Energy

Did you start reading Ch 4 for today?

A) Yup, 4.1 and 4.2

B) Sort of... parts of it

C) No, but I plan to (after the exam!)

D) No, I tend to just read what I have to for solving homework problems

E) (None of these – something else best describes my answer here)

When I do read for class, I usually

A) Read the Taylor assignment (only)
B) Read the online lecture notes (only)
C) Both the above!
D) Mix of the above (but not really both, usually)
E) Other: none of the above really describes my answer.

In which of the following situations is nonzero net work done on the specified object?

I- A *book* is pushed across a table at constant speed
II- A *car* goes around a corner at a constant 30 mph
III- An *acorn* is falling from a tree (still near the top)

A) i only B) ii only C) iii only
D) i and iii only E) Other/not sure

An *acorn* is falling from a tree (still near the top) What is the SIGN of the work done on the acorn by gravity over a short time interval?

A) +

B) -

C) 0

D) It depends on your choice of coordinate system.

A) Done with page 1
B) Done with p. 2
C) Done with page 3
D All done!

When you're done, try this:
A pretzel is dipped in chocolate.
Its shape is a quarter circle (R=2 cm)
(Straight from the origin to (2,0), a circular arc to (0,2), and straight back to the origin)
The linear chocolate density is $\lambda=c(x^2 + y^2)$,
where $c = 3\ g/cm^3$.

How much chocolate is on a pretzel?

A) Done with page 1 (pretzels), got it!
B) Done with p. 1 (but not so confident)
C) Done with page 2 (E field), got it!
D) Done with p. 2 (but not so confident)

How was last Thursday’s test for you?

A) *Way* too hard - no fair!

B) Hard, but fair

C) Seemed reasonable.

D) Easy/fair enough, thanks!

E) Almost too easy, really should make it harder next time!

Consider the vector field $\vec{\mathbf{E}} = E_0\,\hat{\mathbf{i}}$, where E_0 is a constant, and consider the closed square path L shown.

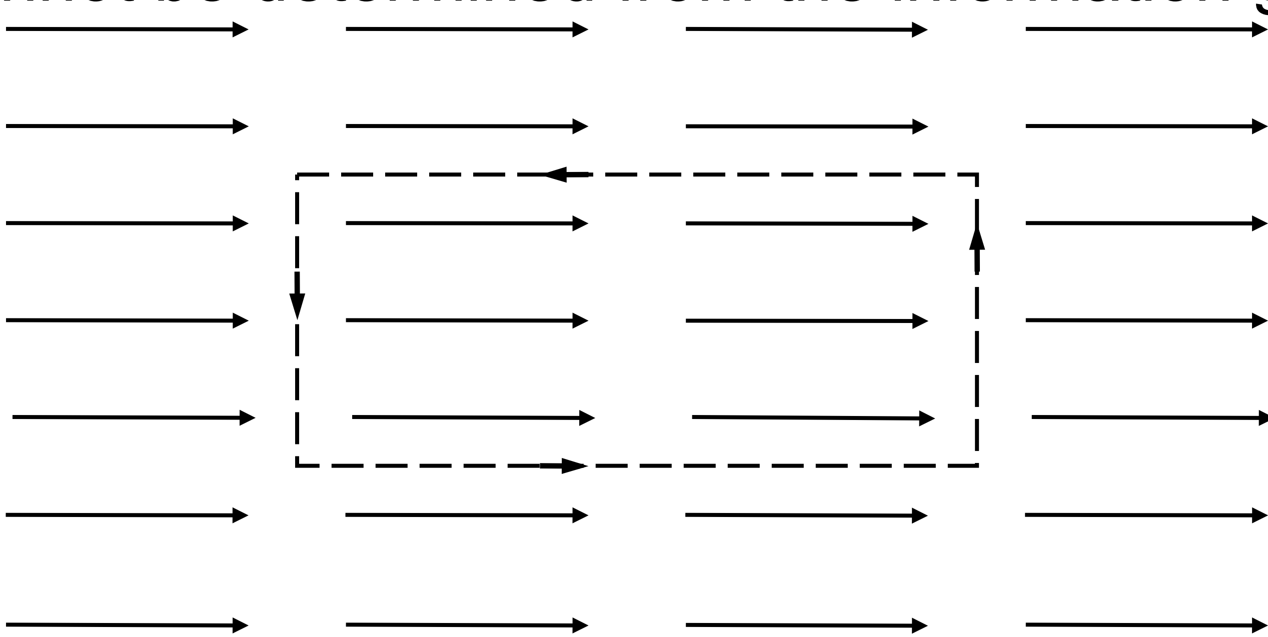
What can you say about the line integral $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$?

A) It is positive

B) It is negative

C) It is zero

D) It cannot be determined from the information given!

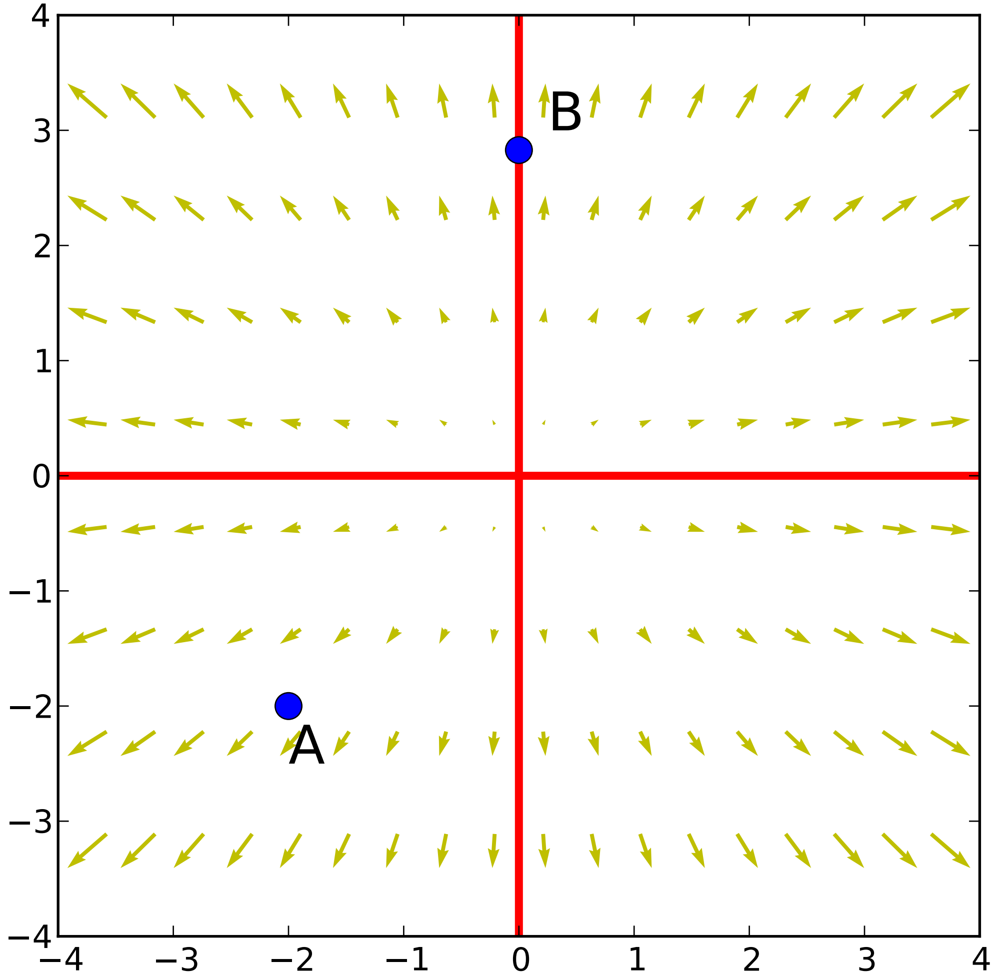


$$\vec{f} = x\hat{i} + y\hat{j}$$

$$\vec{f} = x\hat{i} + y\hat{j}$$

B=(0, Sqrt[8])

A=(-2,-2)



When doing line integrals in cartesian (2D), we start from

$\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$ and thus use $d\vec{\mathbf{r}} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}}$

whenever evaluating work done by F: $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$

In plane-polar coordinates, $\vec{\mathbf{r}} = r\,\hat{r}$

What should we use for d**r**?

A) $d\vec{\mathbf{r}} = dr\,\hat{r}$

B) $d\vec{\mathbf{r}} = \hat{r}\,dr\,d\phi$

C) $d\vec{\mathbf{r}} = \hat{r}\,r\,dr\,d\phi$

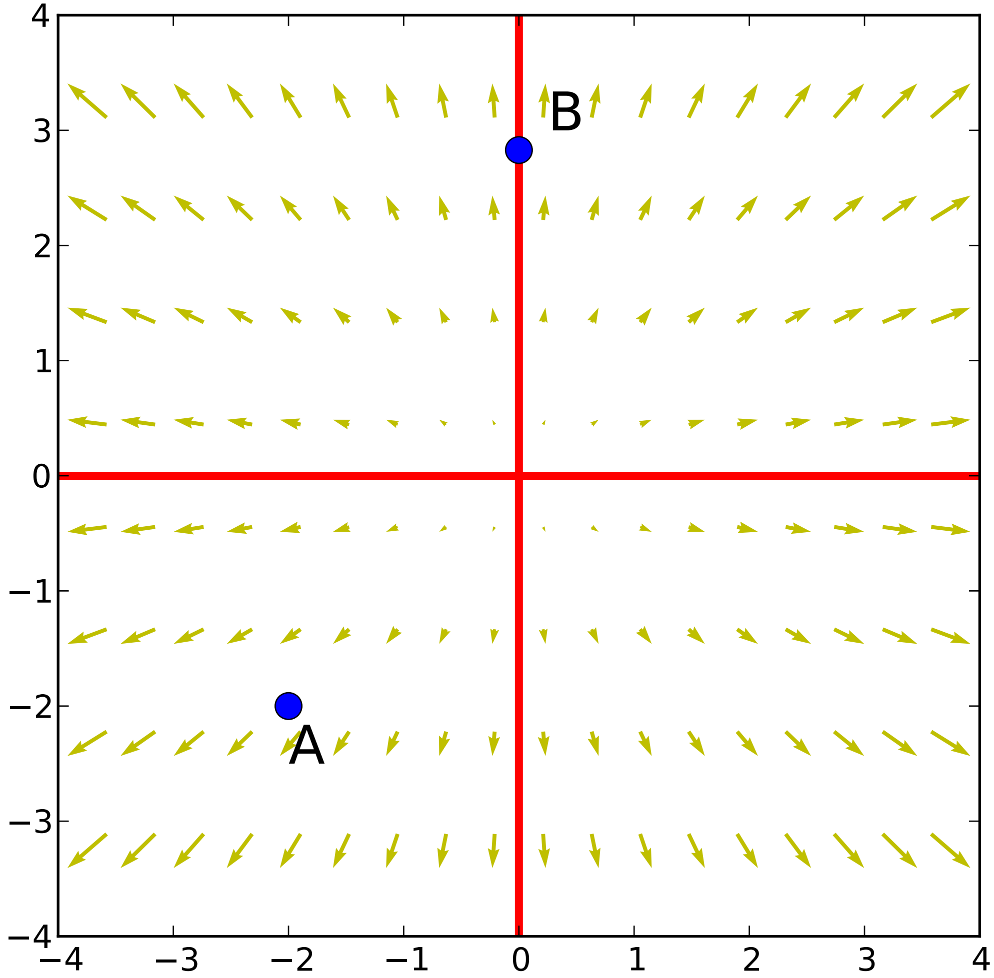
D) $d\vec{\mathbf{r}} = dr\,\hat{r} + d\phi\,\hat{\phi}$

E) $d\vec{\mathbf{r}} = dr\,\hat{r} + r\,d\phi\,\hat{\phi}$

$$\vec{f} = x\hat{i} + y\hat{j}$$

B=(0, Sqrt[8])

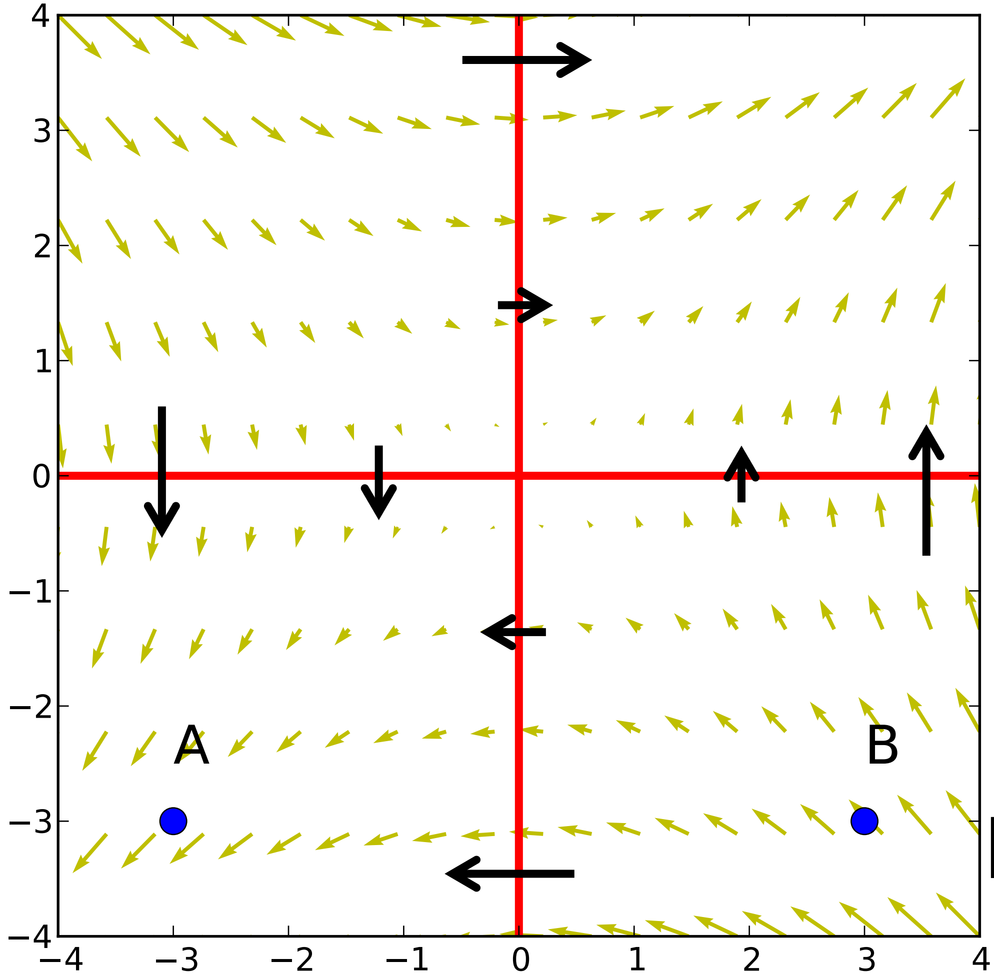
A=(-2,-2)



$$\vec{f} = y\hat{i} + x\hat{j}$$

$$\vec{f} = y\hat{i} + x\hat{j}$$

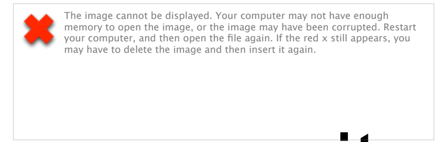
A=(-3,-3)



