Oscillations

How many initial conditions are required
to fully determine the general solution
to a 2nd order linear differential equation?

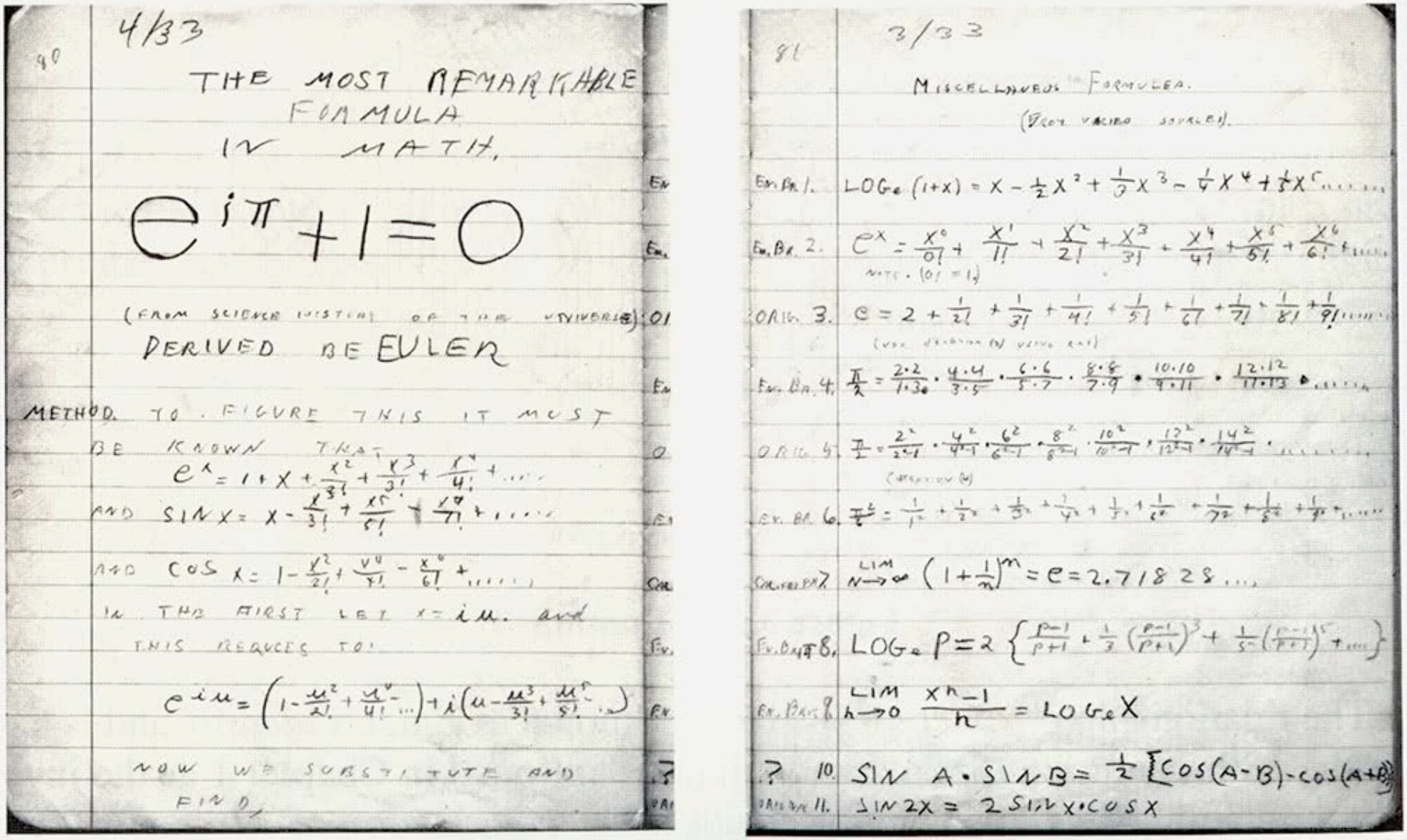
Since cos(ωt) and cos(-ωt) are both solutions of

$$\ddot{x}(t) = -\omega^2 x(t)$$

can we express the general solution as
$x(t)=C_1\cos(\omega t) + C_2\cos(-\omega t)$?

