Taylor Ch 2: Projectiles (and drag) and, basic ODE math.

Classify this ODE:
$y'' = \sin(x)\, y$

A) 1^{st} order, nonlinear
B) 1^{st} order, linear
C) 2^{nd} order, nonlinear
D) 2^{nd} order, linear
E) None of these

Classify this ODE:

$$y''(x) = \sin(x)\, y(x)$$

A) 1st order, nonlinear
B) 1st order, linear
C) 2nd order, nonlinear
D) 2nd order, linear
E) None of these

Classify this ODE:

$$y'' + x^2 y + 1 = 0$$

A) Linear (not Homogeneous)
B) Homogeneous (not Linear)
C) Linear and Homogeneous
D) Nonlinear and Inhomogeneous

Classify this ODE:

$$y''(t) + ty(t) + 1 = 0$$

A) Linear (not Homogeneous)
B) Homogeneous (not Linear)
C) Linear and Homogeneous
D) Nonlinear and Inhomogeneous

Classify this ODE:

$$y''(t) = (t+1)y(t)$$

A) Linear (not Homogeneous)
B) Homogeneous (not Linear)
C) Linear and Homogeneous
D) Nonlinear and Inhomogeneous

Classify this ODE:

$$y' = \sin(y) + 1$$

A) Linear (not Homogeneous)
B) Homogeneous (not Linear)
C) Linear and Homogeneous
D) Nonlinear and Inhomogeneous

Is this ODE homogeneous?

$$y'' = (x+1)y$$

A) Yes
B) No
C) ???

Consider the ODE: $dN/dt - kN = 0$
with $k>0$ and $N(t=0)=N_0>0$.
How does $N(t)$ behave as $t \rightarrow \infty$?

A) $N(t)$ decays to zero
B) $N(t)$ approaches a constant value
C) $N(t)$ stays constant the whole time
D) $N(t)$ diverges (approaches ∞)
E) Not enough info given!

The magnetic force on a particle (charge q, velocity v) is **F** = q **v** x **B**.
If $q>0$, $v(t=0) = +v_0\,\hat{\imath}$, and $B(x,y,z) = B_0\,\hat{\jmath}$, how does the particle move?

A) straight line motion
B) circular orbit in xy plane
C) circular orbit in yz plane
D) circular orbit in xz plane
E) helical motion

In the last problem, suppose v now has a component in the **B** direction (ĵ), and a perp. component (î). It starts in the y=0 plane and crosses the $y=y_{final}$ plane at time T. If you increase only the initial perpendicular *component* of v, what happens to the "passage time" T.?

A) T is independent of v_{perp}
B) T increases as v_{perp} increases
C) T decreases as v_{perp} increases

The magnetic force on a particle (charge q, velocity v) is $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$.
If $q>0$, $v(t=0) = +v_0\,\hat{\imath}$, and $B(x,y,z) = B_0\,\hat{\imath}$, how does the particle move?

A) straight line motion
B) circular orbit in xy plane
C) circular orbit in yz plane
D) circular orbit in xz plane
E) helical motion

In a homework problem, a student derives a formula for position.

The problem involves gravity and a “drag force” F= -cv, where v is speed. They get a term that looks like

$$y(t) = y_{init} \ln(1 + \frac{cv_{init}}{g})$$

Is there any way to tell if they made a mistake, without carefully looking over all their work?

A) Yes, I see a nice check
B) There probably is but I’m not seeing the “trick”
C) With a formula this messy, the only way to check is to redo the work (or compare with someone else, or the book, or google it, or …)

In a Phys 1110 exam, a student produced the following solution.
- Is the final solution correct?
- If NOT, does that mean the initial equation must have been wrong?
- If NOT, and assuming the initial equation is NOT wrong, find the error using dimensional analysis.

$[M] = M$ (mass)

$[g] = L/T^2$ (distance/time^2)

$[h] = L,$

$[\omega] = 1/T$

$[v] = L/T,$

$[R] = L$

$[I] = ML^2$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\omega^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\left(\frac{v^2}{R}\right)^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{2}v^4$$

Which of these ODEs for y(t) are separable?

i) $y' = \frac{y^2}{t} - t$

ii) $y' = e^t \frac{y+1}{\sqrt{t}}$

iii) $y' = 3 - t$

A) none
B) i & ii
C) ii & iii
D) i & iii
E) all

Classify this ODE:

$$\dot{v} = -g\left(1 - \frac{v^2}{v_{\text{terminal}}^2}\right)$$

A) 1st order, nonlinear
B) 1st order, linear
C) 2nd order, nonlinear
D) 2nd order, linear
E) None of these

Classify this ODE:

$$\dot{v} = -g\left(1 - \frac{v^2}{v_{\text{terminal}}^2}\right)$$

A) linear, homogeneous
B) Nonlinear, homogeneous
C) linear, inhomogeneous
D) Nonlinear, inhomogeneous

To think about: What order is it? Is it separable, or not?

Classify this ODE:

$$y'(x) + P(x)\, y(x) = Q(x)$$

Order?
Linear?
Homogeneous?
Separable?

Classify this ODE:

$y'(x) + P(x)\, y(x) = Q(x)$

1^{st} order, linear,
NOT homogeneous (because of $Q(x)$)
NOT separable.

Still, there IS a general solution
(with one undetermined coefficient,
as befits a 1^{st} order linear ODE!)

$$y(x) = e^{-I}\left(\text{constant} + \int Q e^{I}\, dx\right), \text{ with } I = \int P\, dx$$

Consider the equation $dv/dt = -k\, v$,
where v is velocity, t is time, and k is a constant.
What motion does this describe?

A) a mass on a spring
B) a mass in free-fall
C) a moving mass with a drag force
D) a moving mass with a driving force
E) something else entirely!

Suppose you solve an ODE for a particle's motion, and find

$x(t) = c(t-t_0)$ What can you conclude?

A) This particle is responding to a constant force
B) This particle is free (zero force)
C) This particle is responding to a time varying force
D) ???

Suppose you solve an ODE for a particle's motion, and find $x(t) = bt^2$.

What can you conclude?

A) This particle is responding to a constant force
B) This particle is free (zero force)
C) This particle is responding to a time varying force
D) ???

Suppose you solve an ODE for a particle's motion, and find : $x(t) = c(1 - e^{-t/\tau})$

What can you conclude?

A) This particle is responding to a constant force
B) This particle is free (zero force)
C) This particle is responding to a time varying force
D) ???

For an object of “diameter D”,
$f_{linear}=bv=\beta Dv$

$f_{quad} = cv^2 = (1/2)c_0 A \rho_{air} v^2$

For a sphere in air, $f_{quad}/f_{linear} \approx (1600 \text{ s/m}^2) Dv$

Which form of drag dominates most microbiology contexts?

A) linear
B) quadratic
C) ??

For an object of “diameter D”,
$f_{linear}=bv=\beta D$

$f_{quad} = cv^2 = (1/2)c_0 A \rho_{air} v^2$

For a sphere in air, $f_{quad}/f_{linear} \approx (1600\ s/m^2)\ Dv$

Which form of drag dominates most sports events?

A) linear
B) quadratic
C) ??

Where are you now?

A) Done with page 1
B) Done with page 2
C) Done with page 3

If you are done with page 3, try these:

Like in section IIc, find the terminal velocity of an object of mass *m* when air drag force is...

1) ...*quadratic* with respect to speed ($c_1 = 0$, $c_2 \neq 0$)
2) ...a combo of *both* linear *and* quadratic terms ($c_1 \neq 0$, $c_2 \neq 0$)

Finally, if you still have time, find an expression for v(t) from part I. (You sketched this qualitatively in IB). You will need to solve an ODE!

An object falling in air satisfies the ODE (from Newton's 2nd law):

$$m\, dv/dt \;=\; -mg - bv$$

The equation has three dimension-ful parameters (m, g, b)

a) Use those three dimensionful parameters to create "Natural" scales of mass (M_0), length (L_0), and time (T_0)

b) Using these three natural scales, create a natural scale for velocity (V_0)

An object falling in air satisfies the ODE (from Newton's 2nd law):

$$m\, dv/dt = -mg - bv$$

The equation has three dimension-ful parameters (m, g, b)

a) Use those three dimensionful parameters to create "Natural" scales of mass (M_0), length (L_0), and time (T_0)

b) Using these three natural scales, create a natural scale for velocity (V_0)

c) Define a *dimensionless velocity V* by the equation $V=v/V_0$, and rewrite the original ODE as an equation for V instead.

(This equation should contain NO parameters with dimensions, except where you have combinations of quantities that manifestly look dimensionless)

Suppose you solve an ODE (Newton's law!) for a particle's motion, and find

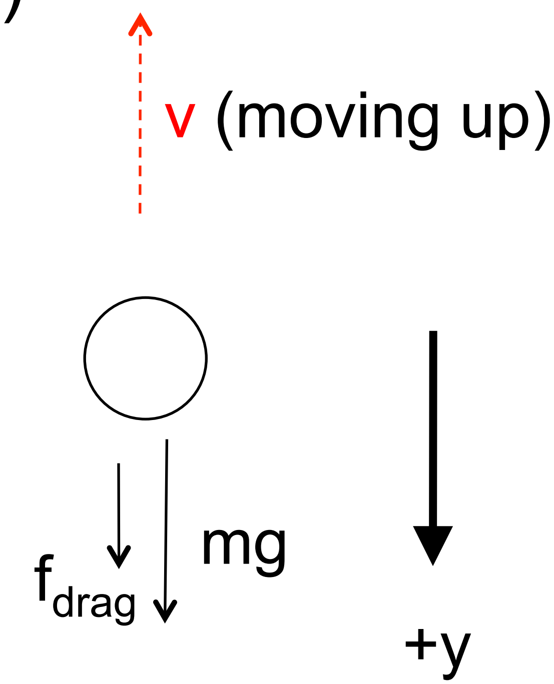
$x(t) = c(t-t_0)$ (where c is a constant)

What can you conclude?

A) This particle is responding to a constant force
B) This particle is free (zero force)
C) This particle is responding to a time varying force
D) ???

