)

13fourier\_series\_forclass.nb (mathematica) Go through this slowly, give them chances to contribute. Draw on board in parallel. E.g. why the a\_n's vanish (formally, by noting that even\*odd function is odd) and why every other b\_n vanishes for the example I had (it was another symmetry argument). Sketch leading sin on board, then visualize additional terms as "trimming" the function.

**Activities:** (none)

**Concept tests**: See powerpoints. Good starting q about frequency (easy), then let them intuit the terms. (Good responses here, not “clicked”)

Delta function definition, followed by long sequence getting at aspects of delta functions. After first, we called them out (I gave everyone about 5-10 seconds to think silently before we shouted it out). They "caught" the issue of needing integration limits to span delta.

The integral of delta(2-x) was hard for some (see .ppt notes too) . (Last year I got questions about "what if you have delta(sin(x))". Revisited when talking about delta(kx), I suggested "u-substition" (but the multiple zeros makes it a little subtler)

Ended with the units clicker question, which was hard for them. (No time)

**Lecture #** (Thurs Apr 19) **PDE's and Separation of variables.**

**Coverage: PDE notes: 1-8.**   
**Summary:** PDE's. Remind about ODE's. (one v'ble) What if you have multiple variables, and partials? Laplacian (reminder/discussion of notation). Didn't derive it: many contexts. Discuss x''=-omega^2 x (ODE, has a general solution, BC's or init conditions constrain it). PDE is different: BC's determine \*whether\* you can solve it, there is no "generic solution") More ex's: Poisson's, diffusion (temp, odor, neutrons), Physical motivation :rapidly varying odor (or temperature) leads to more rapid rate of change at a point. Schrod Eqn (similarity to diffusion) Wave equation. Physics - e.g. for a guitar string, strong curvature of the string leads to rapid time variations (high pitch). Derive heat diffusion (with their help): H depends on Delta T, dx, area. Minus sign on H ~ -dT/dx. "Heat capacity". Derive diffusion ODE. What does "steady state" do? Reduce to 1D and solved it (not a PDE, though) Set up/motivate separation of variables method,

**Props/demos:** None today. (Use table top as prop for our T(x,y) problem)

Used whiteboards a lot.

**Activities:** The discussion/derivation of diffusion equation was very participatory, I had them decide e.g. what H depends on, what you should name the (conduction) constant, why the minus sign on H~-dT/dt, units of heat capacity, etc.

Many "non-clicker" powerpoint questions (semi Tutorial like, with time and encouragement for small-group thinking, but kept everyone together. See powerpoints

**Concept tests**: Only 3 today (but lots of non-clicked activity questions). See PPT slides for pretty detailed notes about how I facilitated these conversations.

Pre-class: review sol'n of usual ODE's.

When solving the 1D version of Laplace, had them find soln to d^2T/dx^2=0 on whiteboards. Took a long wait time before I got a+bx.

**Lecture #** (Tues Apr 24) **Separation of Variables**

**Coverage: PDE 9-15 (roughly)**

**On board: The setup for separation of variables, down to the ODE’s.**   
 **Summary:** Continue/review method of separation of variables. set up a "prototypical problem" (2D, BC's on edges) Focus discussion on sign of separation constant, idea that if f(x)+g(y)=0, both are constants. Semi-infinite problem (T=0 on left and right) as a concrete example. Emphasize that the "c" in X(x) and Y(y) are the same number. Go through BC's one at a time. Superposition of solutions (linearity), idea that summing up generates a Fourier series. (Review how you find the coefficients, there was confusion about limits of integration and normalization constant, so we worked that out) Finding A\_n is not the end goal, still need to "sum it all up", those are just numbers (not the function T(x,y) we're looking for!)

Possibly End with intro to Fourier transforms, motivated by thinking about the "spectrum" of frequencies represented in the sum, and pointing out that if T gets bigger, you get a "denser" spectrum that starts closer and closer to omega=0.

**Props/demos:** None

A**ctivities:** Whiteboards several times today.

Tutorial: 14SepofVar.doc (about 15-20 minutes for this sequence)

Walking through the canonical "square plate" problem with commentary and feedback at each page (see concept test files)

Whiteboard activities: What are the general solutions of X’’(x)=+/- cX(x)?

What are the equations for the boundary conditions?

If boundary is sin(5 Pi x/L), what is the full solution?

**Concept tests**: See ppts’ for many more comments.

Tutorial-wrapup question was about how to characterize the "infinite set" of solutions you get (A sin(n pi x/L)) It's more than a little ambiguous.

Quicky on which variable gets sinusoidal solutions was fine.

The question near the end about what “two zero” BC's tell us got very mixed results, though "E, none of the above" turned out to be the right answer, which is always a bit mean.