B=(3,-3)

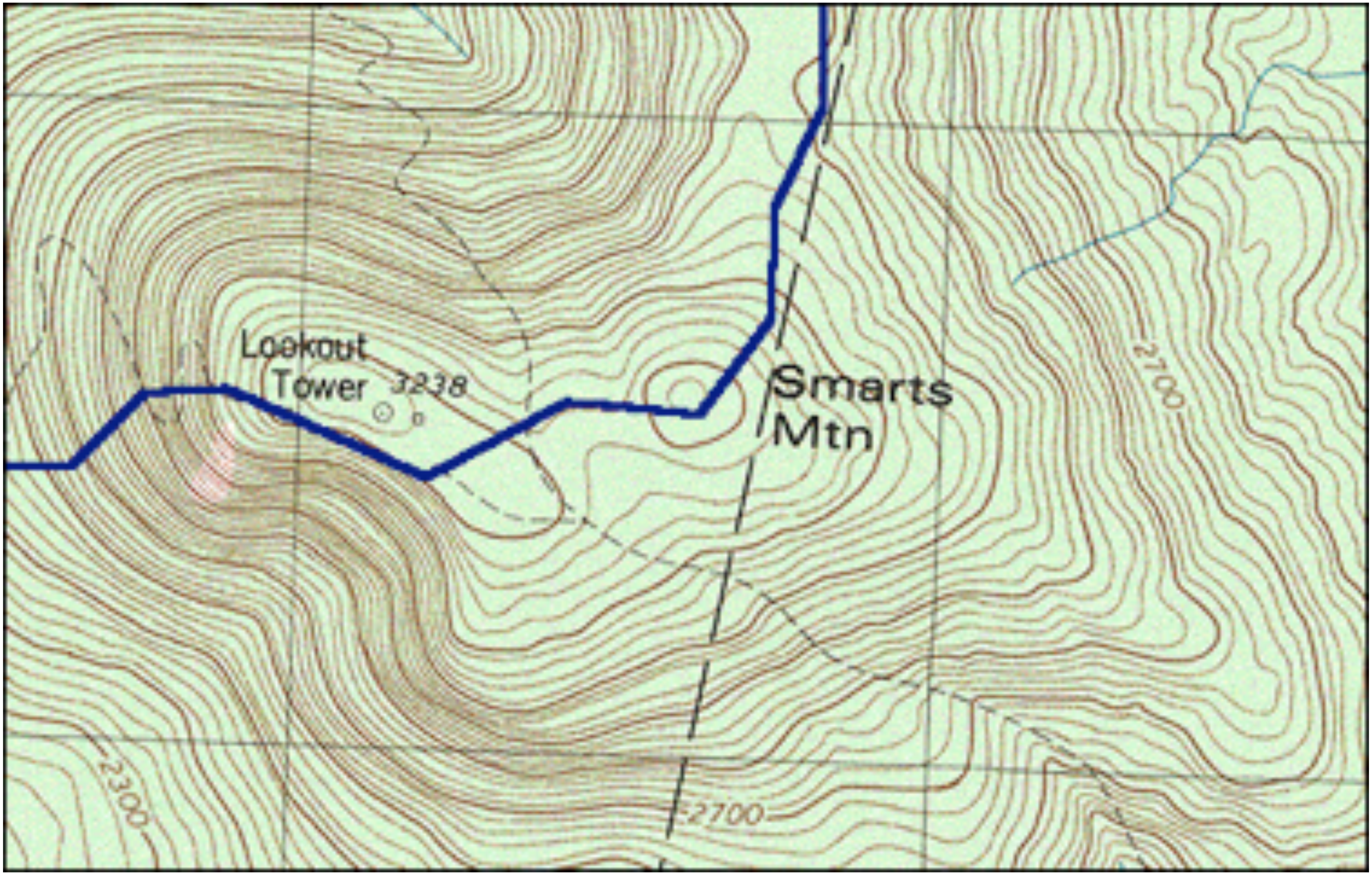
A pretzel is dipped in chocolate.
Its shape is a quarter circle (R=2 cm)
(Straight from the origin to (2,0), a circular arc to (0,2), and straight back to the origin)
The linear chocolate density is $\lambda = c(x^2 + y^2)$, where $c = 3\ g/cm^3$.

How much chocolate is on a pretzel?

Consider an Electric field given by 

. Compute the work done on a charge +q as it moves around that same "pretzel" path, in the CCW direction.

(Is this E field conservative?)


Lookout
Tower
3238
Smarts
Mtn
2700
2300
2700

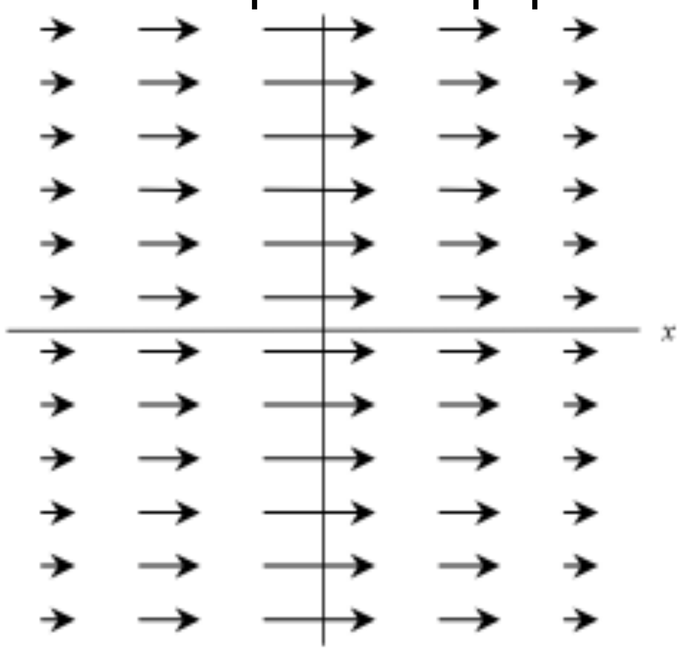
Click A when done with p. 1, etc. **If you finish early .....**

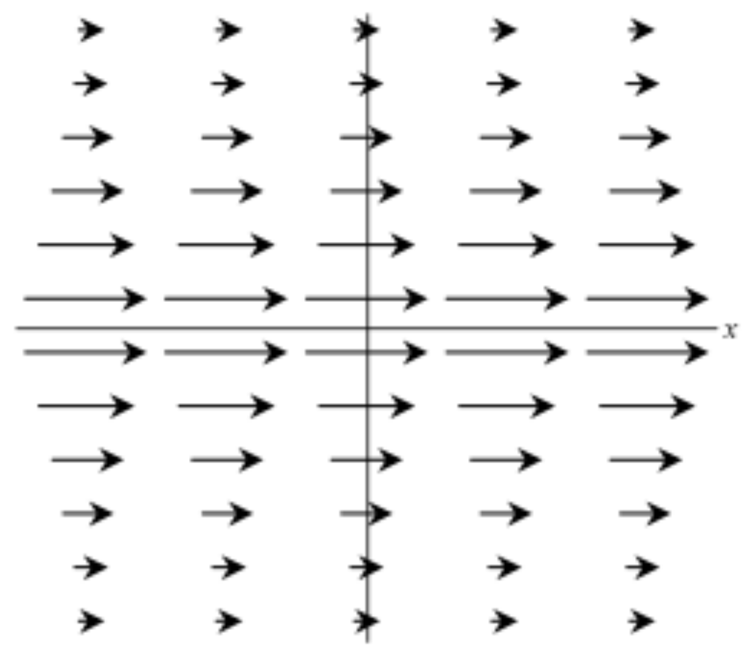
Do you agree or disagree with the following statements?

1) "For a conservative force, the magnitude of the force is related to potential energy. The larger the potential energy, the larger the magnitude of the force."

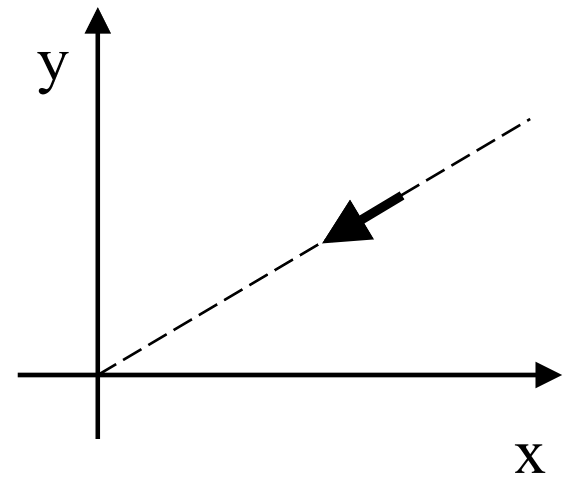
2) "For a conservative force, the magnitude of the force is related to potential energy. For any equipotential contour line, the magnitude of the force must be the same at every point along that contour."

Can you come up with equipotential lines for the 2 force fields below?





Consider an infinitesimal "step" directed radially inward, toward the origin as shown. In spherical coordinates, the correct expression for d**r** is:



A) $d\vec{r} = +dr\,\hat{r}$

B) $d\vec{r} = -dr\,\hat{r}$

C) Neither of these.

cartesian: $d\vec{r} = dx\,\hat{x} + dy\,\hat{y}$

spherical: $d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

Do you agree or disagree with the following statements?

1) "For a conservative force, the magnitude of the force is related to potential energy. The larger the potential energy, the larger the magnitude of the force."

2) "For a conservative force, the magnitude of the force is related to potential energy. For any equipotential contour line, the magnitude of the force must be the same at every point along that contour."

A) Agree with 1 and 2
B) Agree only with 1
C) Agree only with 2
D) Disagree with both

Which of the following are conservative forces?
I- friction (velocity **in**dependent, like μN)
II- Gravity (non-uniform case, e.g. in astronomy)
III- the normal force (between two solid, frictionless objects)

A) i only   B) ii only   C) iii only
D) i and ii only   E) Other/not sure

Which of the following are conservative forces?
I- friction (velocity **in**dependent, like μN)
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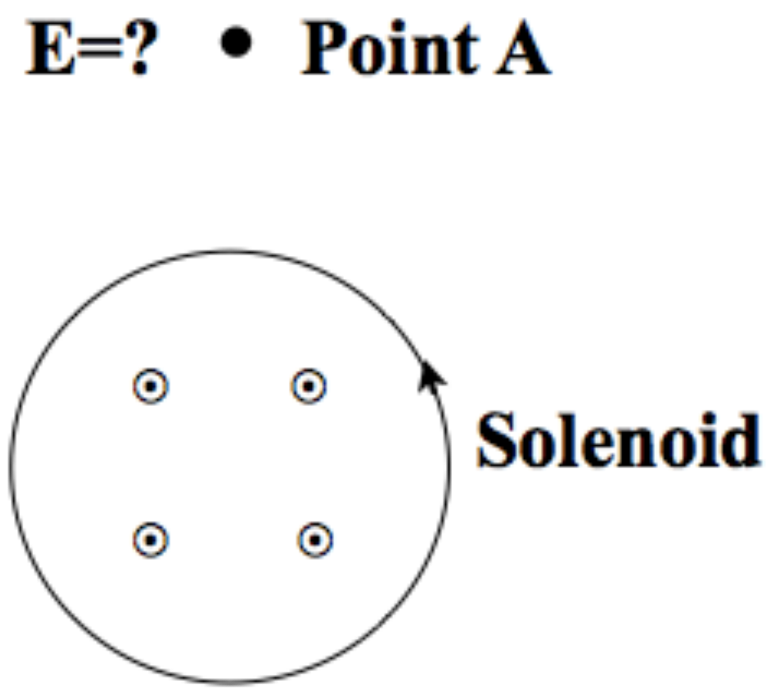
III- the normal force (between two solid, frictionless objects)

A) i only B) ii only C) iii only
D) i and ii only E) Other/not sure

Consider the E-field at Point A, just outside a solenoid, which has a B-field that is increasing with time. Does this E-field produce a conservative force?
(Hint: Consider the motion of positive charge in a wire loop that passes through point A.)

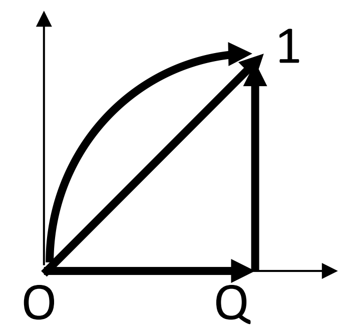
A) Yes, it produces a conservative force
B) No, it is not conservative



Last Class:

A. Work $\Delta T = T_2 - T_1 = \int_1^2 \mathbf{F}\cdot d\mathbf{r} \equiv W(1 \to 2)$

Line integral along a specific path



B. Conservative Forces

1. F(r)
2. Work independent of the path

C. Tutorial

Concepts: Equipotential surface, PE,Gradient, Force

Motivation: Force is proportional to gradient of PE but opposite direction

This Class:

- Rigorous definition of potential energy U(**r**)
- Formal derivation of the relation between force and potential energy

  Given U find F

  Given F find U

- Rigorous understanding of conservative forces.

A charge q sits in an electric field $\mathbf{E} = E_0\,\mathbf{i}$.

What is the potential energy U(r)?