Drag force is: $\vec{f}_D = -b\vec{v} - cv^2\hat{v}$

Consider a mass moving UP
(Let's define DOWN as the +y direction)
Which eq'n of motion is correct?



A) $m\, dv_y/dt = +mg - bv_y - cv_y^2$

B) $m\, dv_y/dt = +mg - bv_y + cv_y^2$

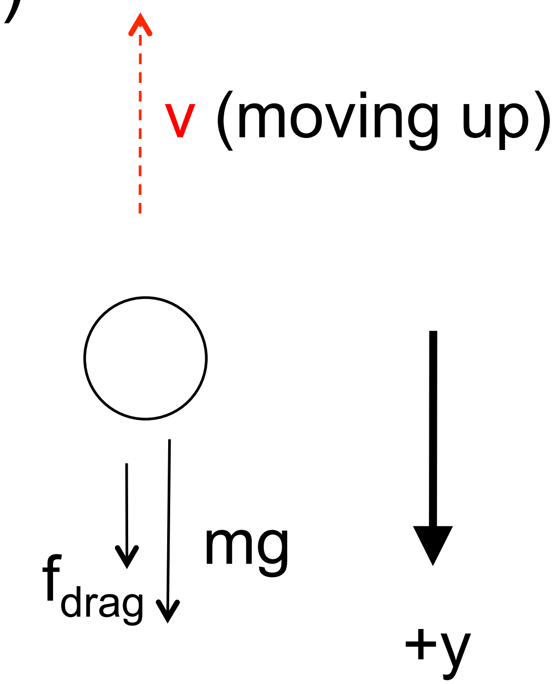
C) $m\, dv_y/dt = +mg + bv_y - cv_y^2$

D) $m\, dv_y/dt = +mg + bv_y + cv_y^2$

E) Other!

Drag force is: $\vec{f}_D = -b\vec{v} - cv^2\hat{v}$

Consider a mass moving UP
(Let’s define DOWN as the +y direction)
Which eq’n of motion is correct?



A)

B) $m\,dv_y/dt = +mg - bv_y + cv_y^2$

C)

D) $m\,dv_y/dt = +mg + bv_y + cv_y^2$

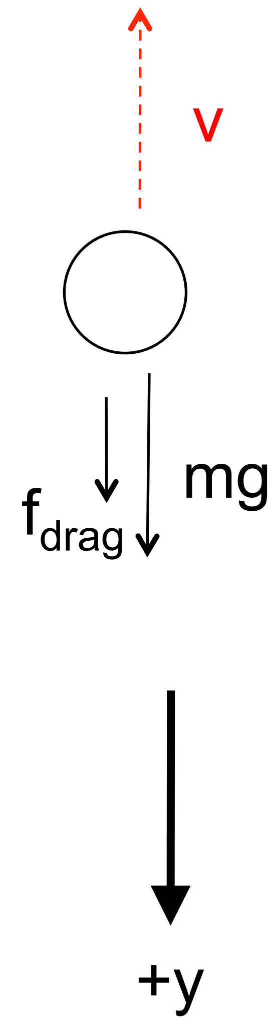
E) Other!

Drag force is $\vec{f}_D = -b\vec{v} - cv^2\hat{v}$

(Let's define DOWN as the +y direction)

While moving up, the correct expression is:

$$m\, dv_y/dt = +mg - bv_y + cv_y^{\,2}$$



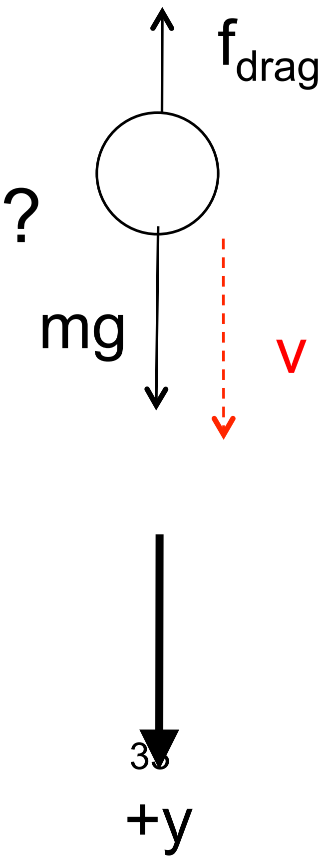
Drag force is $\vec{f}_D = -b\vec{v} - cv^2\hat{v}$

(Still define DOWN as the +y direction)
While moving *up*, the correct expression was:

$$m\, dv_y/dt \;=\; +mg - bv_y \;+cv_y{}^2$$

If the object is now moving DOWN,
which term(s) in that equation will change sign?

A) mg (only)
B) the linear term (only)
C) the quadratic one (only)
D) *more* than one term changes sign
E) NONE of the terms changes sign.

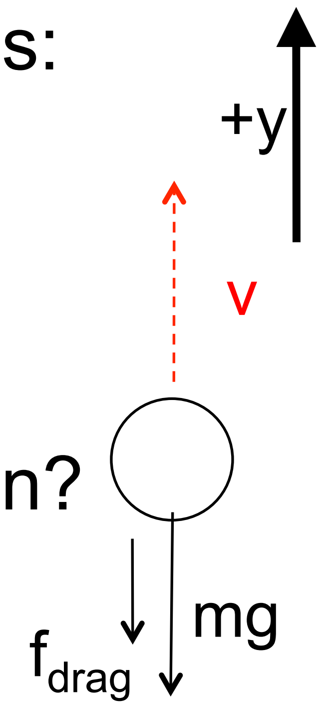


Drag force is $\vec{f}_D = -b\vec{v} - cv^2\hat{v}$

Back to moving *up*, the correct expression was:

$$m\, dv_y/dt \;=\; +mg - bv_y \;+cv_y^{\;2}$$

But, let's now define UP as the +y direction)



Which term(s) in that equation will change sign?

A) mg (only)

B) the linear term (only)

C) the quadratic one (only)

D) *more* than one term changes sign

E) NONE of the terms changes sign.

Assuming you have two solid spheres made of the *same material*, but one has a larger diameter. When dropped in air, which one will reach the higher terminal velocity, the bigger one or the smaller one?

A) The bigger one
B) The smaller one
C) both are the same
D) ???