A) yes
B) no
C) ???/depends

Richard Feynman's notebook (Age 14)



4/33

THE MOST REMARKABLE FORMULA IN MATH.

$$e^{i\pi}+1=0$$

(FROM SCIENCE LISTING OF THE UNIVERSE)

DERIVED BE EULER

METHOD. TO FIGURE THIS IT MUST BE KNOWN THAT

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

AND $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

AND $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

IN THE FIRST LET $x = iu$. and THIS REDUCES TO:

$$e^{iu} = \left(1 - \frac{u^2}{2!} + \frac{u^4}{4!} - \dots\right) + i\left(u - \frac{u^3}{3!} + \frac{u^5}{5!} \dots\right)$$

NOW WE SUBSTITUTE AND FIND,

3/33

MISCELLANEOUS FORMULEA.

(FROM VARIOUS SOURCES).

EX. PR. 1. $\log_e(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 \dots$

EX. PR. 2. $e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \dots$

NOTE. $(0! = 1)$

ORIG. 3. $e = 2 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} \dots$

(VER. DERIVED BY VERY EAS.)

EX. PR. 4. $\frac{\pi}{2} = \frac{2\cdot2}{1\cdot3}\cdot\frac{4\cdot4}{3\cdot5}\cdot\frac{6\cdot6}{5\cdot7}\cdot\frac{8\cdot8}{7\cdot9}\cdot\frac{10\cdot10}{9\cdot11}\cdot\frac{12\cdot12}{11\cdot13}\cdot\dots$

ORIG. 5. $\frac{\pi}{2} = \frac{2^2}{2^2-1}\cdot\frac{4^2}{4^2-1}\cdot\frac{6^2}{6^2-1}\cdot\frac{8^2}{8^2-1}\cdot\frac{10^2}{10^2-1}\cdot\frac{12^2}{12^2-1}\cdot\frac{14^2}{14^2-1}\cdot\dots$

(DERIVED BY)

EX. PR. 6. $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \frac{1}{8^2} + \frac{1}{9^2} + \dots$

CONVERGENT 7. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e = 2.71828\dots$

EX. PR. 8. $\log_e p = 2\left\{\frac{p-1}{p+1} + \frac{1}{3}\left(\frac{p-1}{p+1}\right)^3 + \frac{1}{5}\left(\frac{p-1}{p+1}\right)^5 + \dots\right\}$

EX. PR. 9. $\lim_{h\to 0}\frac{x^h-1}{h} = \log_e x$

? 10. $\sin A \cdot \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

IDENTITY 11. $\sin 2x = 2\sin x \cdot \cos x$

Courtesy of the Archives, California, Institute of Technology.

Figure 3.1–3 Pages From Feynmann's Notebooks.

For a simple harmonic oscillator (mass on a spring), what happens to the period of motion if the spring constant is increased?

A) Increases
B) decreases
C) unchanged
D) It depends!

For SHM, what happens to the period of motion if the mass is increased by 4?

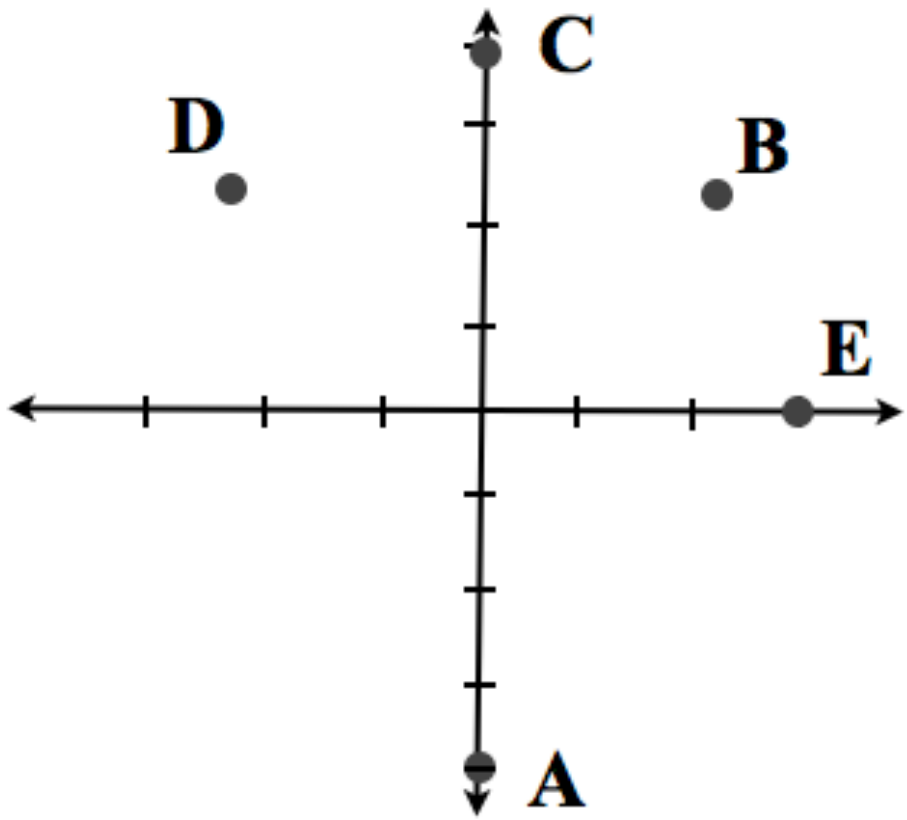
A) Increases by 2x
B) Increases by 4x
C) unchanged
D) Decreases by 2x
E) Decreases by 4x