(Assume U(0)=0)

A) $+qE_0 x\,\hat{\mathbf{i}}$

B) $-qE_0 x\,\hat{\mathbf{i}}$

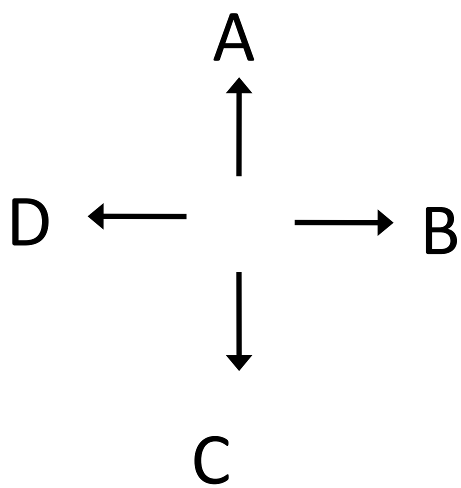
C) $+qE_0 x$

D) $-qE_0 x$

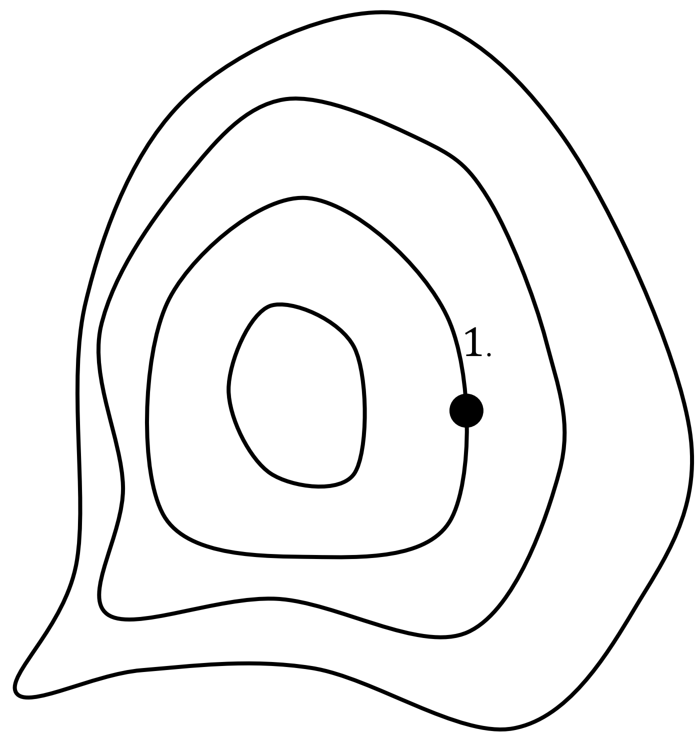
E) Something else!

Consider the contour plot of a function u(x,y), where the central contour corresponds to the largest value of u.

Which way does ∇u point, at point 1?



E) Other/???



Can you explain why?

What is the potential energy of M in terms of ϕ? (Take U=0 at ϕ=0)

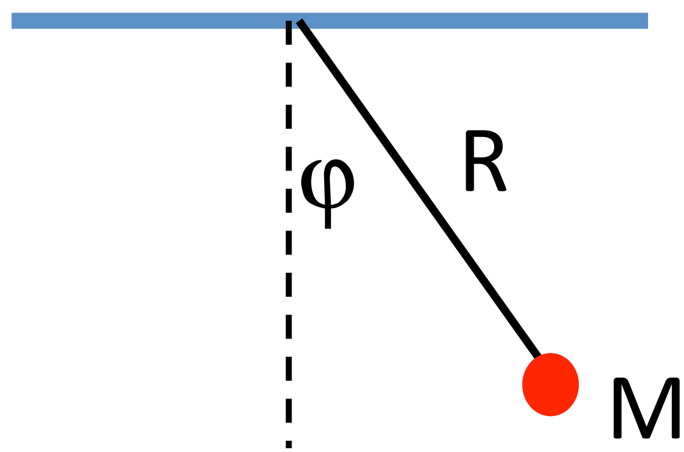
A) MgRcosϕ

B) MgR(cos(ϕ)-1)

C) MgRsin(ϕ)

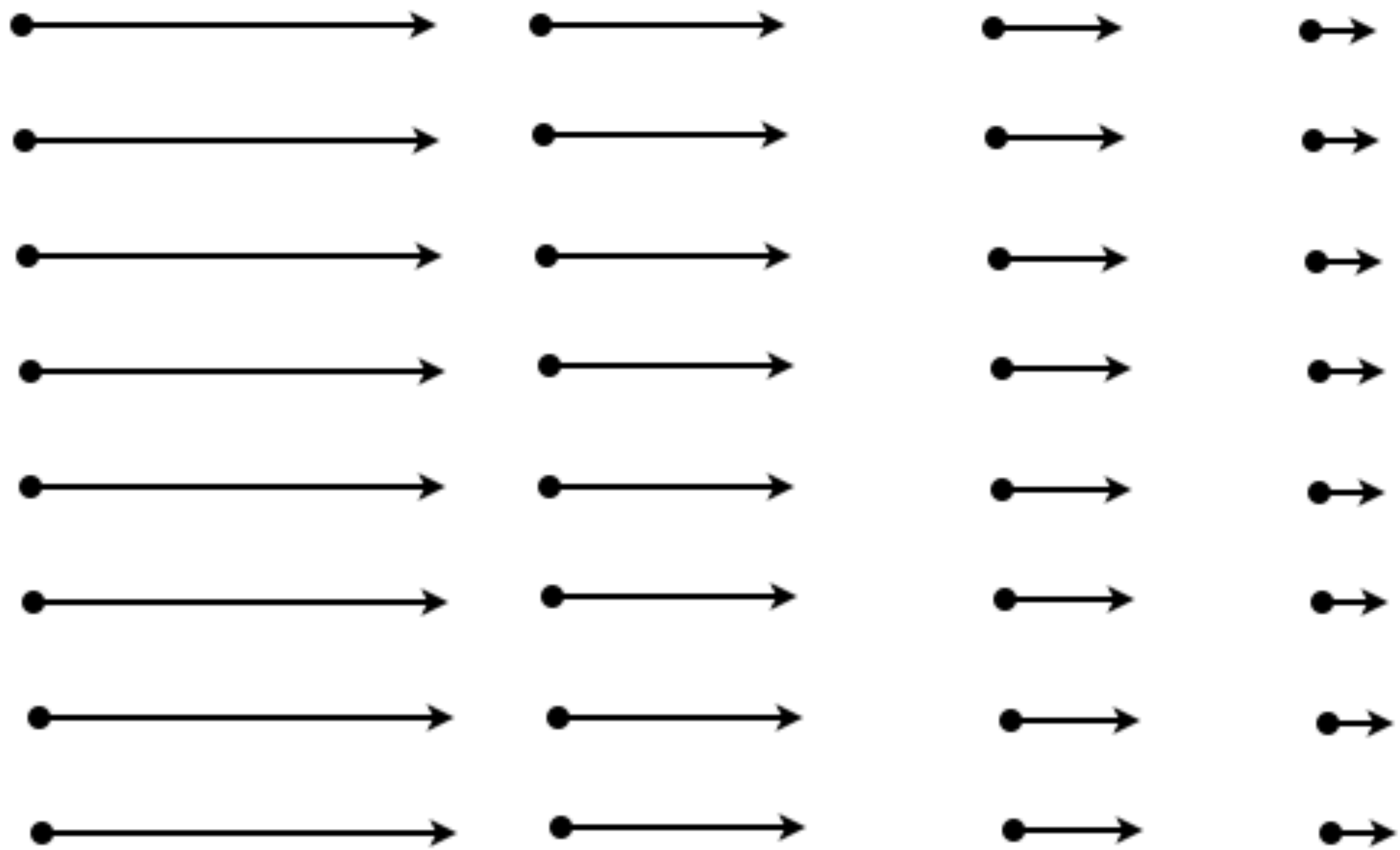
D) MgR(1-cos(ϕ))

E) MgR(1-sin(ϕ))



What is the curl (∇x**F**) of this vector field, **F**?

A) = 0 everywhere
B) $\neq$ 0 everywhere
C) =0 in some places
D) Not enough info to decide



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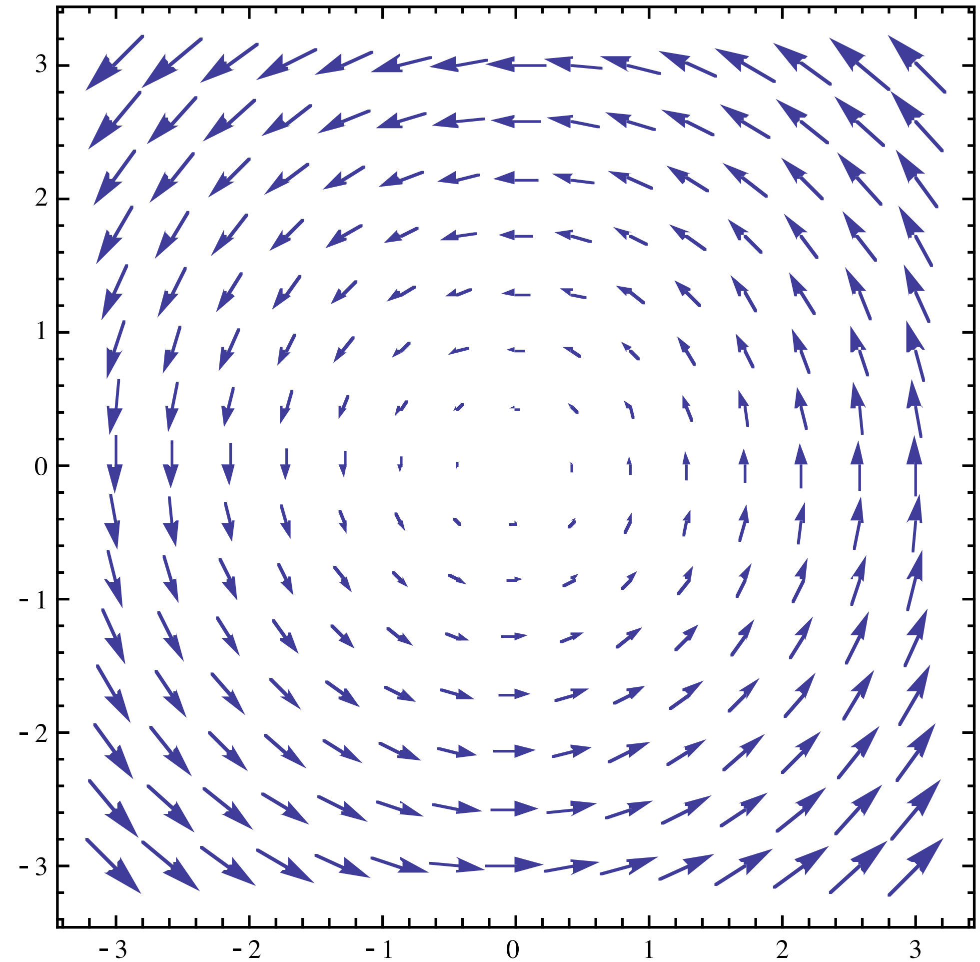
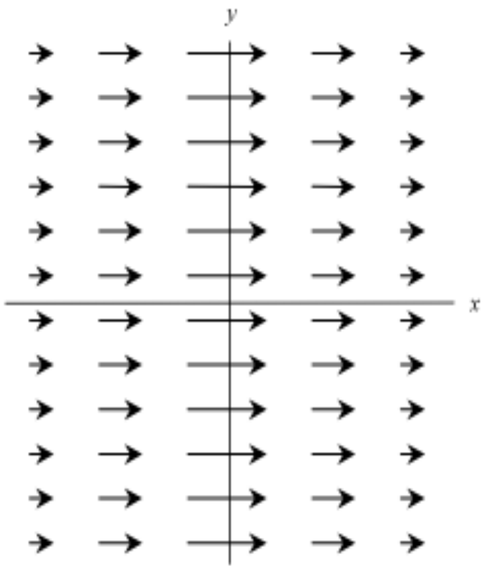
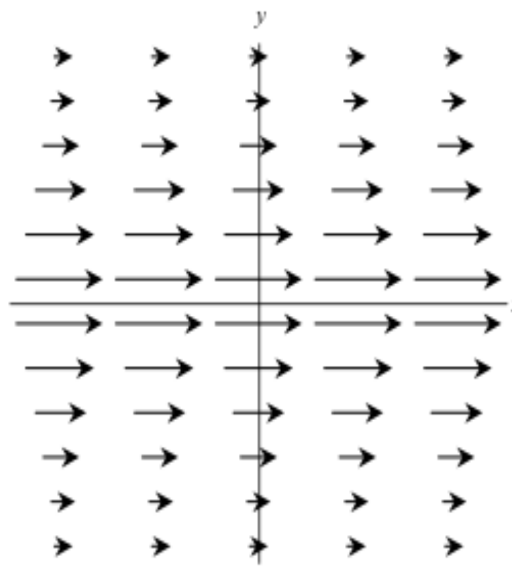
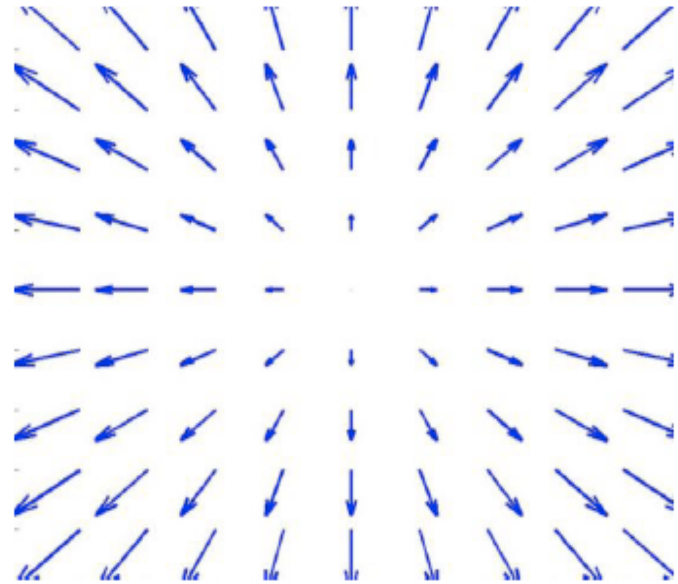


Can you come up with equipotential lines for the 3 force fields below?







Draw it if possible

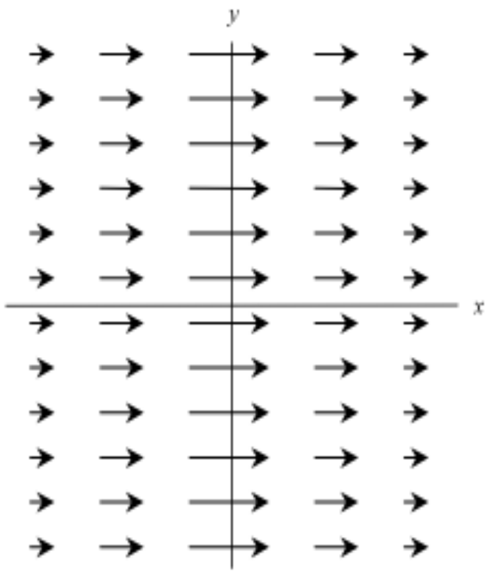
What is the curl (∇x**F**) of this vector field, **F**?