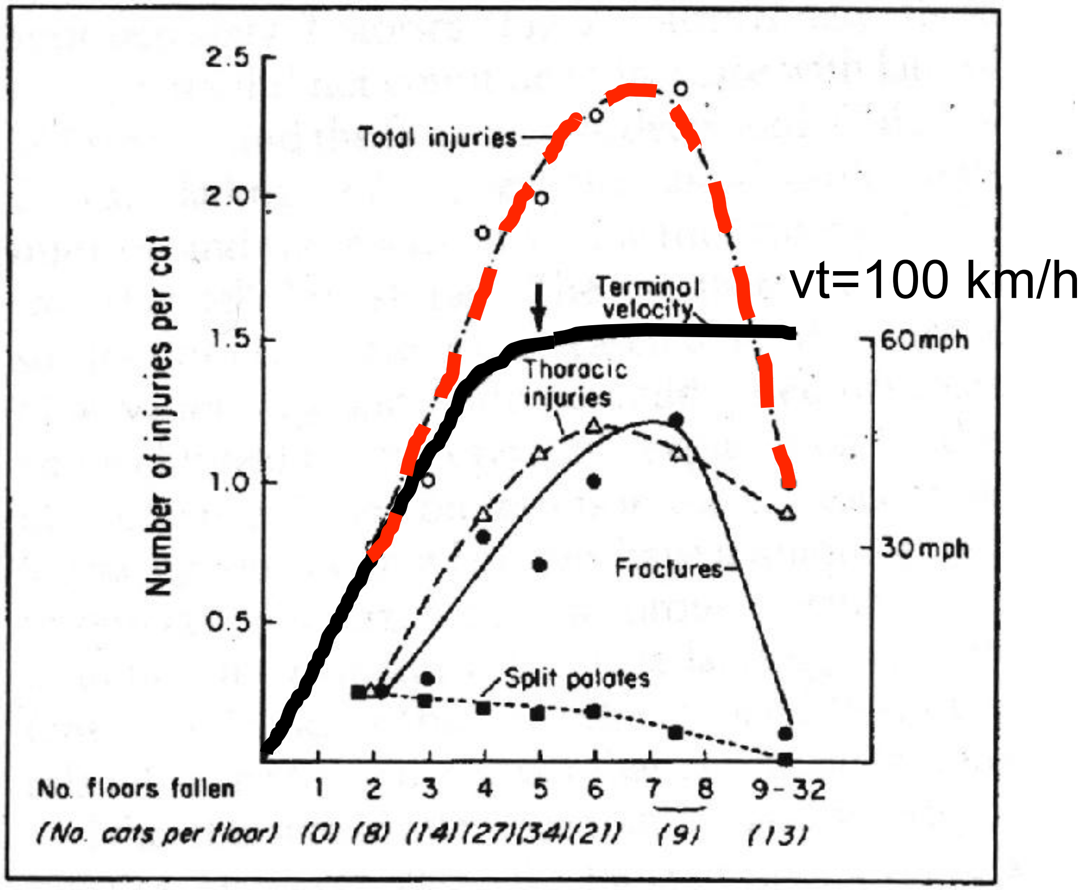


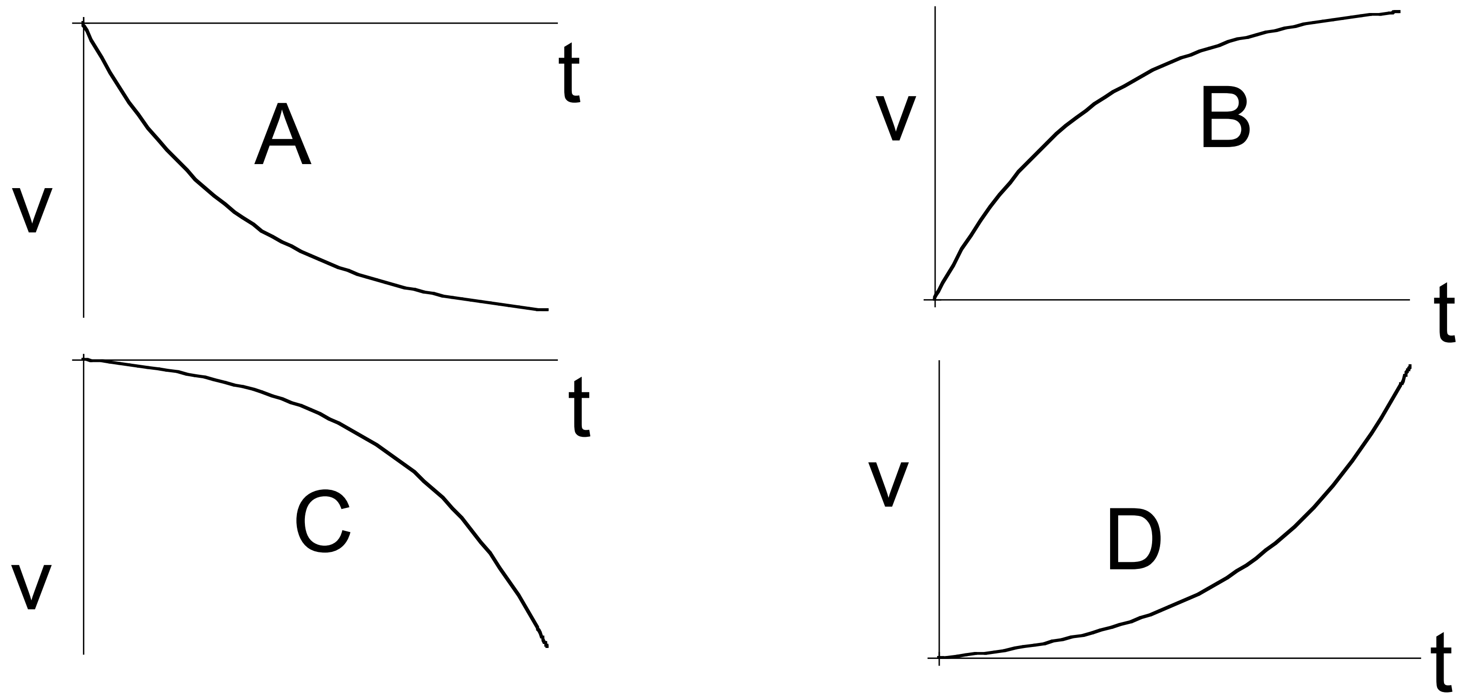
Figure 2—Relationship of injuries to distance fallen and velocity in 132 cats with high-rise syndrome: ↓ points to terminal velocity (——); total number of injuries/cat (○, —·—·—·); number of thoracic injuries (pulmonary contusions +pneumothorax)/cat (△, – – –); number of fractures/cat (●, ——); number of split palates/cat (■, – – – – –).

Journal of the American Veterinary Medical Association, 1978

The solution to the equation describing an object falling from rest with linear air drag was

$$v_y(t) = -v_t(1 - e^{-t/\tau})$$

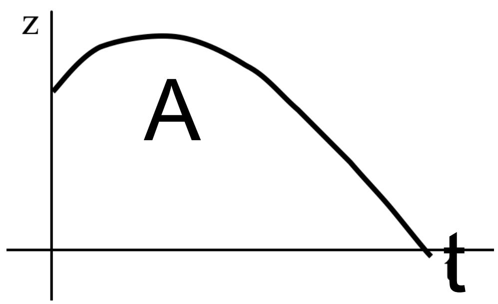
Which figure best shows this sol'n?

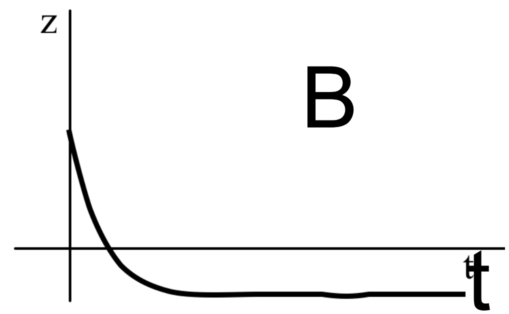


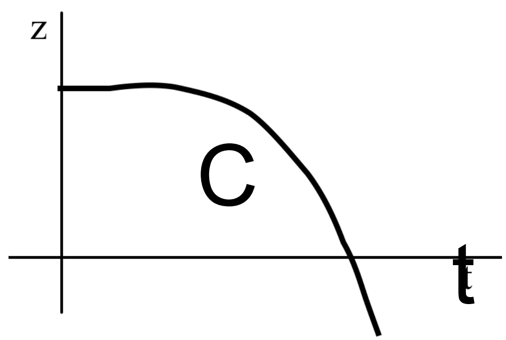
E) Other/???

The solution to the eq'n describing an object falling from rest from h>0 with linear air drag was $$v_y(t) = -v_t(1 - e^{-t/\tau})$$

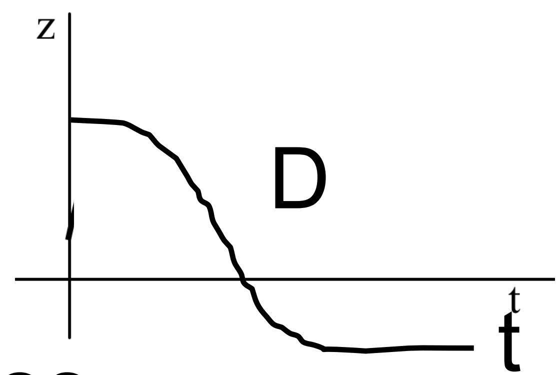
Which figure best shows height, y(t)?





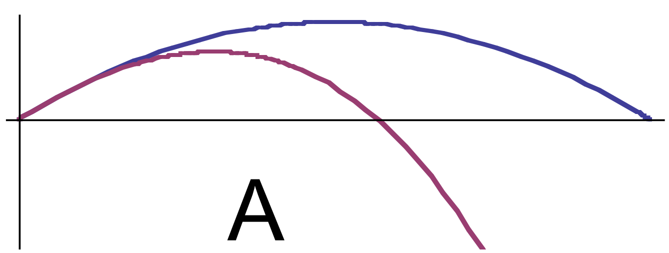


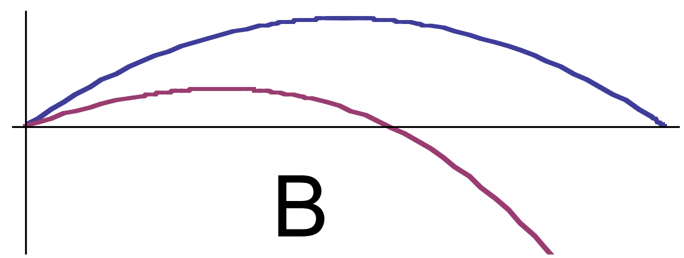
E) Other/???



E) Other/???

Which of these two plots seems more realistic for a tennis ball trajectory, comparing (in each graph) the path with (purple), and without (blue), drag.





C) Neither of these is remotely correct!

With quadratic air drag, $v(t)=v_0/(1+t/\tau)$
where $\tau=m/(cv_0)$, and $c=(1/2)c_0\,A\,\rho_{air}$.
(For a human on a bike, c is of order .2 in SI units)

Can you confirm that c is about 0.2? (c_0 is ~1 for non-aerodynamic things, and ~.1 for very- aerodynamic things.

Roughly how long does it take for a cyclist on the flats to drift down from v_0=10 m/s (22 mi/hr) to ~1 m/s?

A) a couple seconds
B) a couple minutes
C) a couple hours
D) none of these is even close.

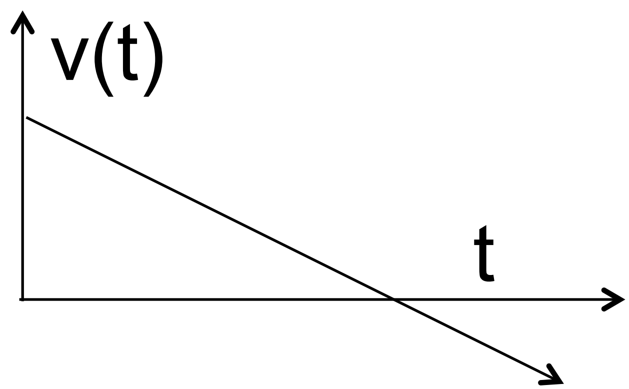
In real life, the answer is shorter than this formula would imply. Why?

A tennis ball is hit directly upwards with initial speed v_0. Compare the time T to reach the top (height H) to the time and height in an ideal (vacuum) world.

A) $T>T_{vacuum}$, $H \approx H_{vacuum}$
B) $T>T_{vacuum}$, $H<H_{vacuum}$
C) $T \approx T_{vacuum}$, $H<H_{vacuum}$
D) $T<T_{vacuum}$, $H<H_{vacuum}$
E) Some other combination!!

A tennis ball is hit directly upwards with initial speed v_0. Compare the time T to reach the top (height H) to the time and height in an ideal (vacuum) world.