For the previous situation, what happens to the period of motion if the initial displacement is increased by 4?

A) Increases by 2x
B) Increases by 4x
C) unchanged
D) Decreases by 2x
E) Decreases by 4x

Complex Numbers

Which point below best represents $4e^{i3\pi/4}$ on the complex plane?



Challenge question: Keeping the general form $Ae^{i\theta}$, do any OTHER values of θ represent the SAME complex number as this? (If so, how many?)

On a white board, draw points showing

i) $e^{i\pi/6}$

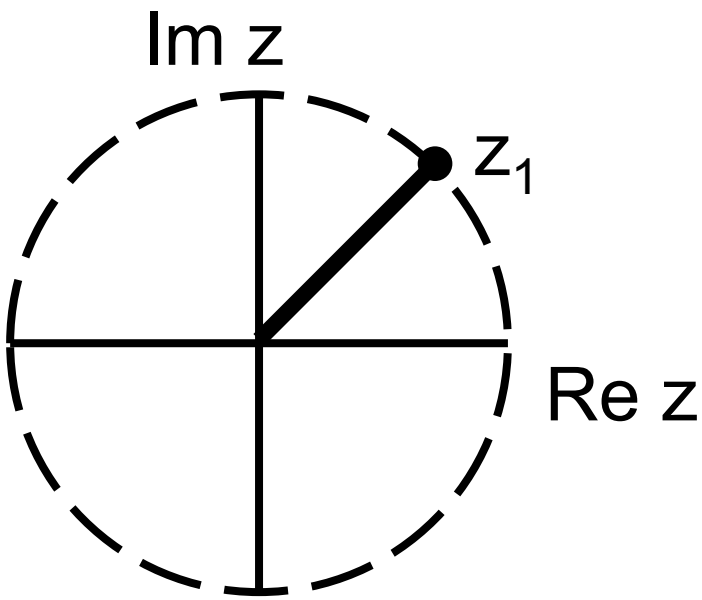
ii) $e^{i\pi/3}$

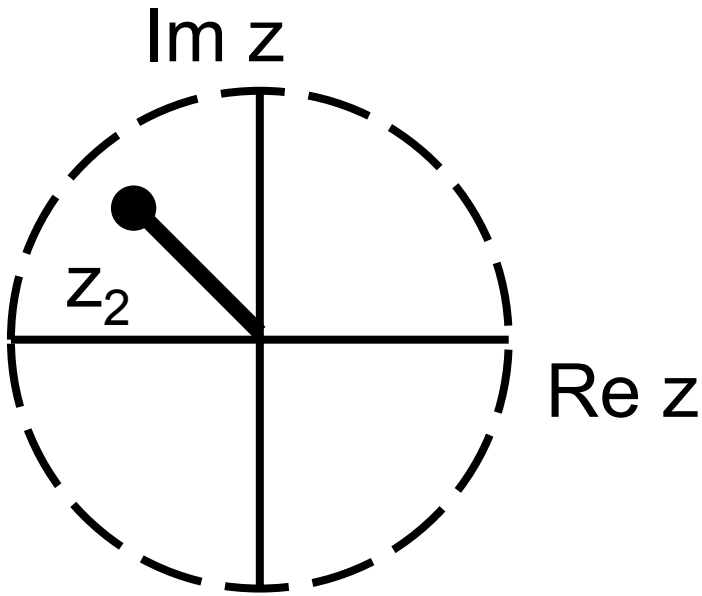
iii) $e^{i\pi/3}\ e^{i\pi/6}$