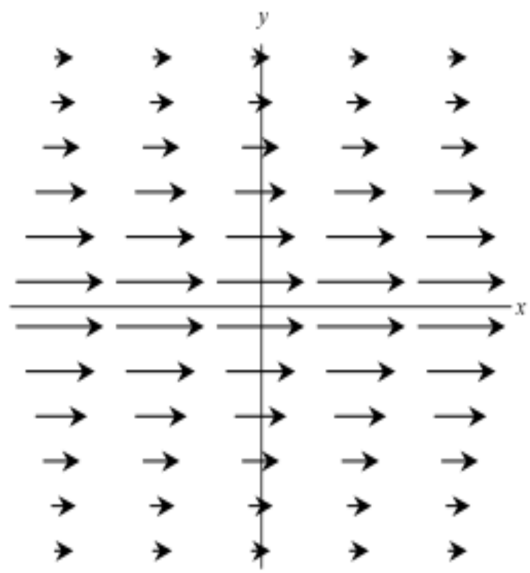
A) = 0 everywhere

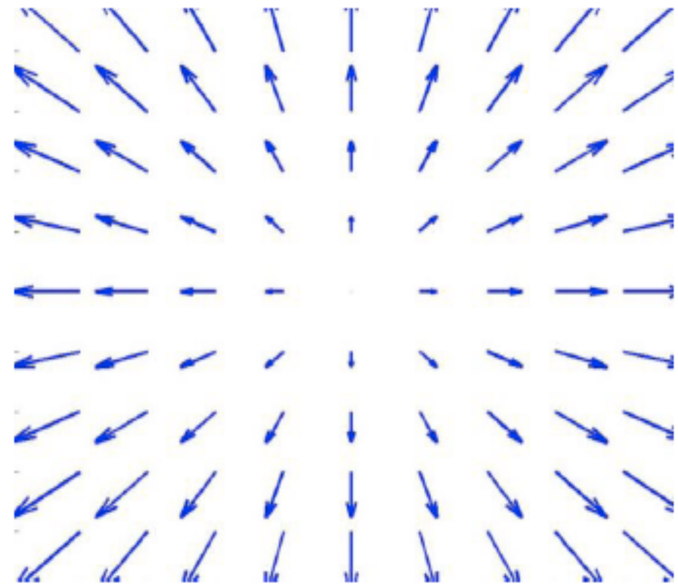
B) ≠ 0 everywhere

C) =0 in some places

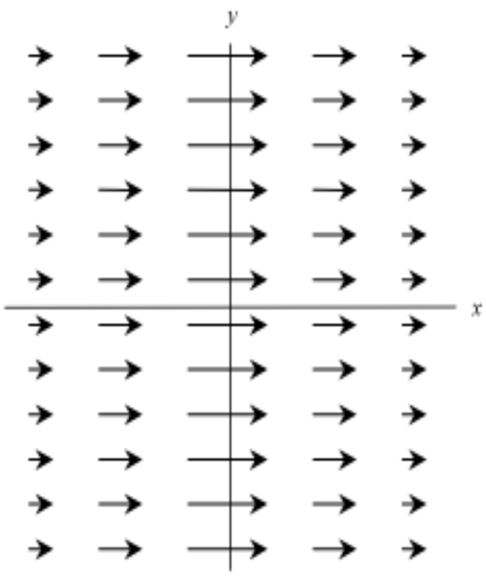
D) Not enough info to decide

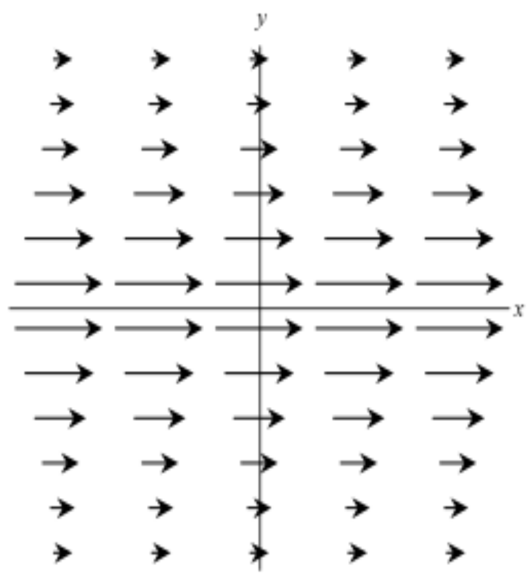


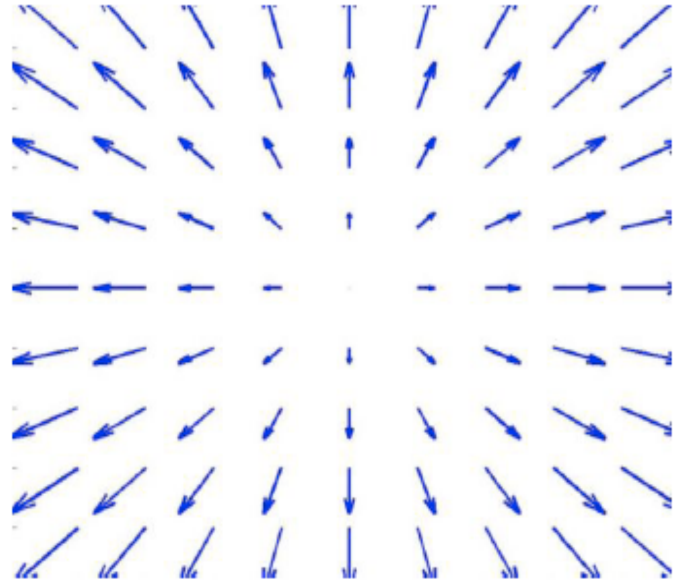




Now think about line integrals about various loops, to convince yourself they are always 0 in the “curl-free” cases.







Discussion questions:

- Why is there a minus sign in $F = -dU/dx$?

- What is the physical meaning of the zero of potential energy?

What is the curl (∇x**F**) of this vector field, **F**?

A) = 0 everywhere
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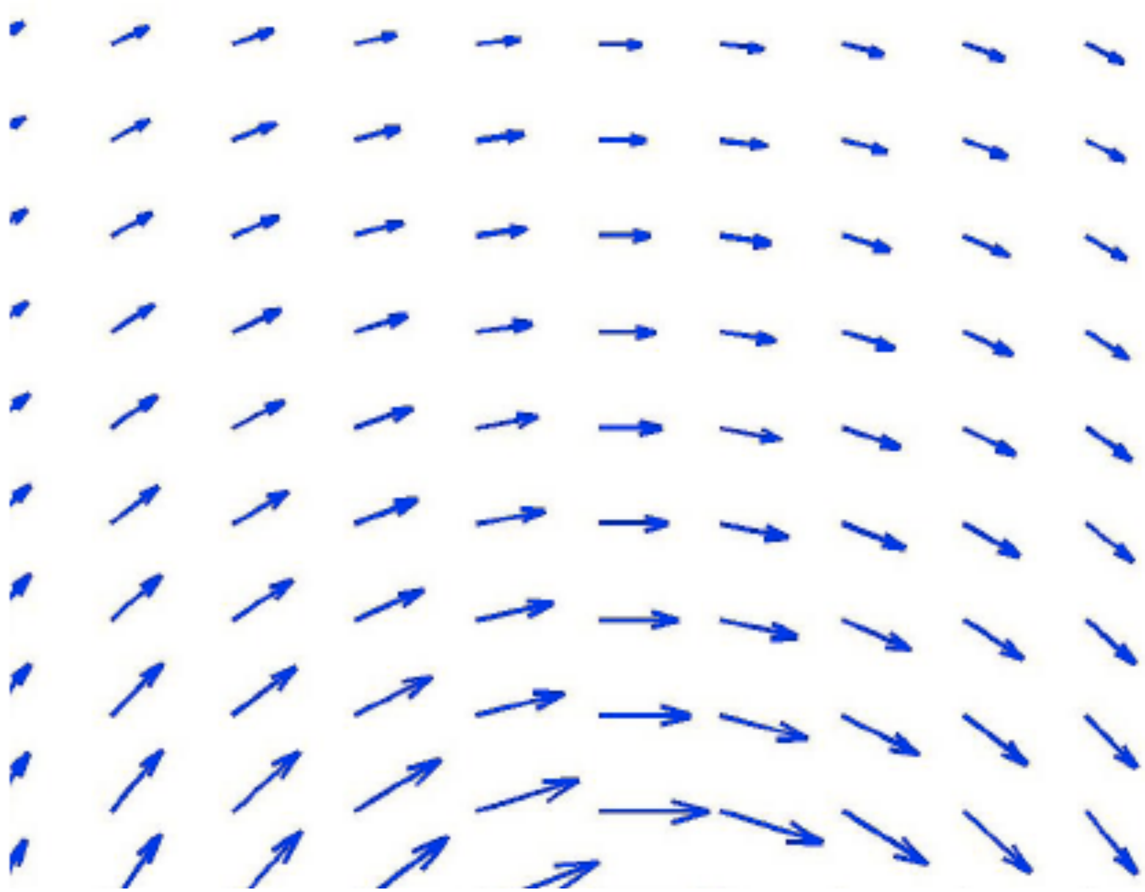
D) Not enough info to decide



What is the curl ($\nabla \times \mathbf{F}$) of this vector field, **F**?

A) = 0 everywhere

B) ≠ 0 everywhere

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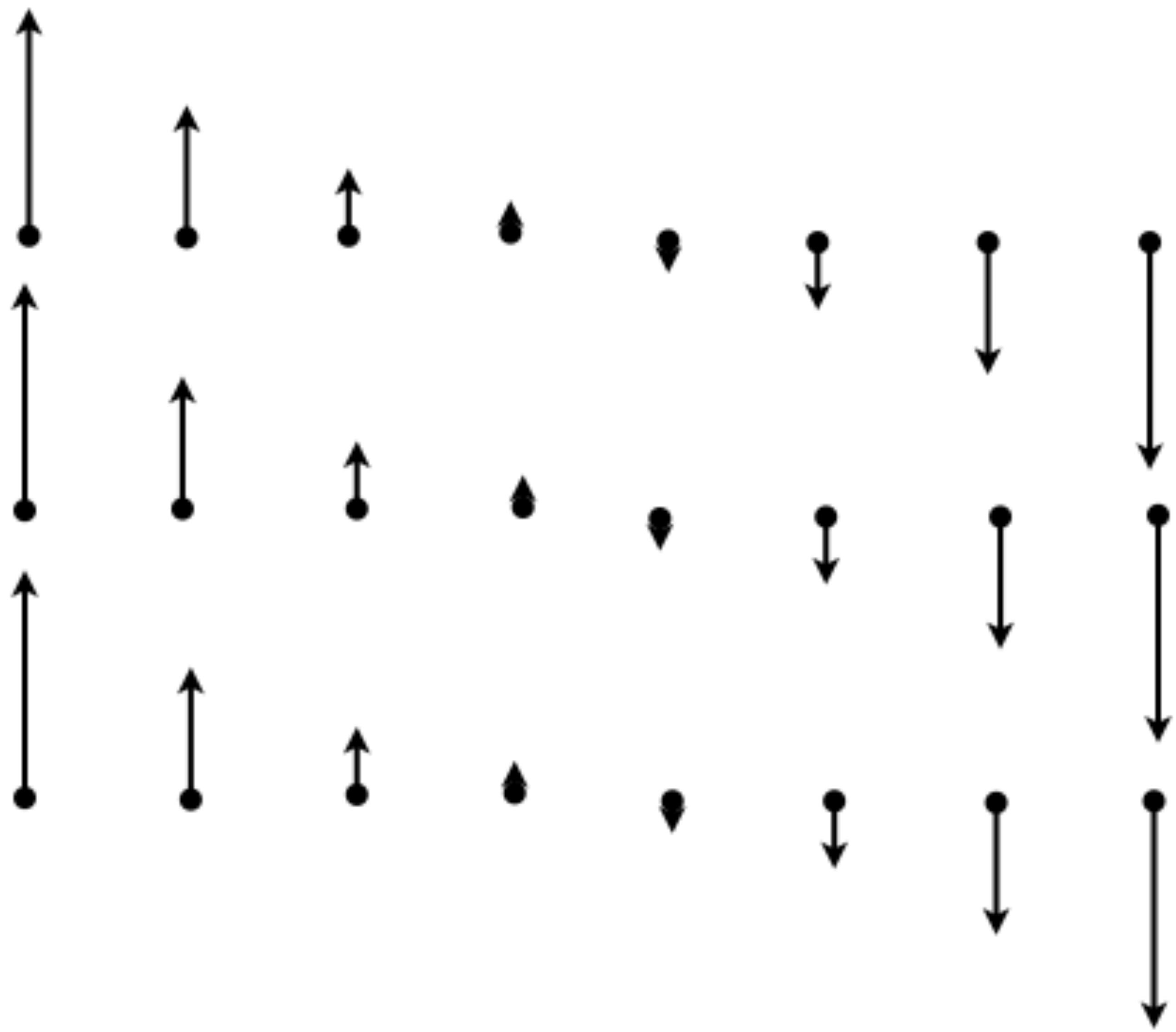