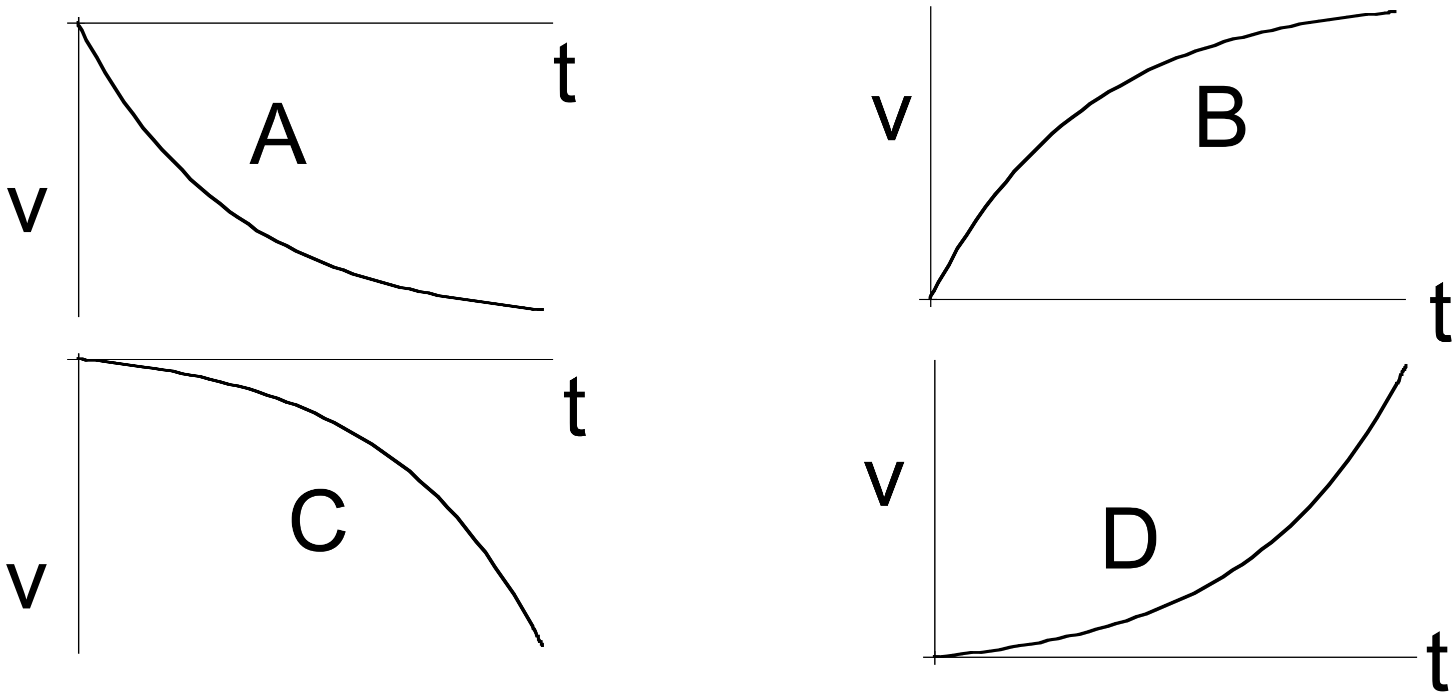
A) $T > T_{vacuum}$, $H \approx H_{vacuum}$
B) $T > T_{vacuum}$, $H < H_{vacuum}$
C) $T \approx T_{vacuum}$, $H < H_{vacuum}$
D) $T < T_{vacuum}$, $H < H_{vacuum}$
E) Some other combination!!



The solution to the equation describing an object falling from rest with quadratic air drag was

$$v_y(t) = -v_t \tanh(gt/v_t)$$

Which figure best shows this sol'n?



E) Other/???

An object is launched directly upwards with initial speed v_0.
Compare the time t_1 to reach the top
to the time t_2 to return back to the starting height.

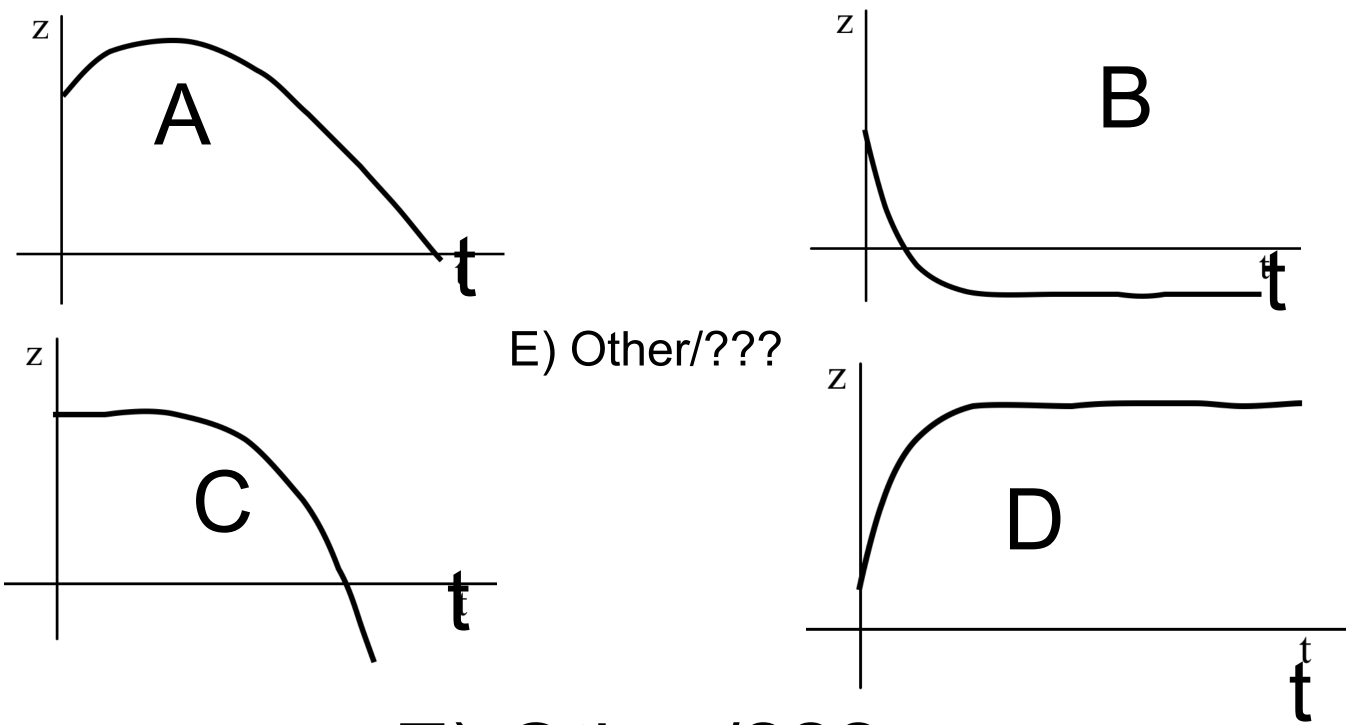
A) $t_1 > t_2$
B) $t_1 = t_2$
C) $t_1 < t_2$
D) Answer depends on whether v_0 exceeds v_t or not.

If you add linear drag, what happens (qualitatively) to the horizontal range of a projectile (with angle fixed)? Why?

The solution to the eq'n describing an object thrown up from h>0 with linear air drag was

$$v(t) = -g\tau + \tau\left(v_0/\tau + g\right)e^{-t/\tau}$$

Which figure best shows height, z(t)?

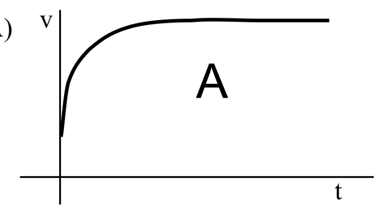


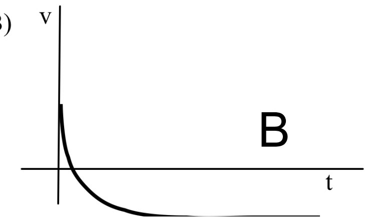
E) Other/???

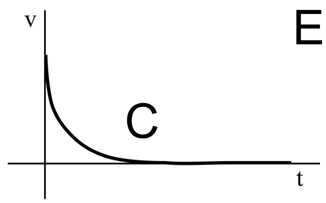
The solution for an object moving horizontally with linear air drag was

$$v(t) = v_0 e^{-t/\tau} \qquad \text{if } v(t=0) = v_0 > 0$$

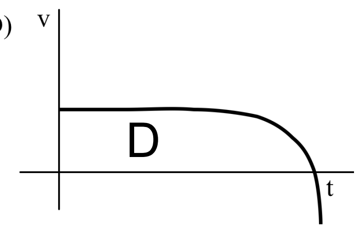
Which figure best shows this sol'n?

A) 

B) 

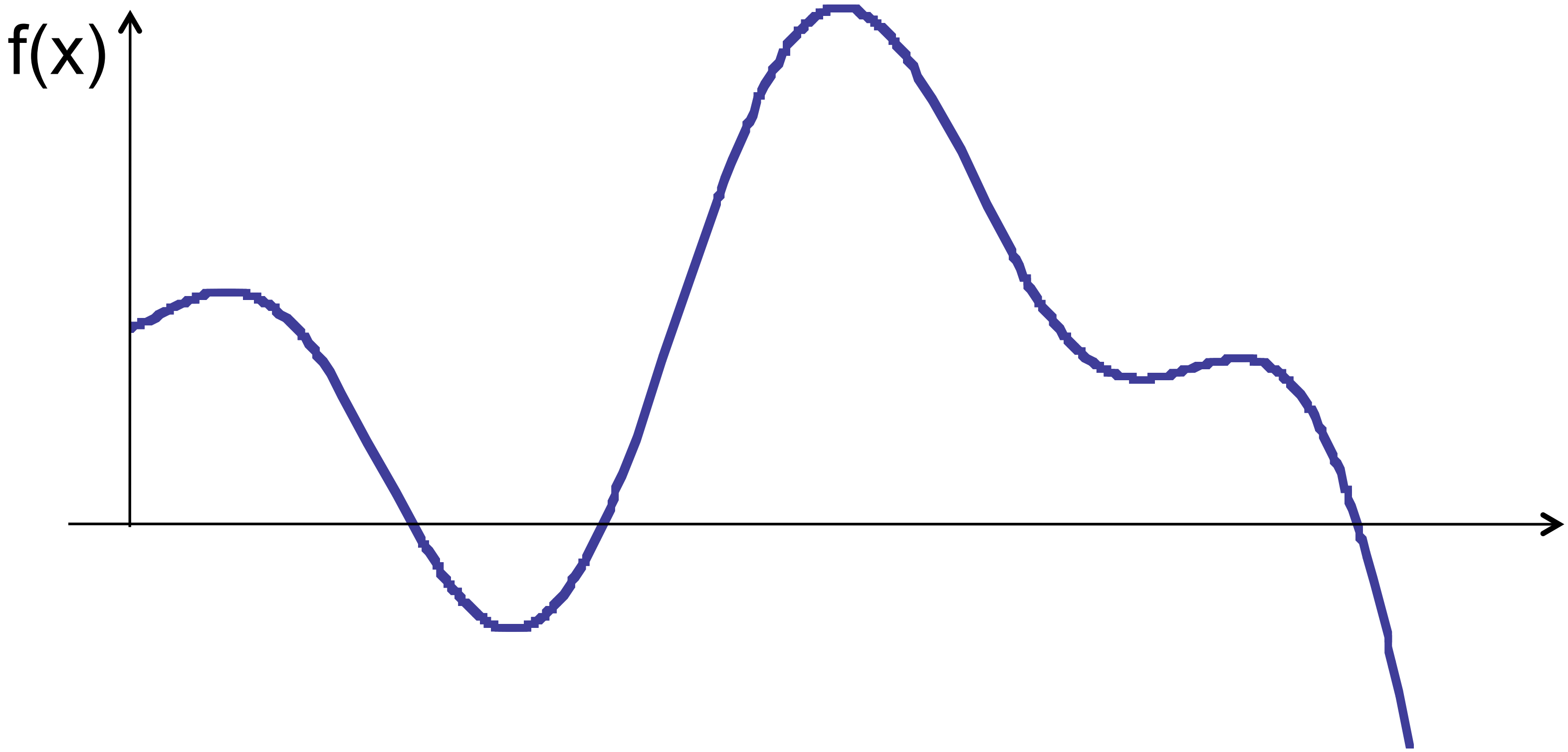
C) 