**Lecture #** (Thurs Apr 26) **Finish Sep of v’bles** and start **Fourier transforms.**

**Coverage:** PDE 15-19, and then Fourier transforms, through page PDE 21

**Summary:** Continuing/reviewing/summarizing Sep of v’bles ideas. Work through the “rectangle” story in detail – one big crux is the step where discover you need to sum up, generating a Fourier series, for the “final boundary” condition. (Review how you find the coefficients, there was confusion about limits of integration and normalization constant, so we worked that out) Finding A\_n is not the end goal, still need to "sum it all up", those are just numbers (not the function T(x,y) we're looking for!) Show MMA notebook, talk through some properties of Laplaces equation (smooth, no local maxima) We spent some time talking about the curve, and also finished the story up by using symmetry arguments to solve the “four different” problems with 3 zero’s and one function on boundary, then superposing to get the fully general solution, and finally how to deal with 3D (or other PDE’s)

Last 15 minutes was an intro to Fourier transforms, motivated by thinking about the "spectrum" of frequencies represented in the sum, and pointing out that if T gets bigger, you get a "denser" spectrum that starts closer and closer to omega=0Review, motivate Fourier Xform (FT) . (Limit as T-> infinity)

**Props/demos:** Mathematica notebook 15\_Sep\_variables.nb

**Activities:** none today. Whiteboards were out, used only briefly during a clicker question.

**Concept tests**: These guided much of the lecture,

Question on Fourier's trick was itself a trick (only small fraction got it, I was mean here, I left out the sinh(n pi L/H) factor in all my answers). Beside this, the discussion was good, they argued about, and then mostly figured out, the otherwise correct Fourier answer.

Superposition question (dealing with multiple edges) - they did fine here, I pretty much gave the answer. I thought this was worthwhile though, discussion was good even though they almost all got it.

Fourier series - after an introduction, and motivation of going from "sum to integral", I asked about the integration variable. 25% are confused – need to talk about dummy variables

**Lecture #** (Tues May 1) **Fourier transform and posttest.**

**Coverage:** Lecture notes on PDE/Fourier transform. (to pp 24, roughly)

**Summary:** Review formalism of FT, do some “Even-odd” sensemaking, physical interpretation of g(omega), discussion of dummy variables and analogy /connection between Fourier series and Fourier transforms. got through a couple of clicker questions (which guided the lecture), but need lots of time for posttest.

**Props/demos:** None

**Activities:** Posttest starting at 12:55 (so they have 50 minutes

**Concept tests**: These guided much of the lecture,

starting with "what are we integrating over/what's a dummy variable", what even f(t) means for FT, (twice, and they needed this) both limits of sinc (before they sketched it), and two delta function review questions. See notes in ppt file.

Fourier series - after an introduction, and motivation of going from "sum to integral", I asked about the integration variable. 25% are confused - talked about dummy variables. (Not sure if they all got it even after my discussion, I think it was a good question to end with) This was too quick - need to go back from start next class.

**Lecture #** (Thurs May 3) **Finish Fourier Transforms, Method of Relaxation, Exam Review,**

**Coverage:** Finish lecture notes on PDE/Fourier transform. (p. 25 on)

**Summary:**  Making FT concrete by doing some integrals, first symmetry arguments, then evaluation of square pulse, then delta function, then Gaussian. Focus at end was on "width" of distributions. Sensemaking throughout, some connections to musical ideas, short laser pulses and at end QM (uncertainty principle) . Brief connection of Fourier transform to optics/slits, (see demo below), but didn’t go into detail. (Some details in my lecture notes, but didn’t seem important to go into on the last day)

Also discussed Alysia’s ppts on “Method of relaxation “ (for about 10 minutes), ending with a little mma notebook that I wrote to solve the same problem we had solved analytically. (A very course grid does a nice job, and fast. Many advantages – any shape, any bc’s, perhaps the main disadvantage is simply that it is not analytic!

Ended class with ~15 minutes of exam review by going through some clicker questions, this was fun and they seemed receptive.

**Props/demos:** Brought a laser pointer - first just to talk about the spot being nearly a delta function in frequency space, but then with some grating caps that generate cute little patterns, which are basically the 2D FT of the grating pattern. ( Didn't have enough time to really explain in detail, but showed it at the very end of class, they seemed amused.)

Went through 16b\_relaxationmethod\_laplace.nb mathematica notebook

**Activities** : Whiteboard: Let them do the integral to get the sinc function, and let them sketch the sinc solution in groups.

**Concept tests**: Several whiteboard activities today, ~5 minutes each (see ppt notes). Ended class with a blitz of 3-4 review questions, they seem to like that.