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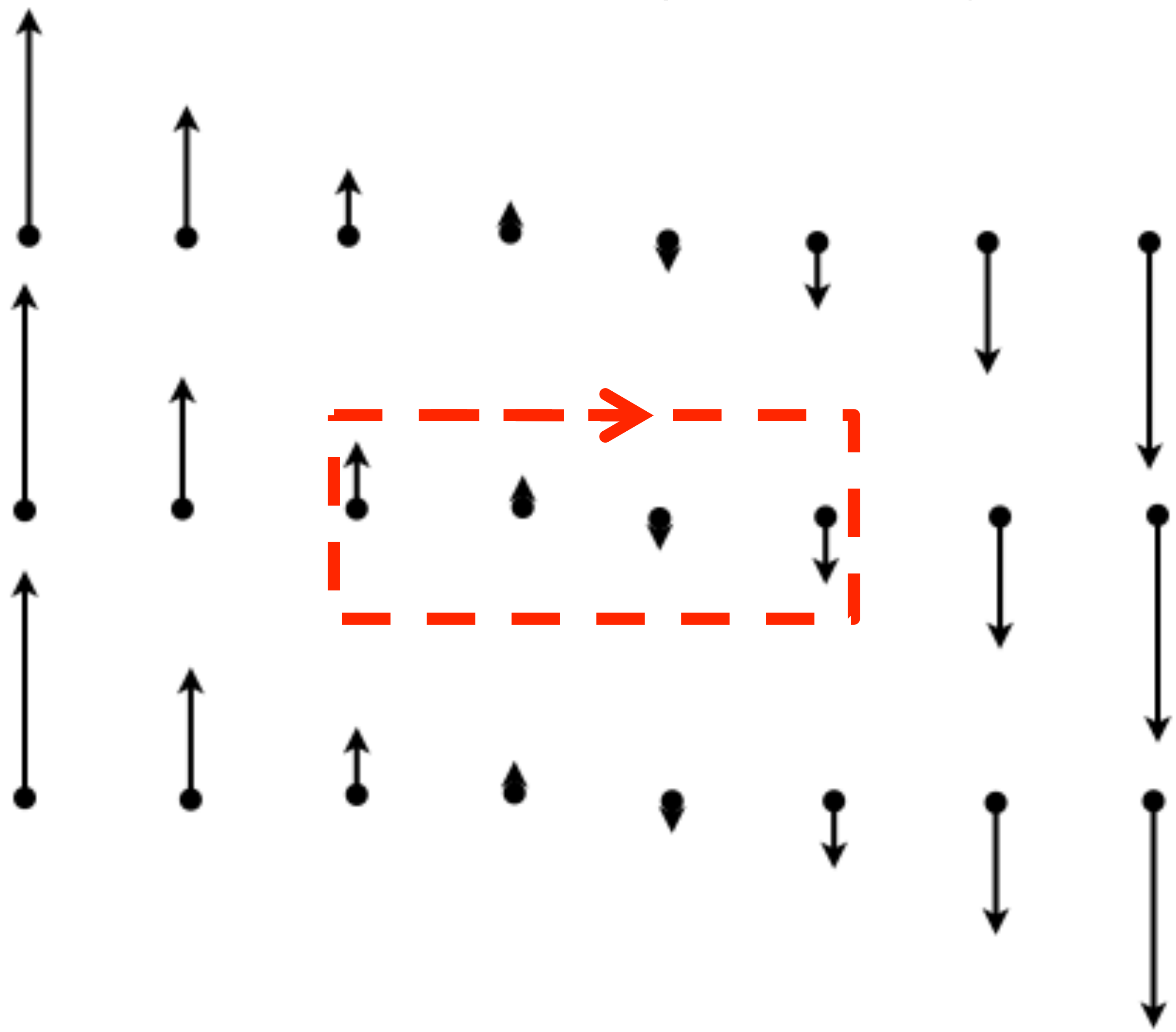
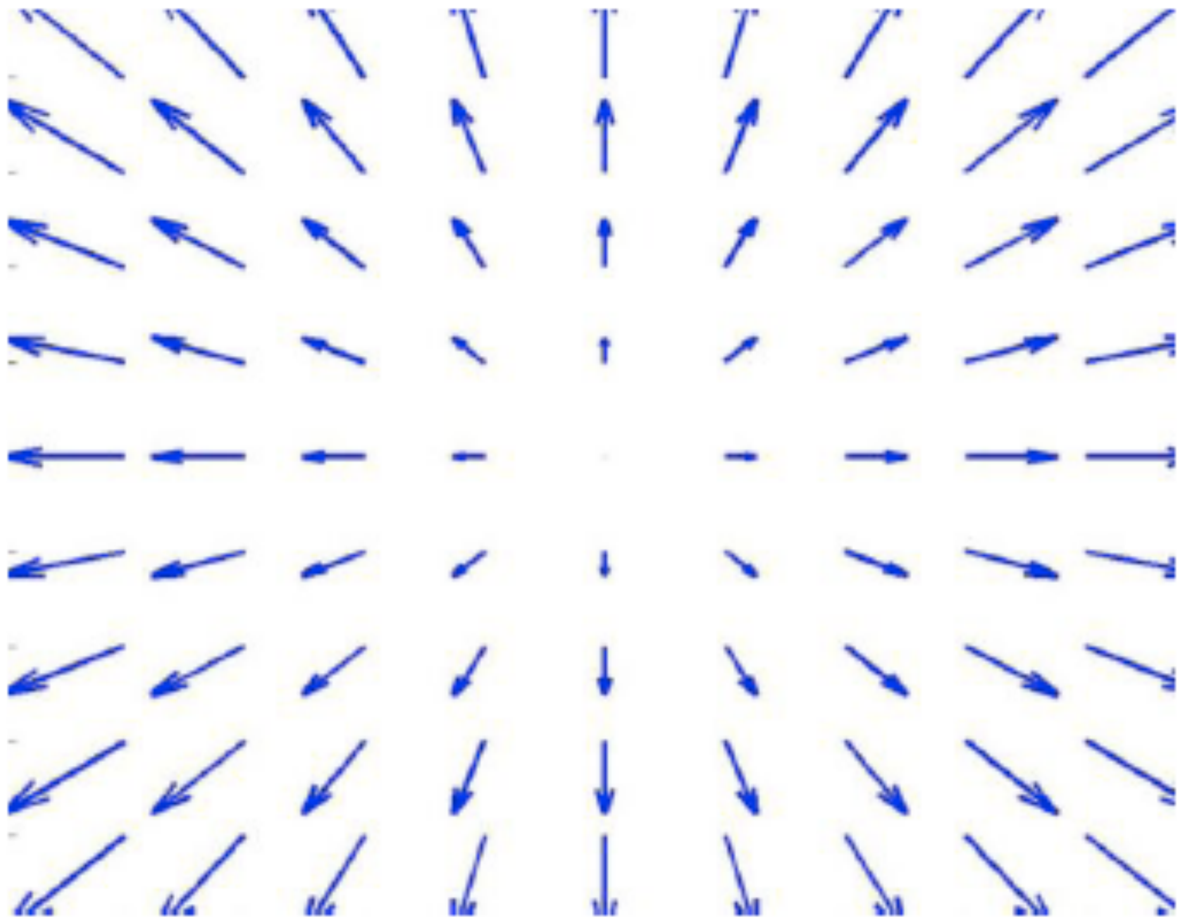
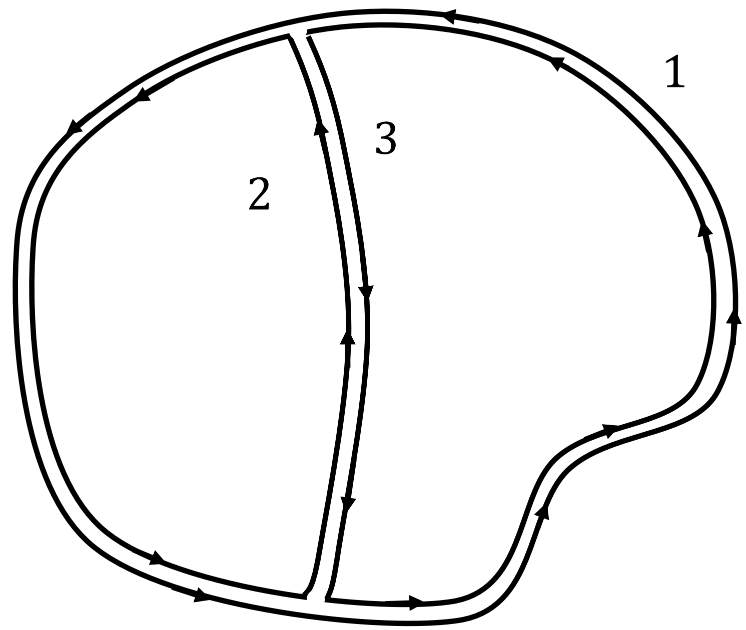


Consider the Coloumb force from a + charge on a positive test charge. Is this a conservative force?

A) Yes
B) No
C) ???



Consider the three closed paths 1, 2, and 3 in some vector field **F**, where paths 2 and 3 cover the larger path 1 as shown. What can you say about the 3 path integrals?



A) $\oint_1 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} > \oint_2 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} + \oint_3 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$

B) $\oint_1 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} < \oint_2 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} + \oint_3 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$

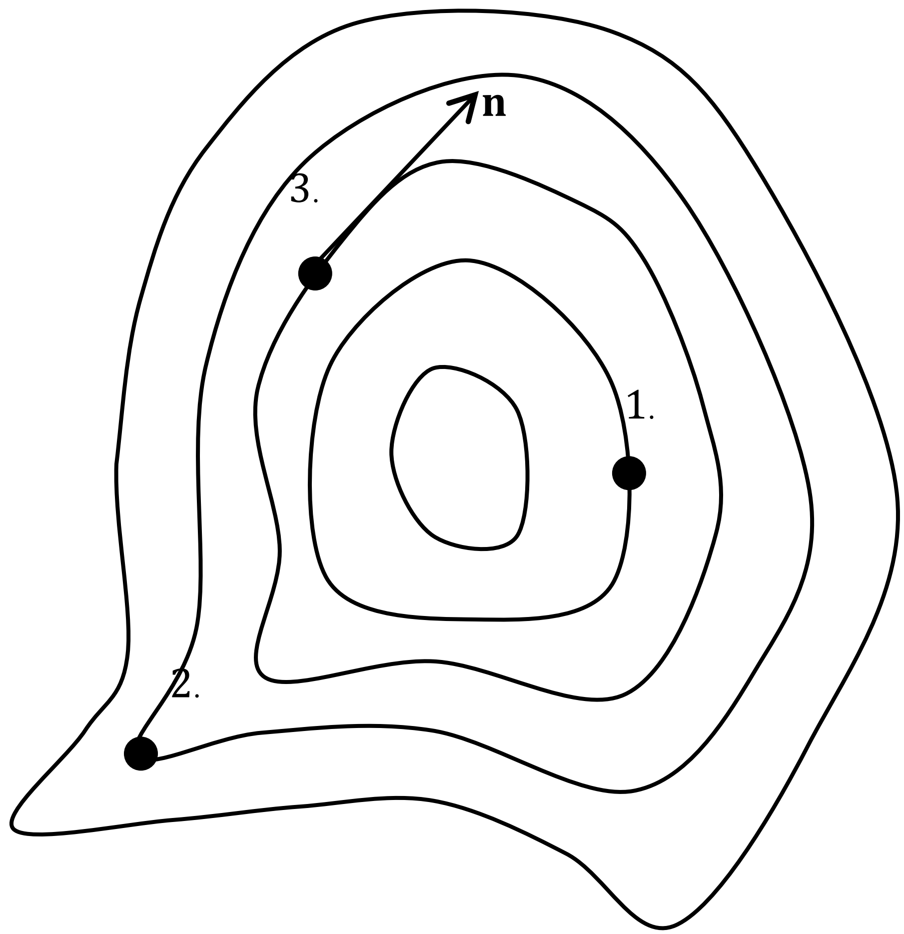
C) $\oint_1 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \oint_2 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} + \oint_3 \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$

D) There is no way to decide without knowing $\vec{\mathbf{F}}$

Consider the contour plot of a function f(x,y), where the central contour corresponds to the largest value of f. What is the sign of the directional derivative of f(x,y) at point 3, in the direction of the unit vector **n** (shown by the arrow)?

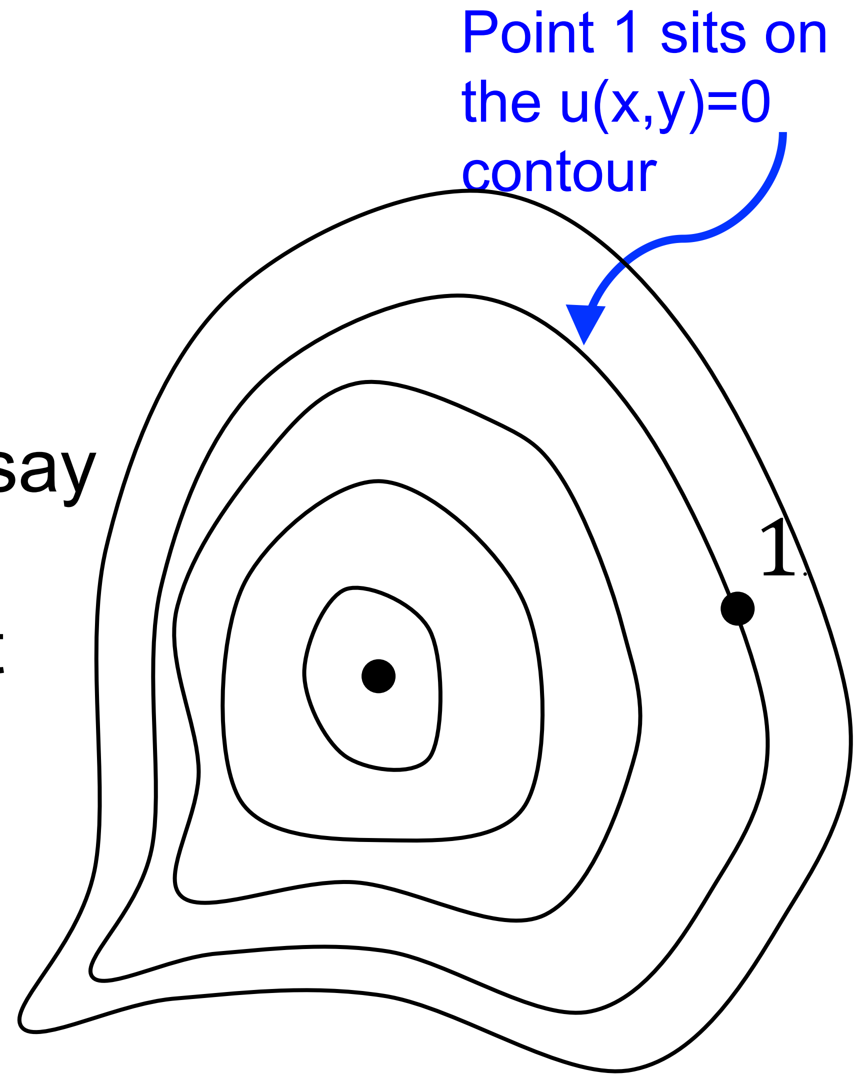
$$\vec{\mathbf{n}} \cdot \nabla f$$

A) >0
B) <0
C) =0
D) ???