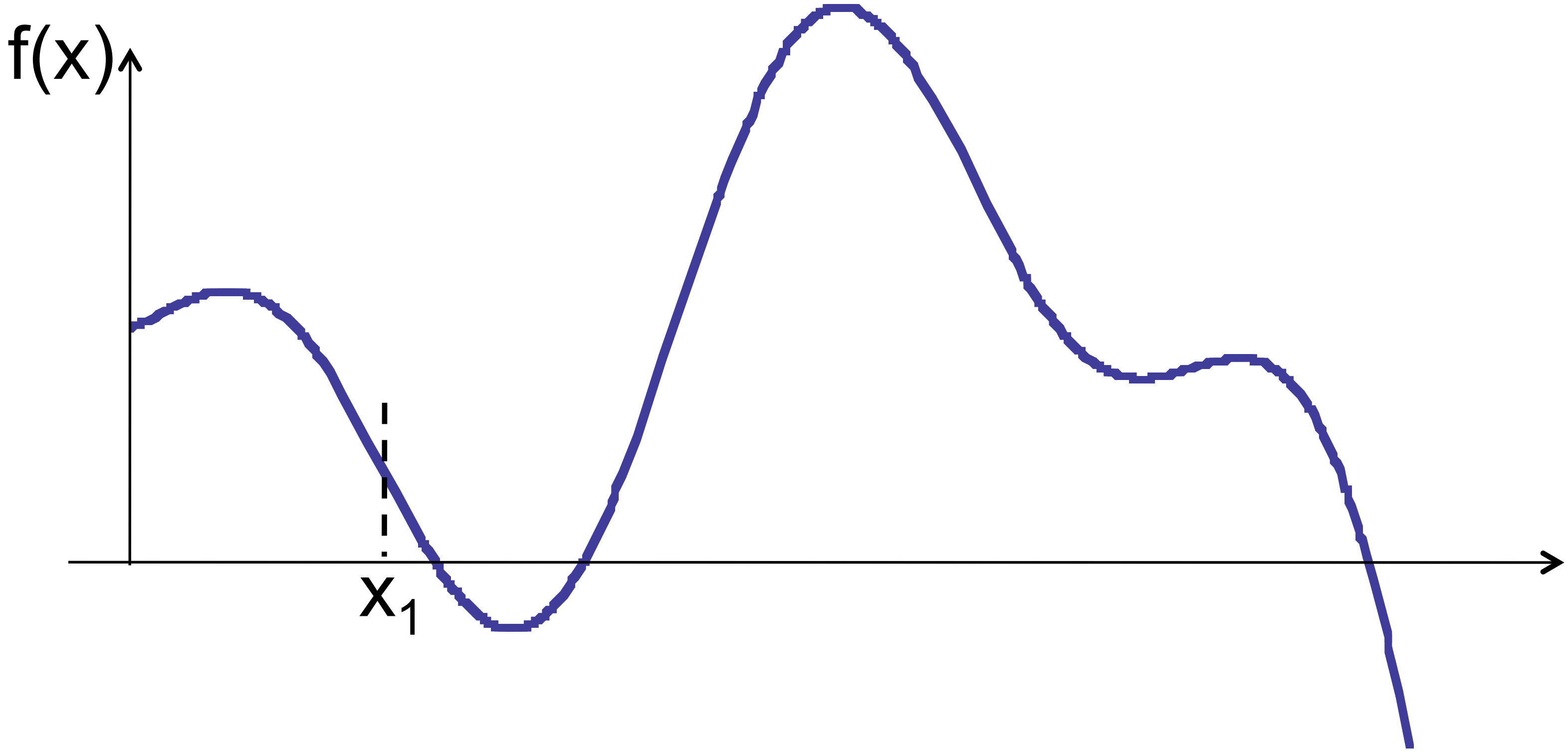
E) Other/???

D) 

If you add linear drag,
what happens (qualitatively) to the horizontal range of a projectile (with angle and v_0 fixed)?

A) It goes down (because of the horizontal drag)
B) It goes up (because it's in the air *longer!*)
C) It could go either way, depending… (the two reasons above make it ambiguous)


f(x)



If we expand f(x) around point x_1 in a Taylor series,

$f(x) = a_0 + a_1 (x-x_1) + a_2 (x-x_1)^2 + \ldots$

What is your best guess for the signs of a_0 and a_1?

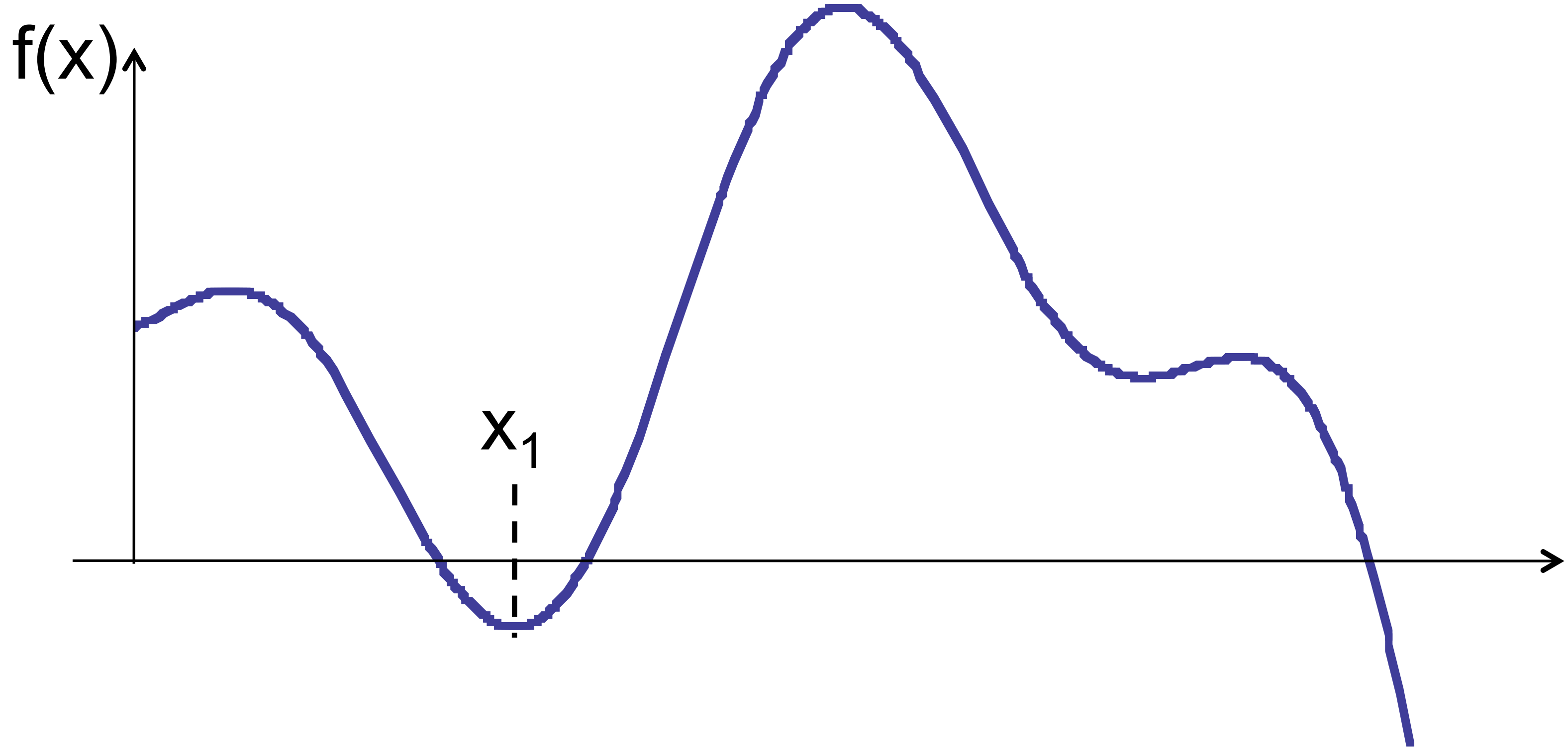
A) a_0 is +, a_1 is +

B) a_0 is +, a_1 is –

C) a_0 is -, a_1 is +

D) a_0 is -, a_1 is –

E) Other! (Something is 0, or it's unclear)



If we expand f(x) around point x_1 in a Taylor series,

$f(x) = a_0 + a_1 (x-x_1) + a_2 (x-x_1)^2 + \ldots$

What is your best guess for the signs of a_0 and a_1?