iv) $e^{i\pi/3}\ /\ e^{i\pi/6}$

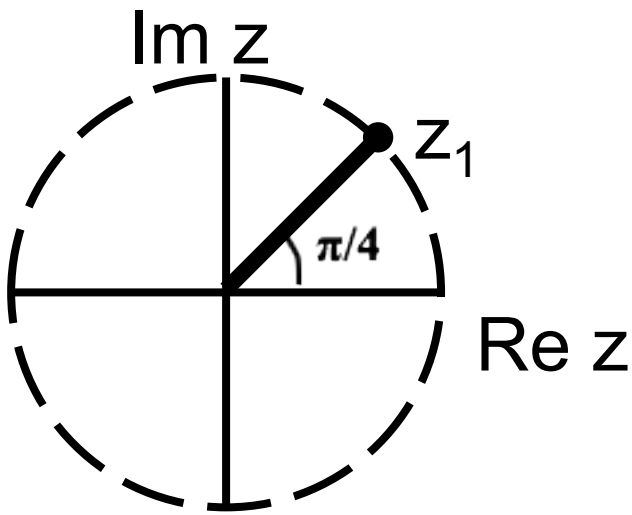
Click A) only when you're done, please.

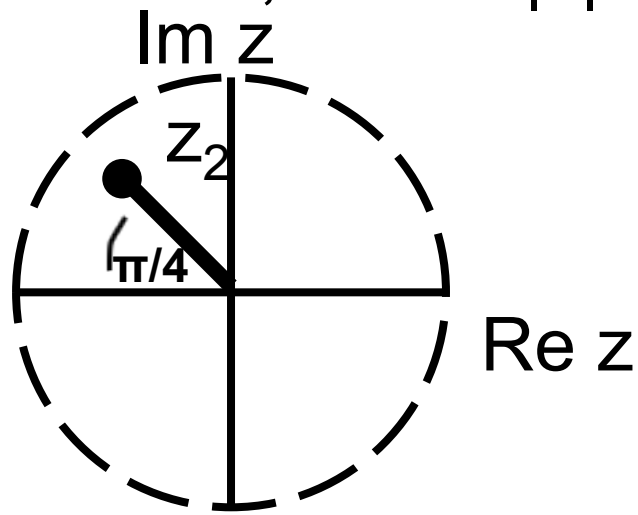
Consider two complex numbers, z_1 and z_2.
The dotted circle shows the unit circle, where $|z|=1$.



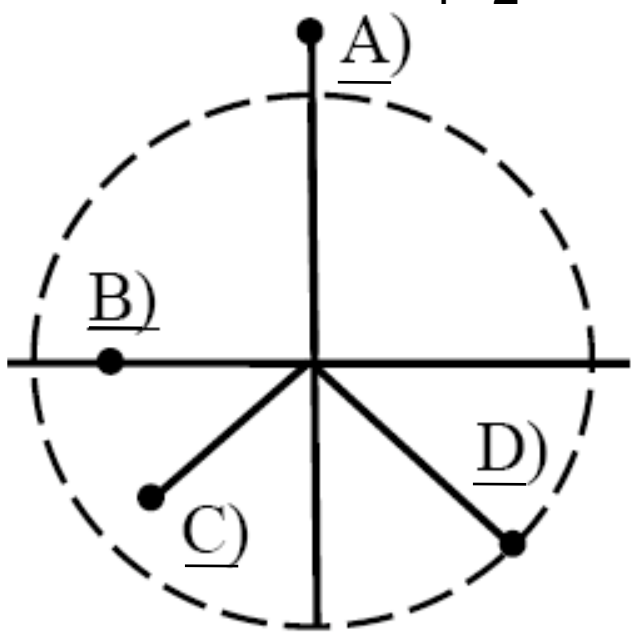


Consider two complex numbers, z_1 and z_2.
The dotted circle shows the unit circle, where $|z|=1$.





Which shows the product $z_1 z_2$?



E) I have no idea

What is 1/(1+i) in “polar” form?

A) $e^{i\,\pi/4}$

B) $e^{-i\,\pi/4}$

C) $0.5\,e^{i\,\pi/4}$

D) $0.5\,e^{-i\pi/4}$

E) Something else!