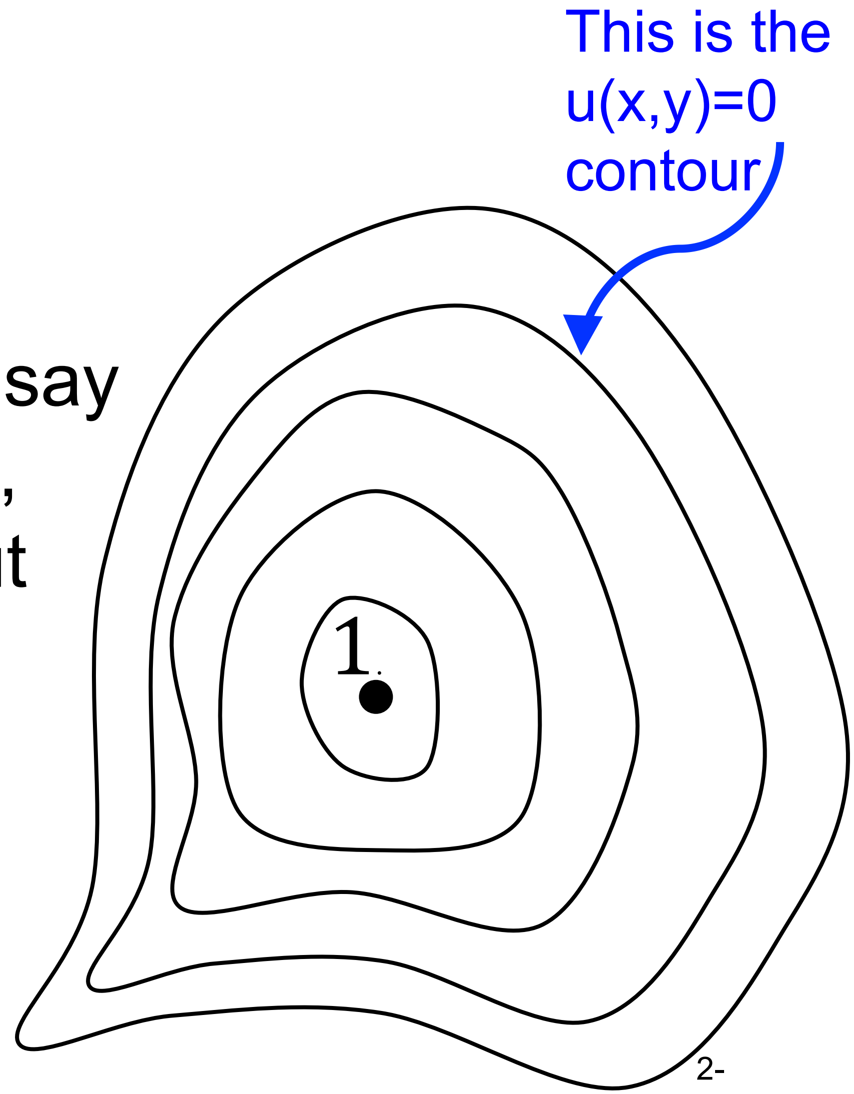
Consider the contour plot of potential energy u(x,y).
Point 1 is somewhere on the u=0 contour.
Can we conclude $\mathbf{F}(r_1)=0$?

A) Yes, it *must* be true
B) It *might* be true, it depends!
C) It *cannot* be true
D) I still don't see how we can say anything about the vector **F**, given only information about the scalar function u



Consider the contour plot of potential energy u(x,y). Point 1 is at an "extremum", and u(1) is nonzero. Can we conclude $\mathbf{F}(r_1)=0$?

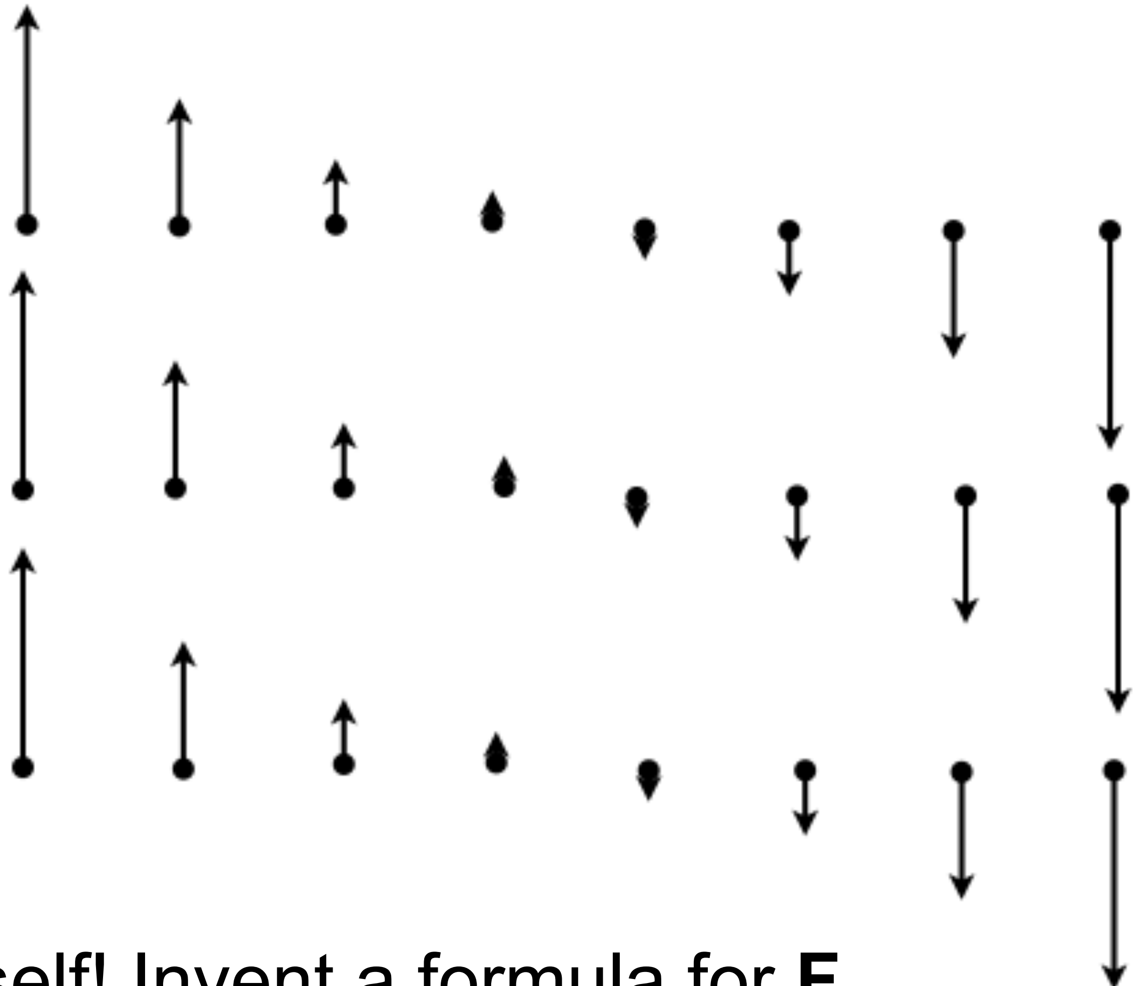
A) Yes, it *must* be true
B) It *might* be true, it depends!
C) It *cannot* be true
D) I still don't see how we can say anything about the vector **F**, given only information about the scalar function u



What is the curl (∇x**F**) of this vector field, **F**?

A) = 0 everywhere B) ≠ 0 everywhere

C) =0 in some places D) Not enough info to decide

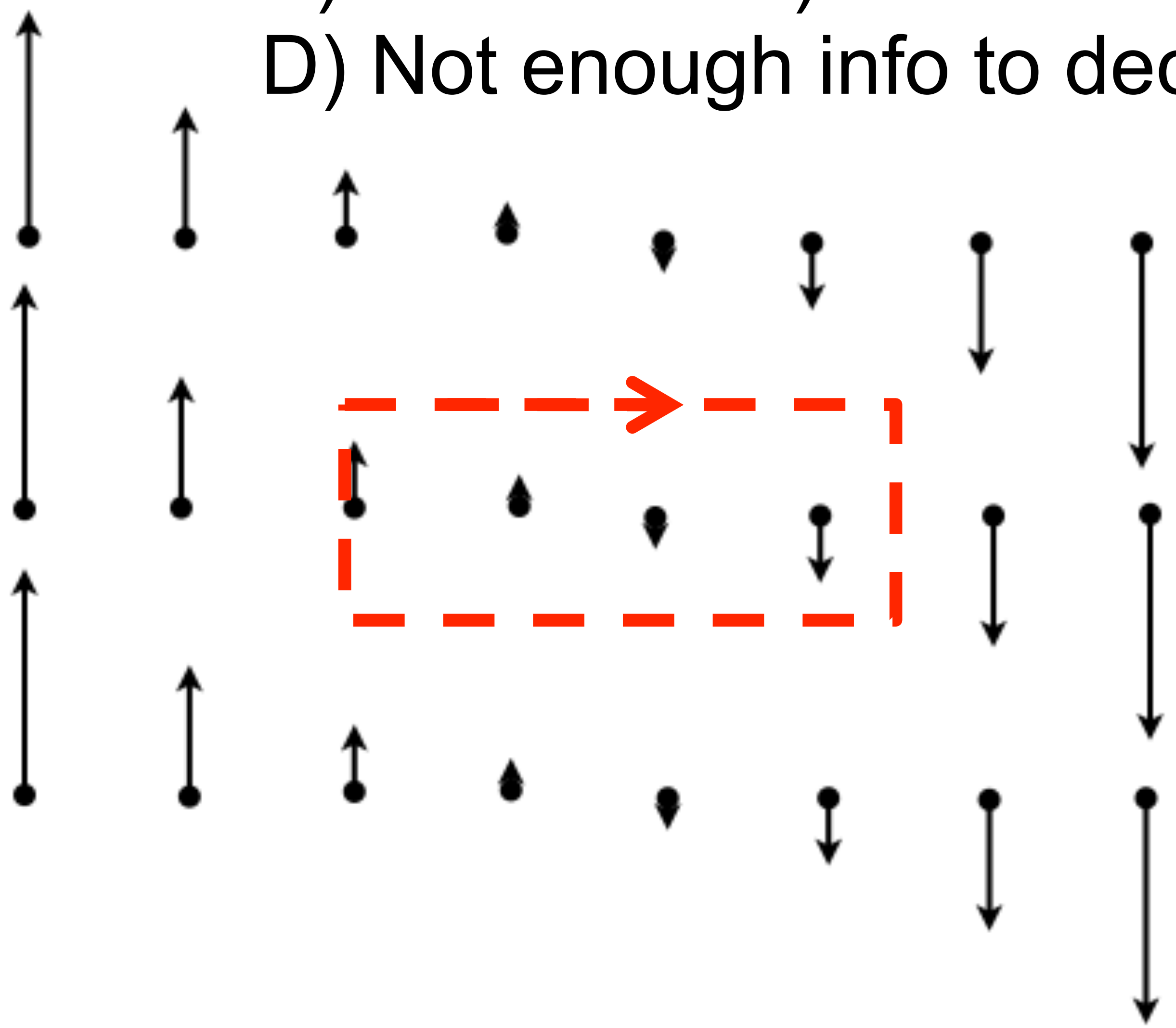


Check yourself! Invent a formula for **F**, and take its curl.

What is the sign of the line integral $\oint \vec{F} \cdot d\vec{r}$ around the red line path shown?

A) = 0 B) >0 C) <0

D) Not enough info to decide



Last class: the following are basically all equivalent!

- F(r) is a conservative force

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- F(r) is a conservative force
- $\int_{r_o}^{r_1} \vec{F}(r') \cdot d\vec{r}'$ is path independent
- PE is well defined $U(r) = -\int_{r_o}^{r} \vec{F}(r') \cdot d\vec{r}'$
- $\vec{F}(r) = -\vec{\nabla} U(r)$
- $\vec{\nabla} \times \vec{F}(r) = 0$
- $\oint \vec{F}(r') \cdot d\vec{r}' = 0$

If the force F(r,t) is *explicitly* time dependent, mechanical energy is not conserved.

Which of the equations below (which were all used in our PROOF that energy is conserved for *conservative* Forces) is incorrect in this case?

A) $dT = \vec{\mathrm{F}} \cdot \mathrm{d}\vec{\mathbf{r}}$

B) $\mathrm{U} = -\int \vec{\mathrm{F}} \cdot \mathrm{d}\vec{\mathbf{r}}$

C) $\mathrm{dU} = \nabla \mathrm{U} \cdot \mathrm{d}\vec{\mathbf{r}}$

D) All of the above are fine, something else is the problem

Of the four labeled points, at which is the force = 0 ?

A) i only B) ii only

C) ii & iv only D) ii, iii and iv only

E) Some other combination!