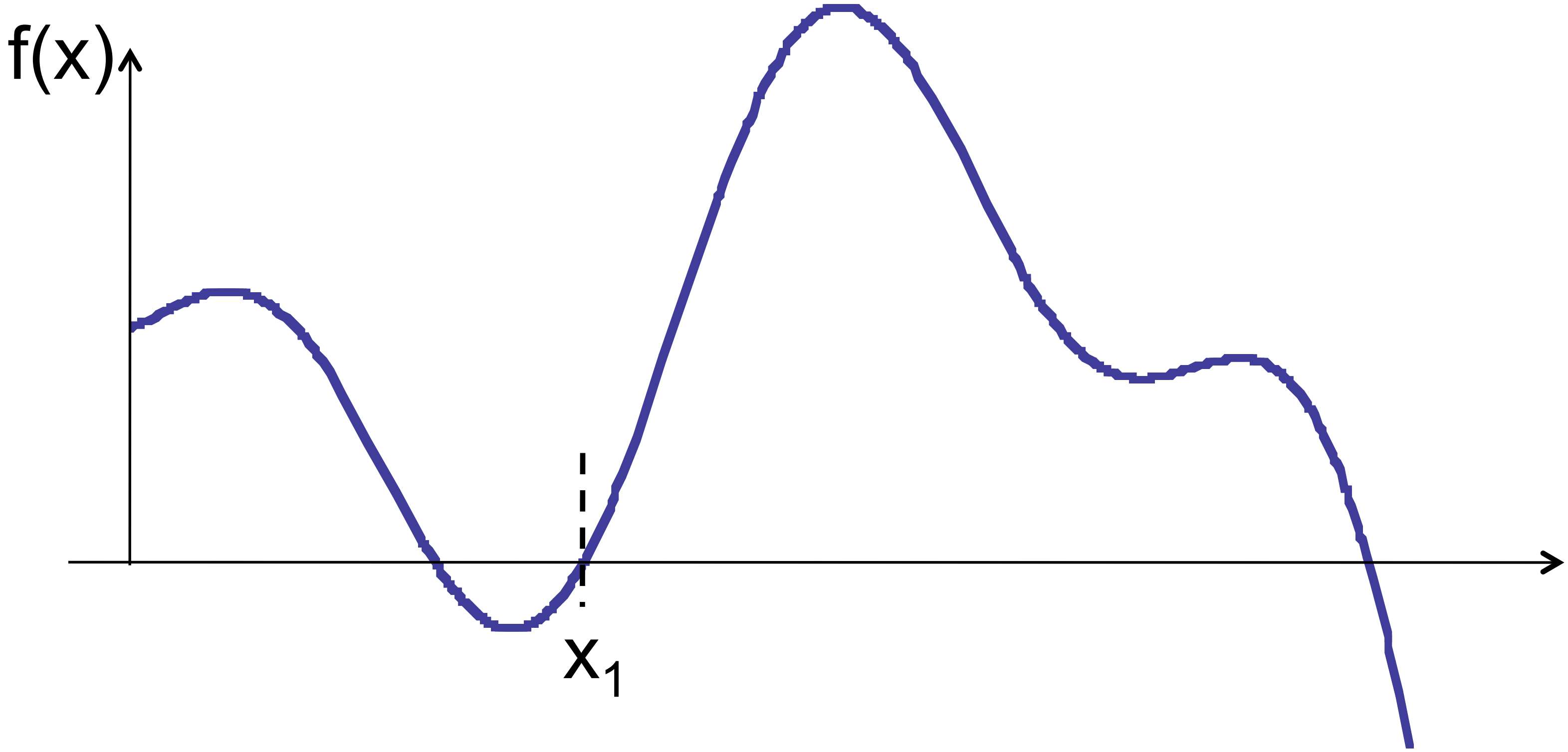
A) a_0 is +, a_1 is +

B) a_0 is +, a_1 is –

C) a_0 is -, a_1 is +

D) a_0 is -, a_1 is –

E) Other! (Something is 0, or it’s unclear)

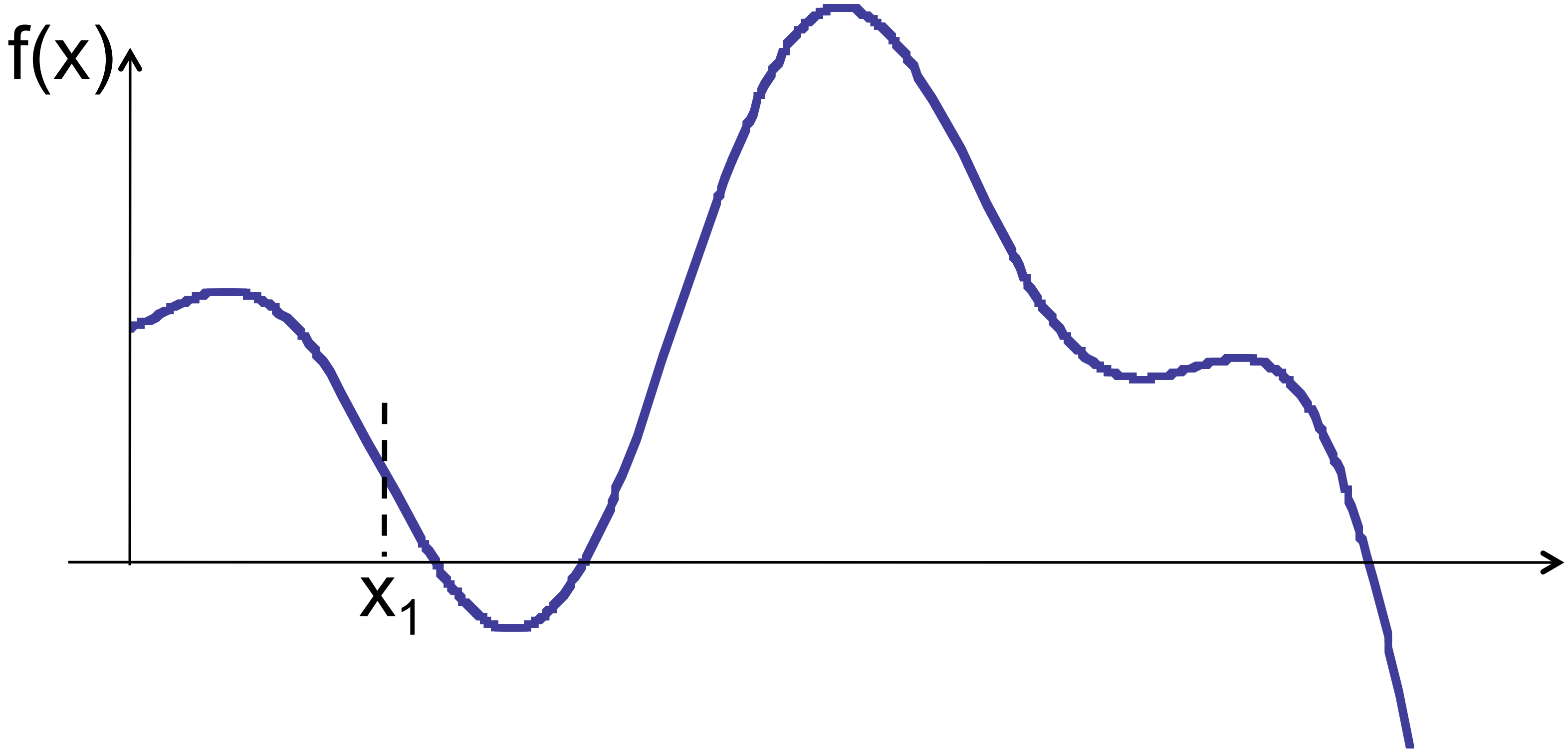


If we expand f(x) around point x_1 in a Taylor series,

$f(x) = a_0 + a_1 (x-x_1) + a_2 (x-x_1)^2 + \ldots$

What is your best guess for the signs of a_0 and a_1?

A) a_0 is +, a_1 is +
B) a_0 is +, a_1 is –
C) a_0 is -, a_1 is +
D) a_0 is -, a_1 is –
E) Other! (Something is 0, or it's unclear)



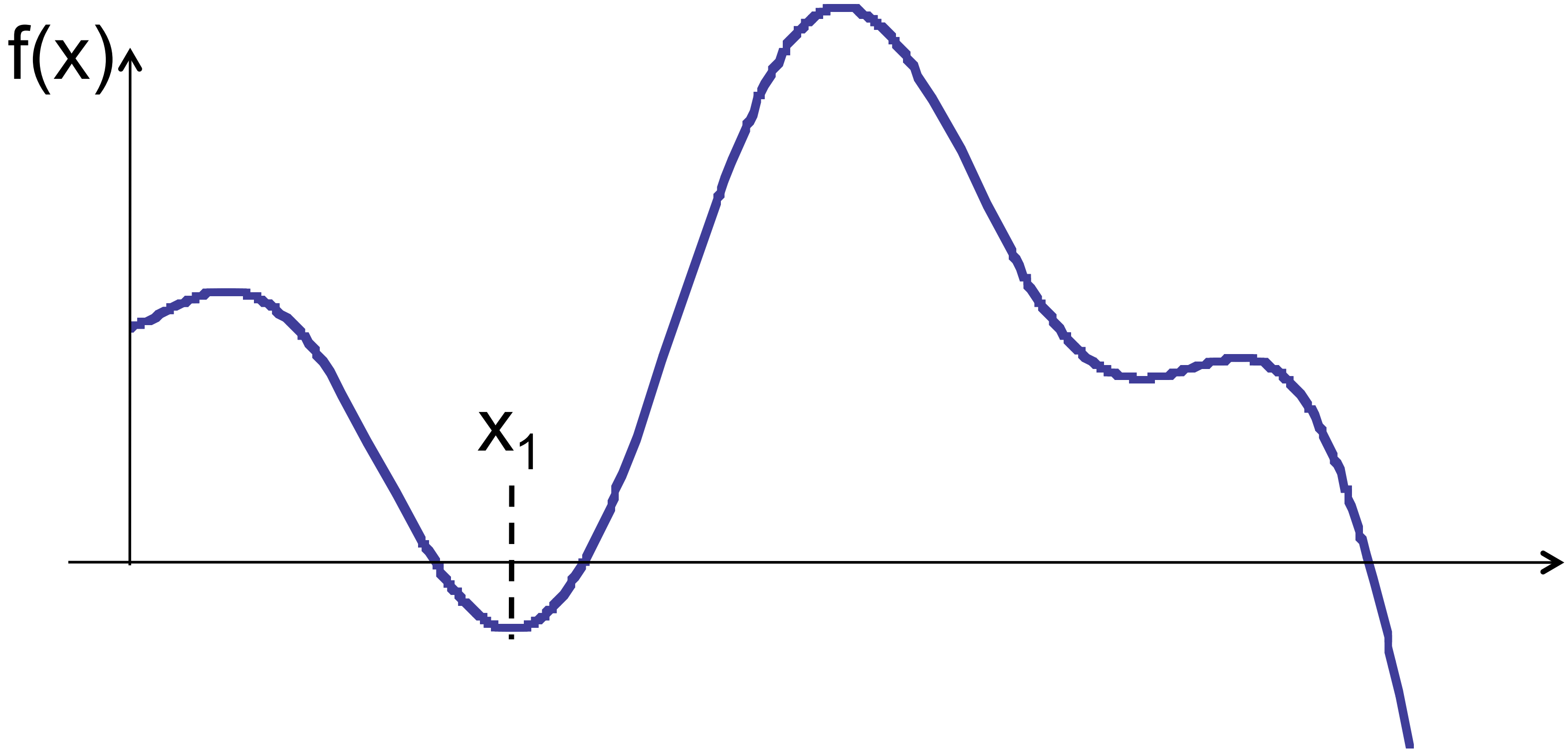
If we expand f(x) around point x_1 in a Taylor series,

$f(x) = a_0 + a_1 (x-x_1) + a_2 (x-x_1)^2 + \ldots$

What is your best guess for the sign of a_2?

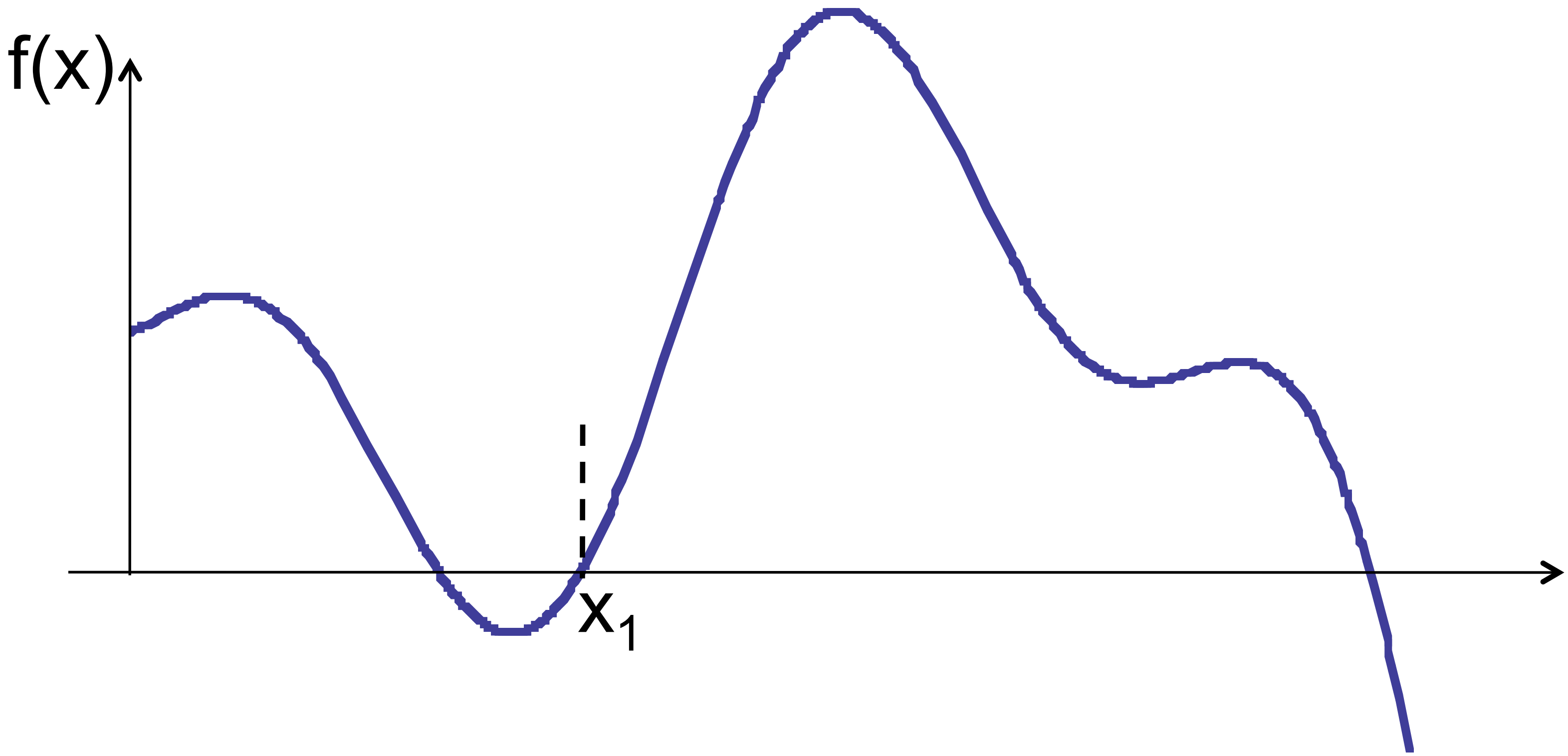
A) +(?) B) -(?) C) 0 (?)

D) ????? How could we possibly tell this without a formula?



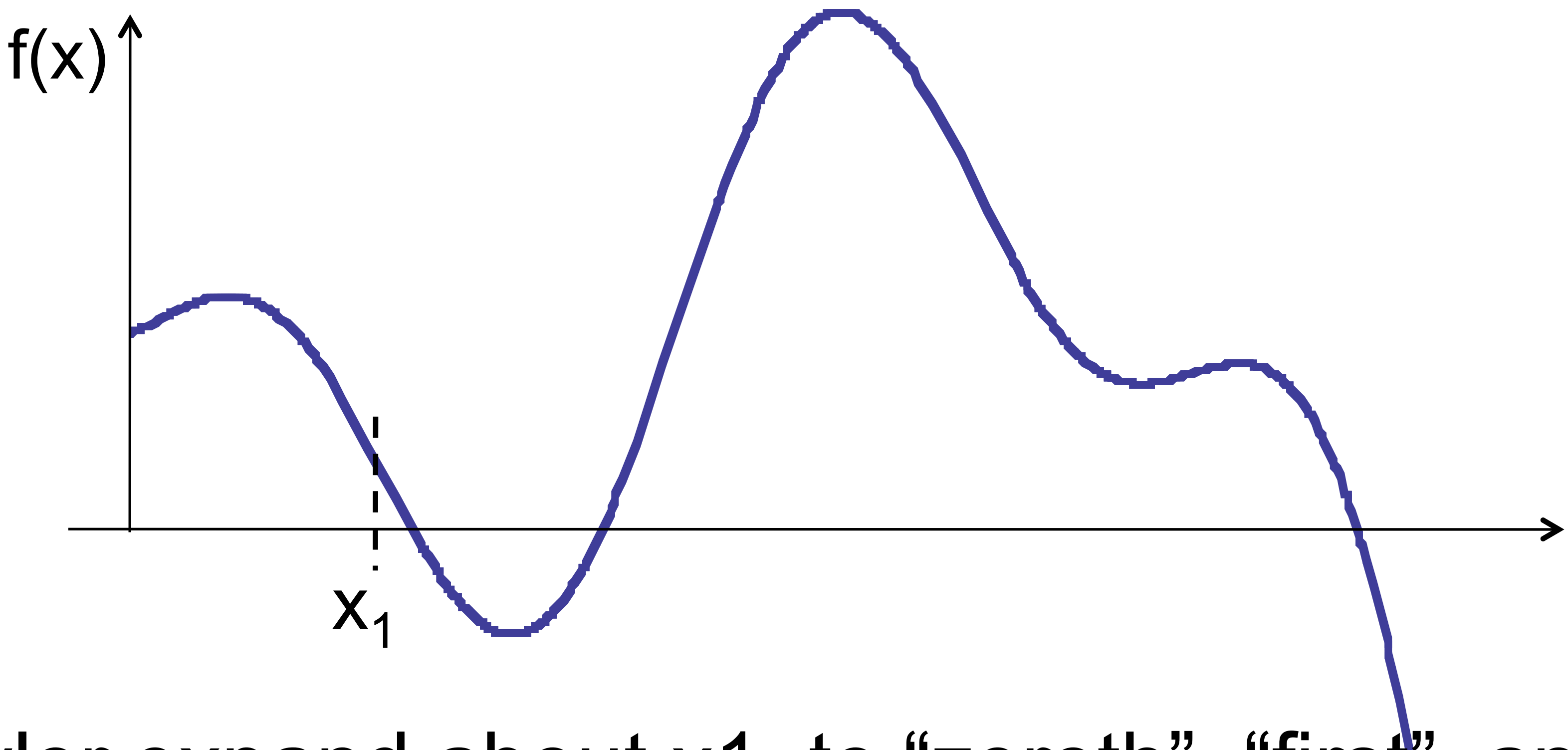
If we expand f(x) around point x_1 in a Taylor series,
$f(x) = a_0 + a_1 (x-x_1) + a_2 (x-x_1)^2 + \ldots$
What is your best guess for the sign of a_2?

A) +(?) B) -(?) C) 0 (?)
D) ????? How could we possibly tell this without a formula?