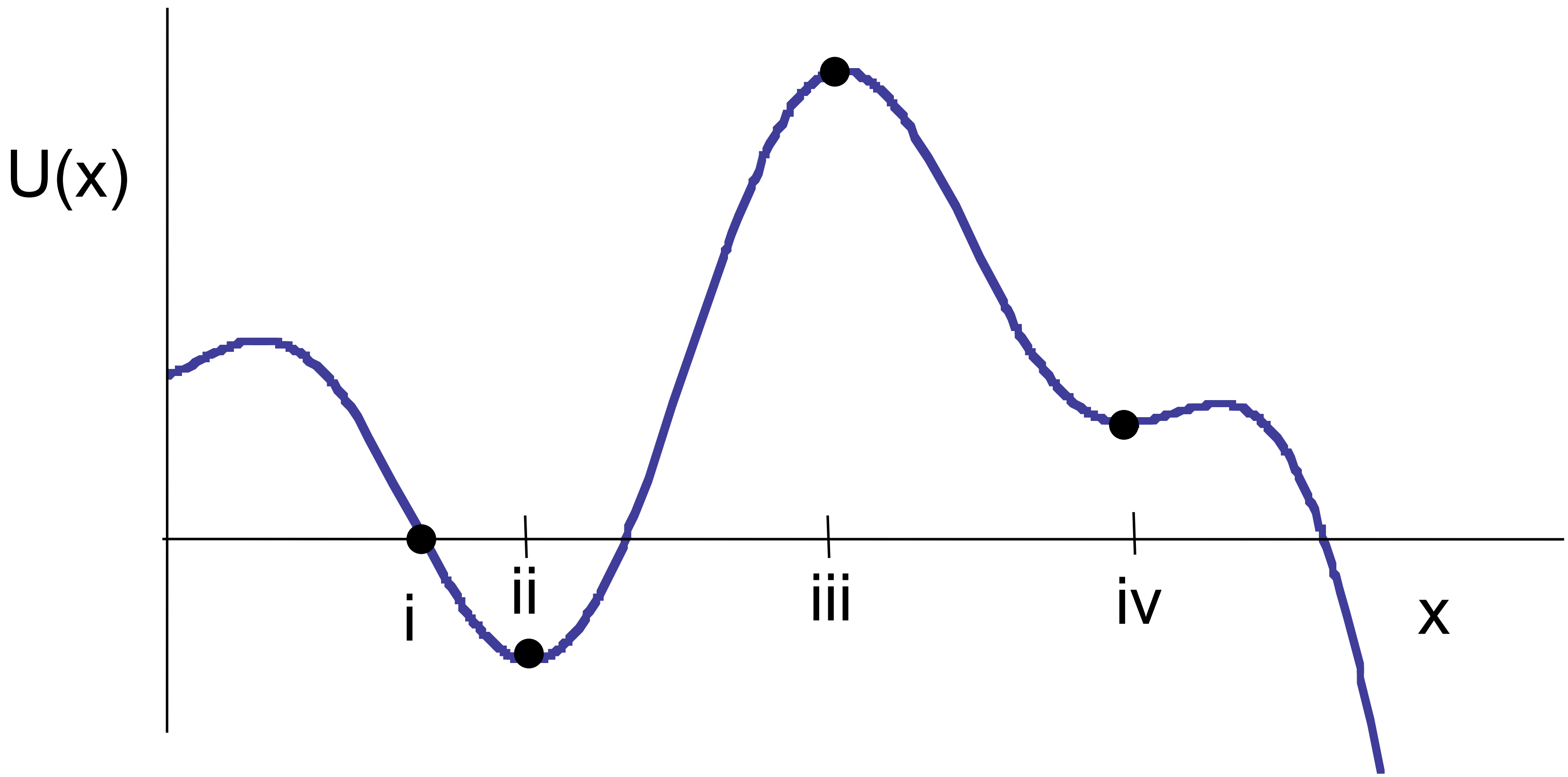
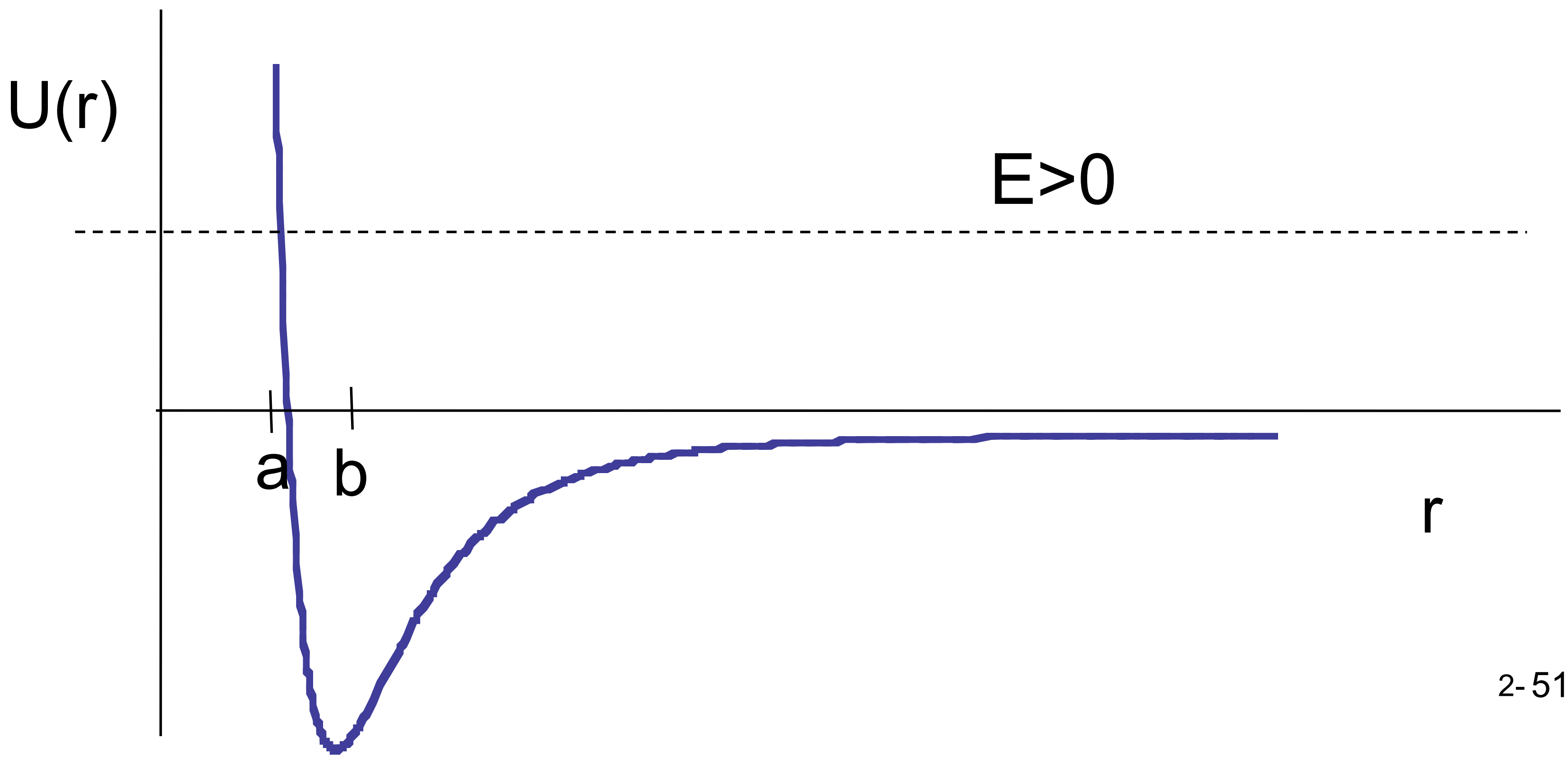


Fig 4.12 of Taylor shows PE of H in an HCl molecule.

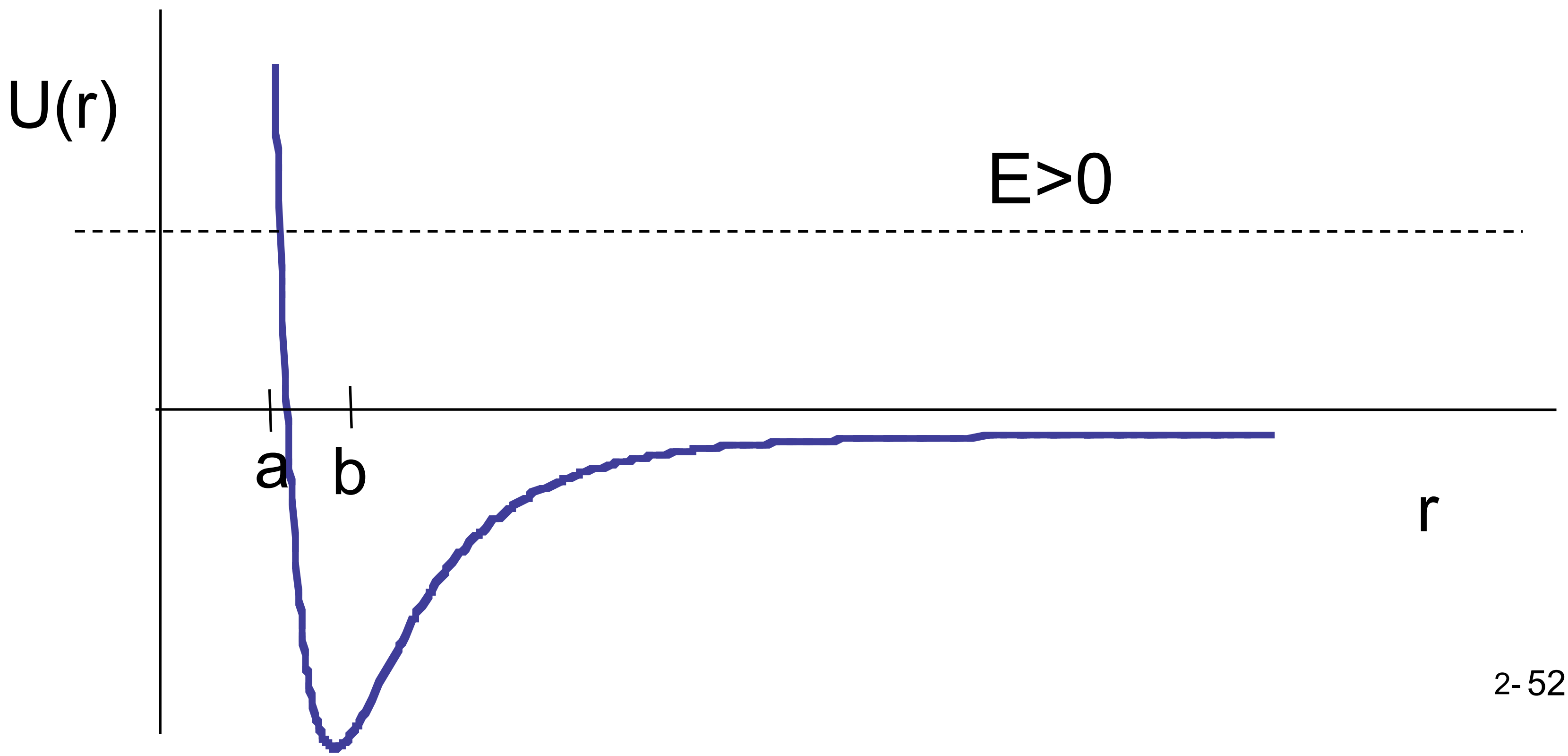
If the mechanical energy, E, is shown (dashed), what's the best description of the motion of the H atom?

A) Trapped, at r=a B) Oscillates around r=b

C) Unbound, the H "escapes" D) Other/????



Given the plot below (with E>0)
Sketch a plot of KE(r) vs r on your whiteboard.
What does this tell you about the motion?



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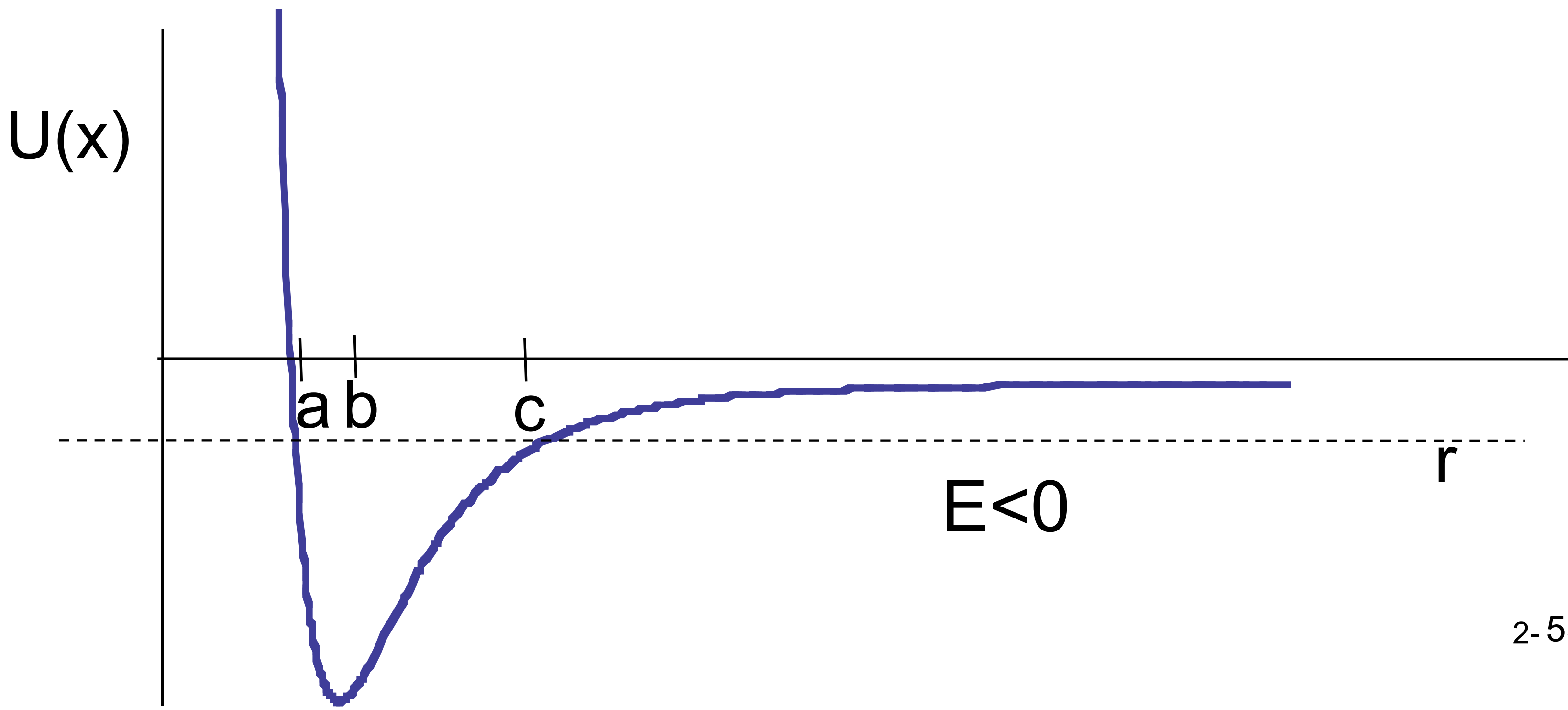
If the energy, E, is shown (dashed), what's the best description of the motion of the H?

A) Trapped, at r=a B) Trapped, at a OR c

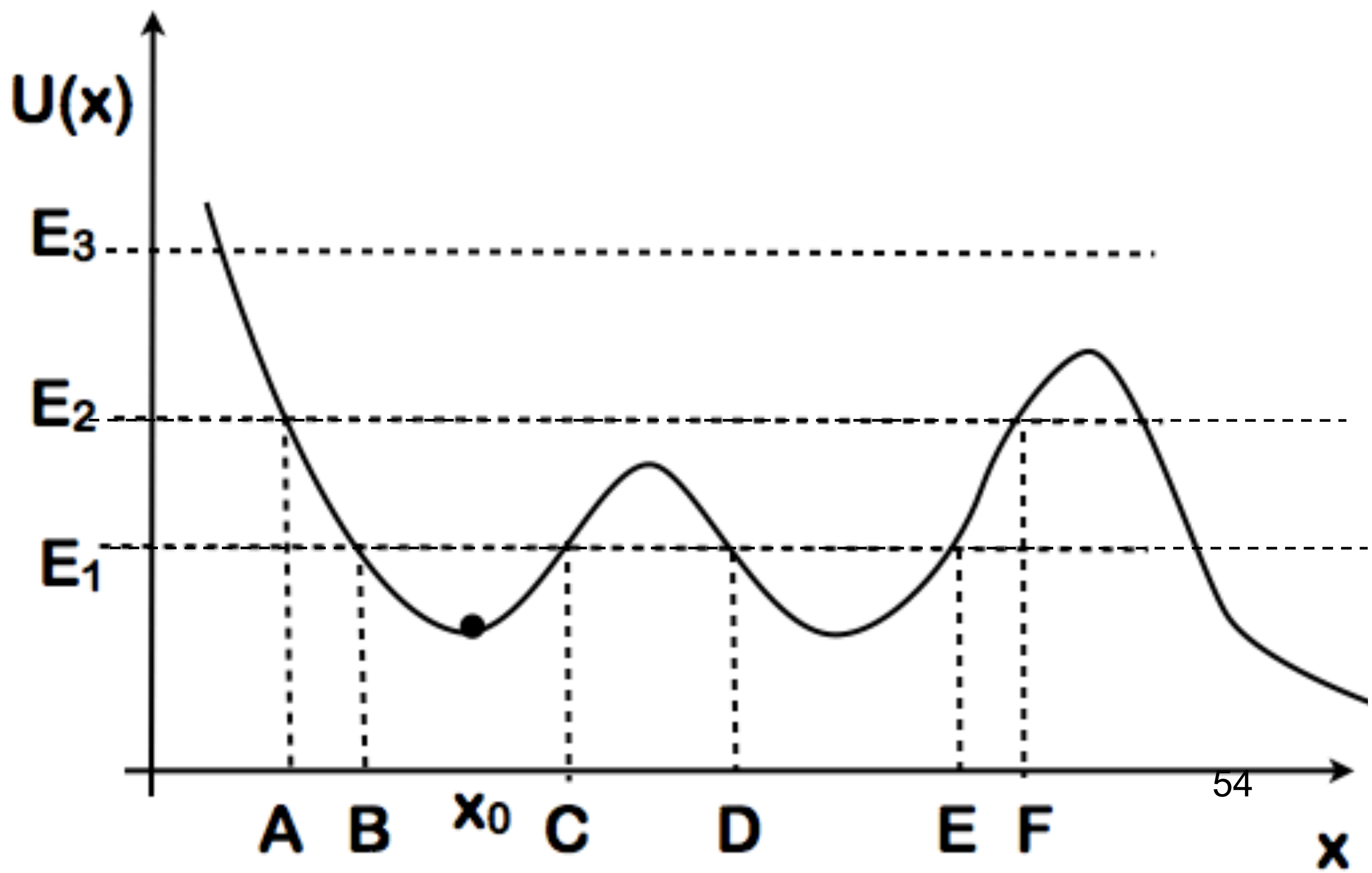
C) Unbound, H "escapes" D) Oscillates around r=b

E) Other/???

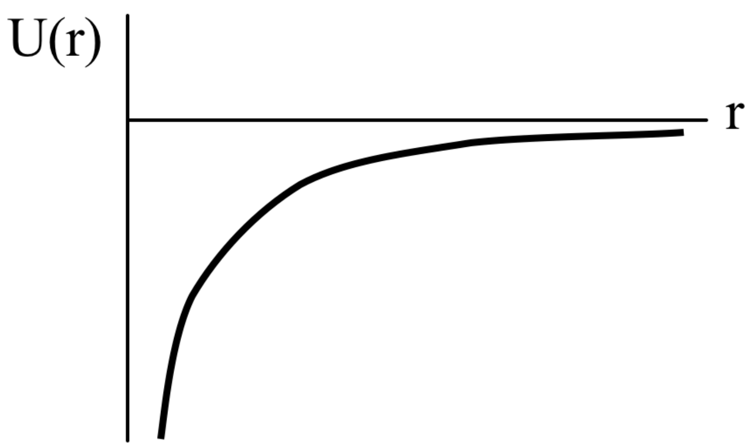


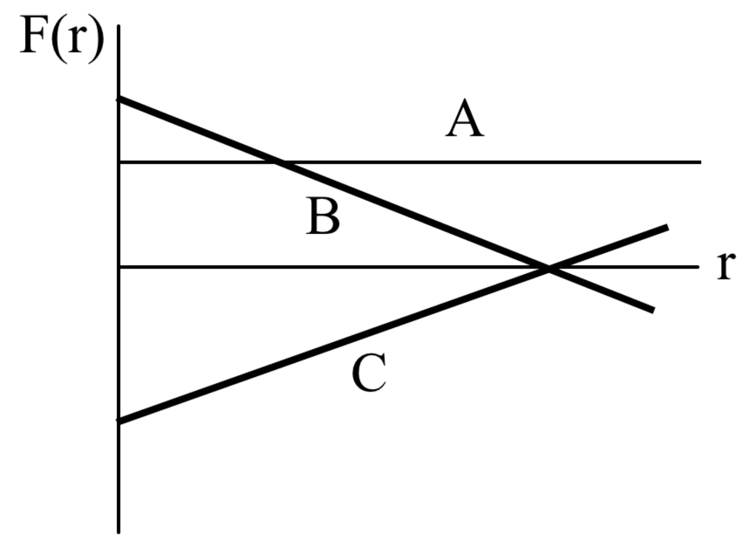
Which statements are true about a particle located at x_o?
I- If it has energy E_1, it can move between B and E
II- If it has energy E_2 it is bounded between A&F, it cannot escape
III- A particle with E_3 is unbounded

A) All
B) II only
C) III only
D) I&II
E) II&III



The potential energy of a test mass is shown as a function of distance from the origin U(r) ~ - 1 / r. Which graph shows the corresponding force as a function of distance?



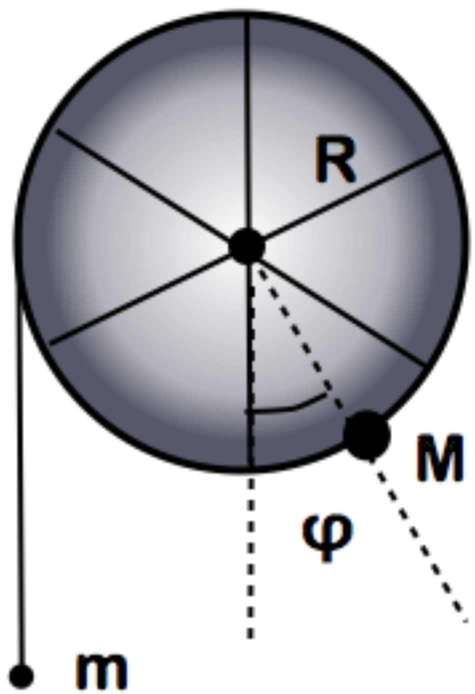


D) None of these!

Consider this massless frictionless wheel.
M is attached to the side, while m hangs from a string wrapped around the wheel.

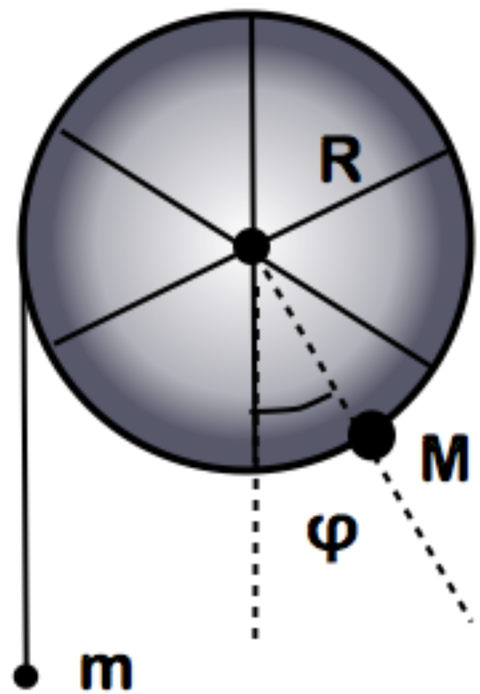
What is the potential energy of m in terms of φ?

A) $+mg\varphi$
B) $-mg\varphi$
C) $+mgR\varphi$
D) $-mgR\varphi$
E) Something else, there's got to be some trig!

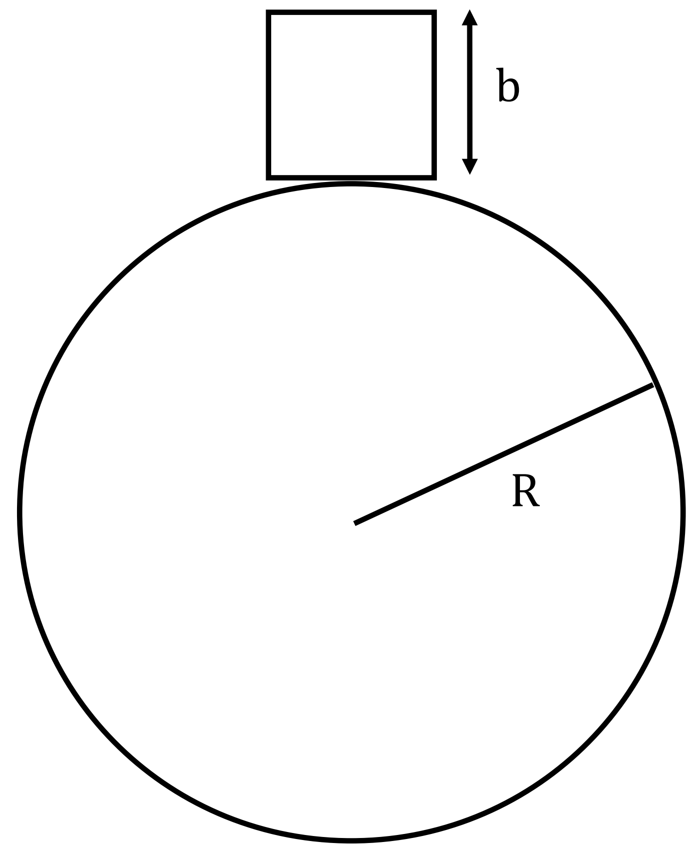


What is the potential energy of M in terms of φ?

A) MgRcos(φ)
B) MgRsin(φ)
C) MgR(cos(φ)-1)
D) MgR(1-cos(φ))
E) Something else!



A square object of edge length b is perched on top of a stationary cylinder of radius R, as shown. The square can roll without slipping on the surface of the cylinder. The cylinder is fixed and cannot move. Is the square in stable equilibrium?



A) Yes, the equilibrium is always stable
B) No, the equilibrium is always unstable
C) The equilibrium is always neutral
D) The nature of the equilibrium depends on the relative size of b and R

A particle oscillates in a one-dimensional potential.
How many (which?) of the following properties guarantee simple harmonic motion?

i) The period T is independent of the amplitude A
ii) The potential $U(x) \sim x^2$
iii) The force $F=-kx$ (Hooke's law)
iv) The position is sinusoidal in time: $x=A\sin(wt+a)$

A) only 1 of these properties guarantees simple harmonic motion
B) exactly 2 properties guarantee simple harmonic motion
C) exactly 3 properties guarantee simple harmonic motion
D) all (any one of these guarantees simple harmonic motion)

A particle oscillates in a potential well which is <u>not</u> a simple $U(x) \sim x^2$ harmonic well.
The well U(x) can be written as a Taylor series expansion about the equilibrium point x_0 (at which dU/dx vanishes) :
The higher derivatives U'' and U''' are nonzero at $x = x_0$.

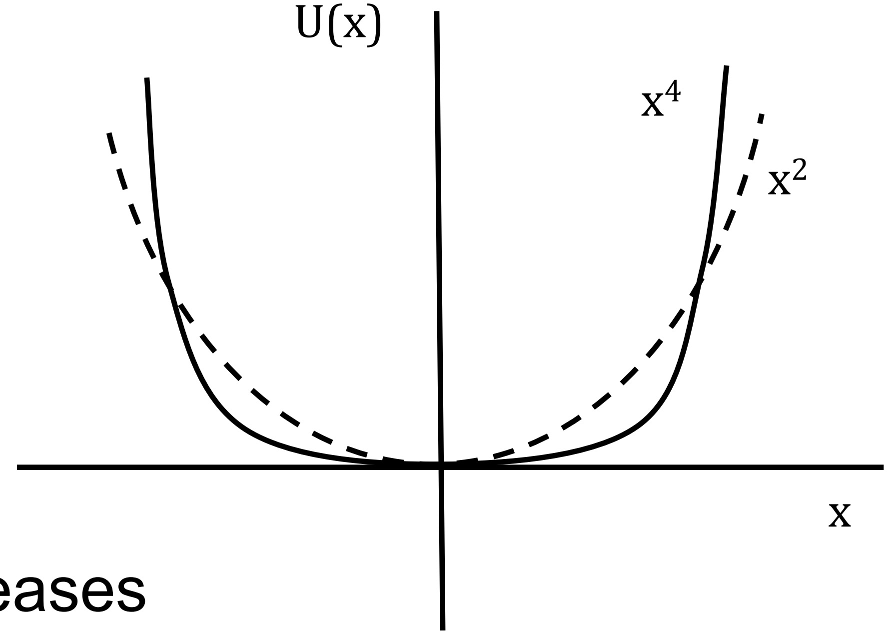
In the limit of very small oscillations, that is, in the limit of , what can you say about this potential well?

A) The well definitely becomes harmonic..
B) The well definitely becomes anharmonic.
C) Whether the well becomes harmonic or anharmonic depends on the function U(x).)

A particle is oscillating, back and forth with amplitude *A*, in a potential well .

Notice that, compared to the harmonic x^2 potential, the anharmonic x^4 potential has a flatter bottom and steeper sides.

When the amplitude of oscillation is increased, what happens to the period T of the oscillation for the anharmonic x^4 potential?



A) period T increases, as A increases

B) period T decreases, as A increases

C) period T remains constant, as A increases

Summary:

1-D systems, U(x) yields F(x)=-dU/dx

Equilibrium when U'(x)=0

Stable Equilibrium if U''(x)>0

Plots of U(x) vs x give us immediate information (about binding, motion, v(x), v(t), equilibrium, ...)

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