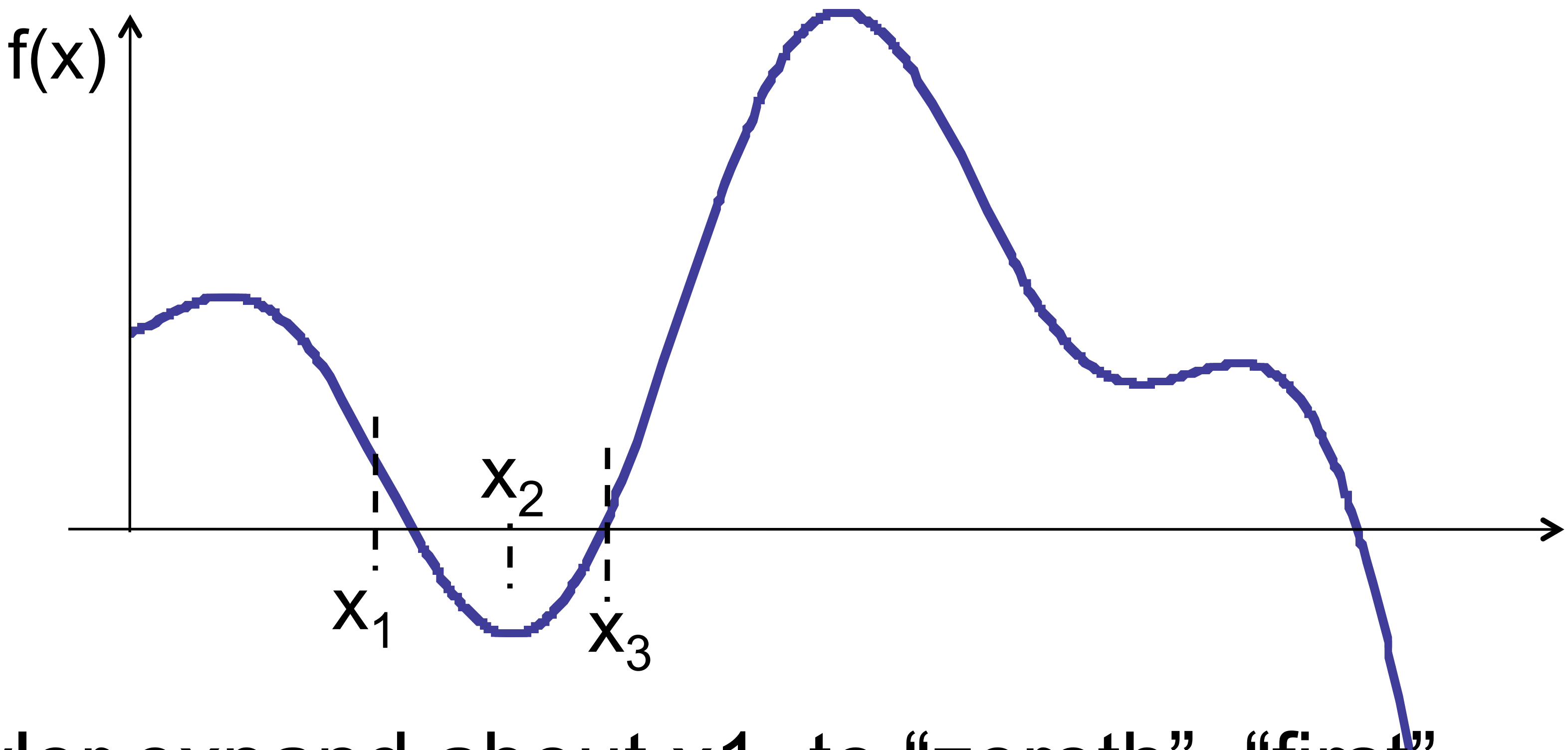
If we expand f(x) around point x_1 in a Taylor series,
$f(x) = a_0 + a_1 (x - x_1) + a_2 (x - x_1)^2 + \ldots$
What is your best guess for the sign of a_2?

A) +(?) B) -(?) C) 0 (?)
D) ????? How could we possibly tell this without a formula?



Taylor expand about x1, to “zeroth”, “first”, and “second” orders. In each case, SKETCH (on top of the real curve) what your “Taylor approximation” looks like

If time – try it about the BOTTOM point of the curve, or the first zero crossing…



Taylor expand about x1, to “zeroth”, “first”, and “second” orders. In each case, SKETCH (on top of the real curve) what your “Taylor approximation” looks like

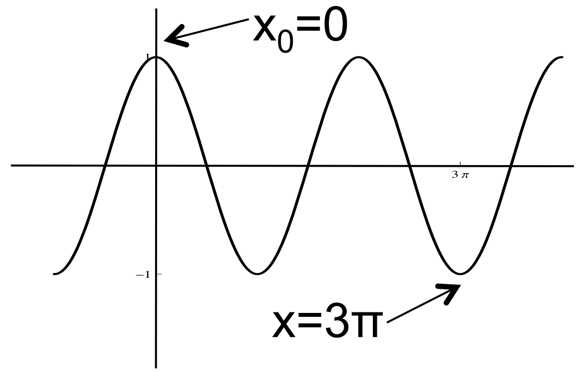
Repeat for the other two points.

f(x)=cos(x).

If x_0=0, will the full Maclaurin series expansion produce the exact value for cos(3π)=-1?

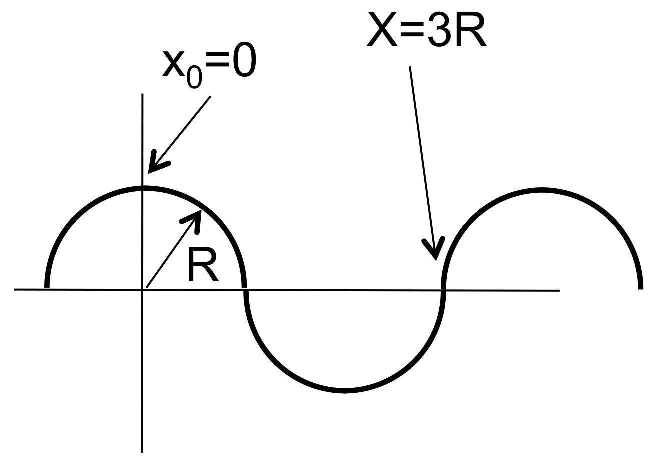
$$\cos(3\pi) \stackrel{?}{=} \cos(0) + \left.\frac{d\cos(x)}{dx}\right|_{x=0}(3\pi) + \frac{1}{2!}\left.\frac{d^2\cos(x)}{dx^2}\right|_{0}(3\pi)^2 + \ldots$$

A) Yes
B) No, not even close
C) Close, but not exact



Consider f(x), composed of an infinite series of semicircles.

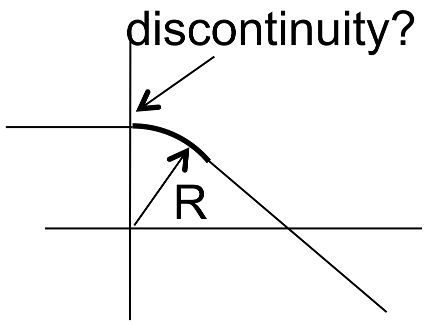
If $x_0=0$, will the Maclaurin series expansion produce the correct value for f(3R)=0?



A) Yes
B) No, not even close
C) Close, but not exact

Consider f(x), which is a quarter-circle joining two straight lines. Near x=0:

$$f(x) = \begin{cases} R, & \mathrm{x} < 0 \\ \sqrt{\mathrm{R}^2 - x^2}, & \mathrm{x} > 0 \end{cases}$$



A) f(x) is discontin. at x=0
B) f'(x) is discontin. at x=0
C) f''(x) is discontin. at x=0
D) Some higher deriv is discontin. at x=0
E) f and all higher derivs are continuous at x=0.

Where are you now?

A) Done with page 1
B) Done with page 2
C) Done with page 3

If you are done with page 3, try these:

Rewrite your code to solve for v[t] for an object falling with quadratic drag force.

(You used NDSolve to do this for the falling penny - now you know, roughly what NDSolve was doing!)

Does the Taylor Series expansion
$\cos(\theta) = 1 - \theta^2/2! + \theta^4/4! + \ldots$
apply for θ measured in

A) degrees
B) radians
C) either
D) neither

What are the first few terms of the Taylor Series expansion for e^x (about x=0)?

A) $e^x = 1 + x^2/2! + x^4/4! + \ldots$

B) $e^x = 1 - x^2/2! + x^4/4! + \ldots$

C) $e^x = 1 + x + x^2/2! + x^3/3! + \ldots$

D) $e^x = 1 - x + x^2/2! - x^3/3! + \ldots$

E) Something else!!

What is the start of the Taylor series expansion for $\sqrt{1+x}$, if x is small?

A) $1+\sqrt{x}+\ldots$

B) $1+\mathrm{x}+\ldots$

C) $1-\frac{1}{2}\mathrm{x}+\ldots$

D) $1+\frac{1}{2}\mathrm{x}+\ldots$

E) Something entirely different!

While you're at it…. What's the NEXT term?

What is the next term in the binomial expansion for $\sqrt{1+\varepsilon} \approx 1+\varepsilon/2+\ldots$?

A) ε^2

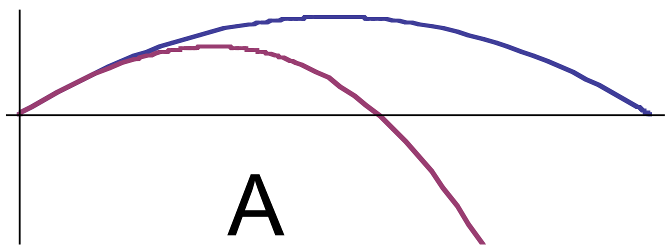
B) $\varepsilon^2/2$

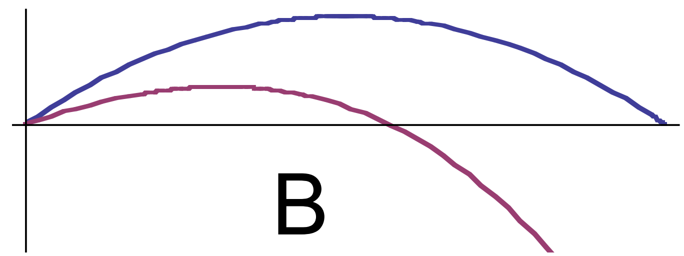
C) $\varepsilon^2/4$

D) $\varepsilon^2/8$

E) Something else

Which of these two plots seems more realistic for a tennis ball trajectory, comparing (in each graph) the path with (purple), and without (blue), drag.





C) Neither of these is remotely correct!