What is $(1+i)^2/(1-i)$

A) $e^{i\,\pi/4}$

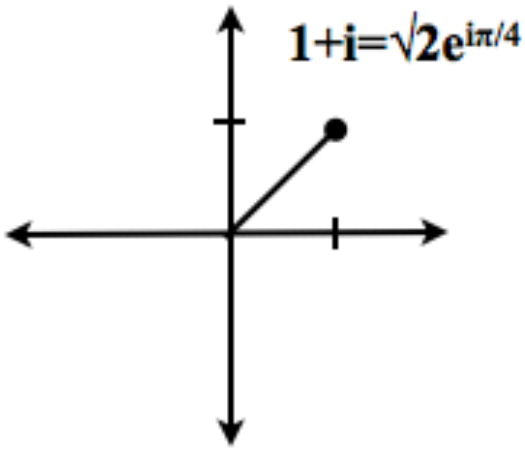
B) Sqrt[2] $e^{i\,\pi/4}$

C) $e^{i\,3\pi/4}$

D) Sqrt[2]$e^{i\,3\pi/4}$

E) Something else!

What is $(1+i)^2/(1-i)$



What is $(1+i)^2/(1-i)$

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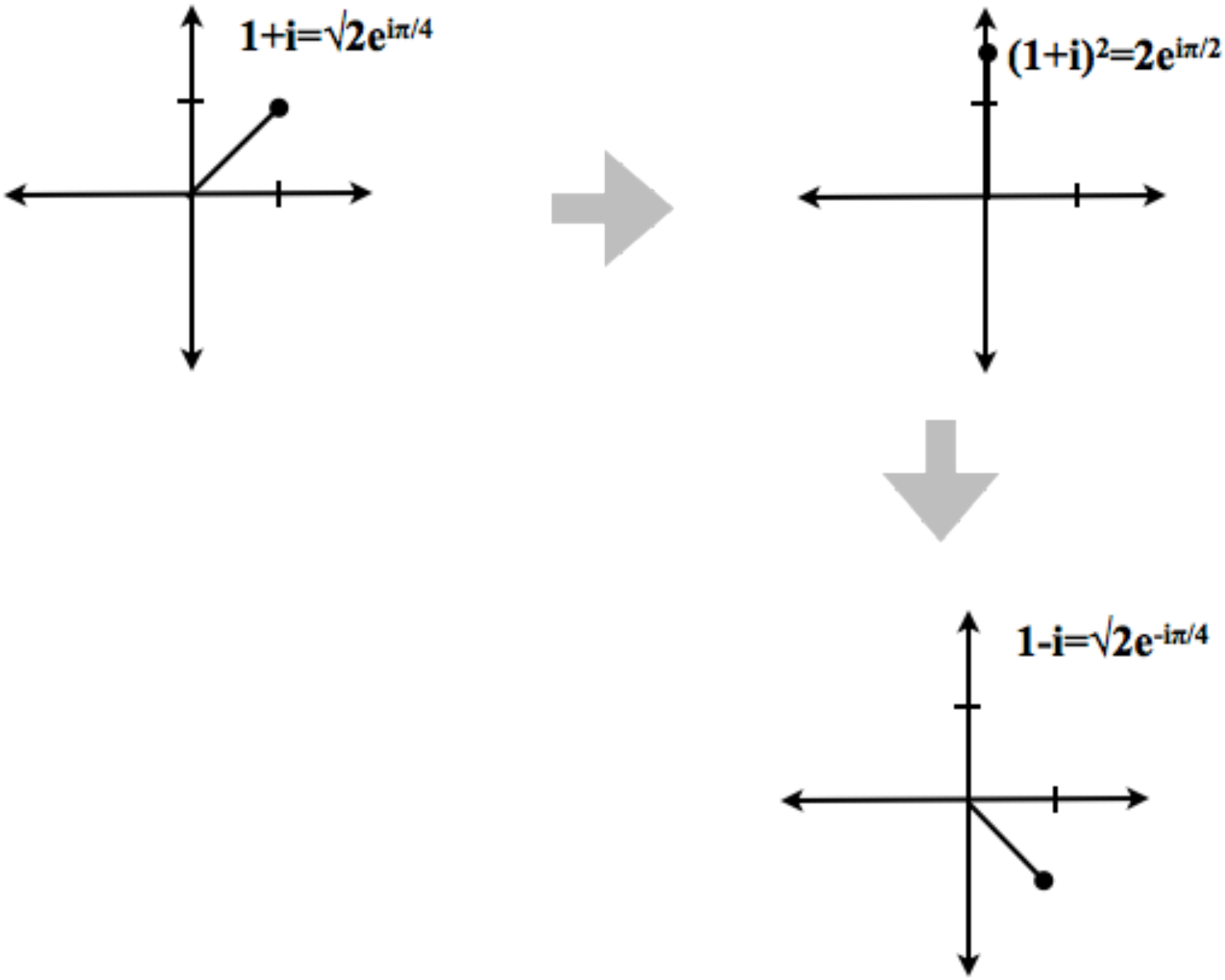
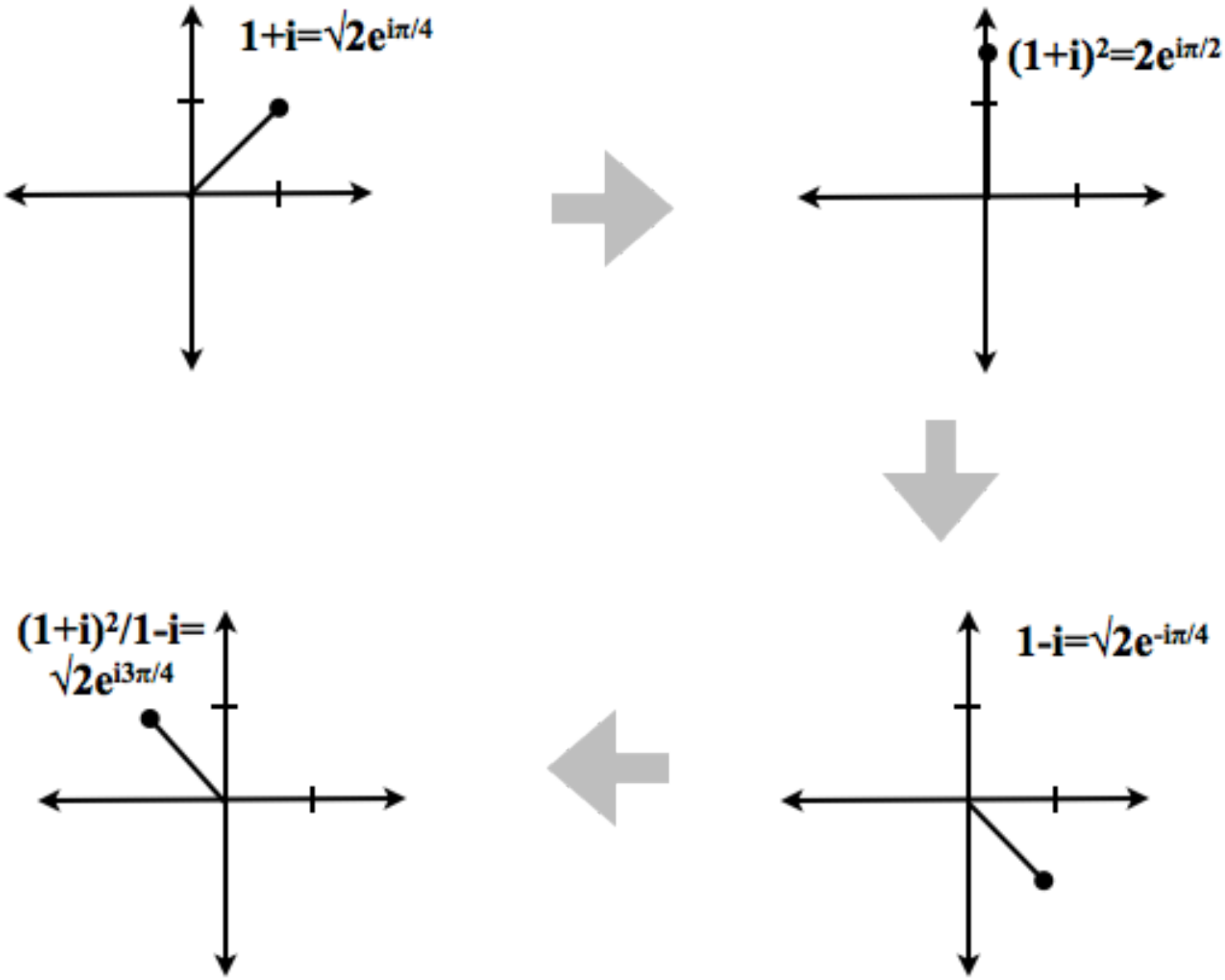
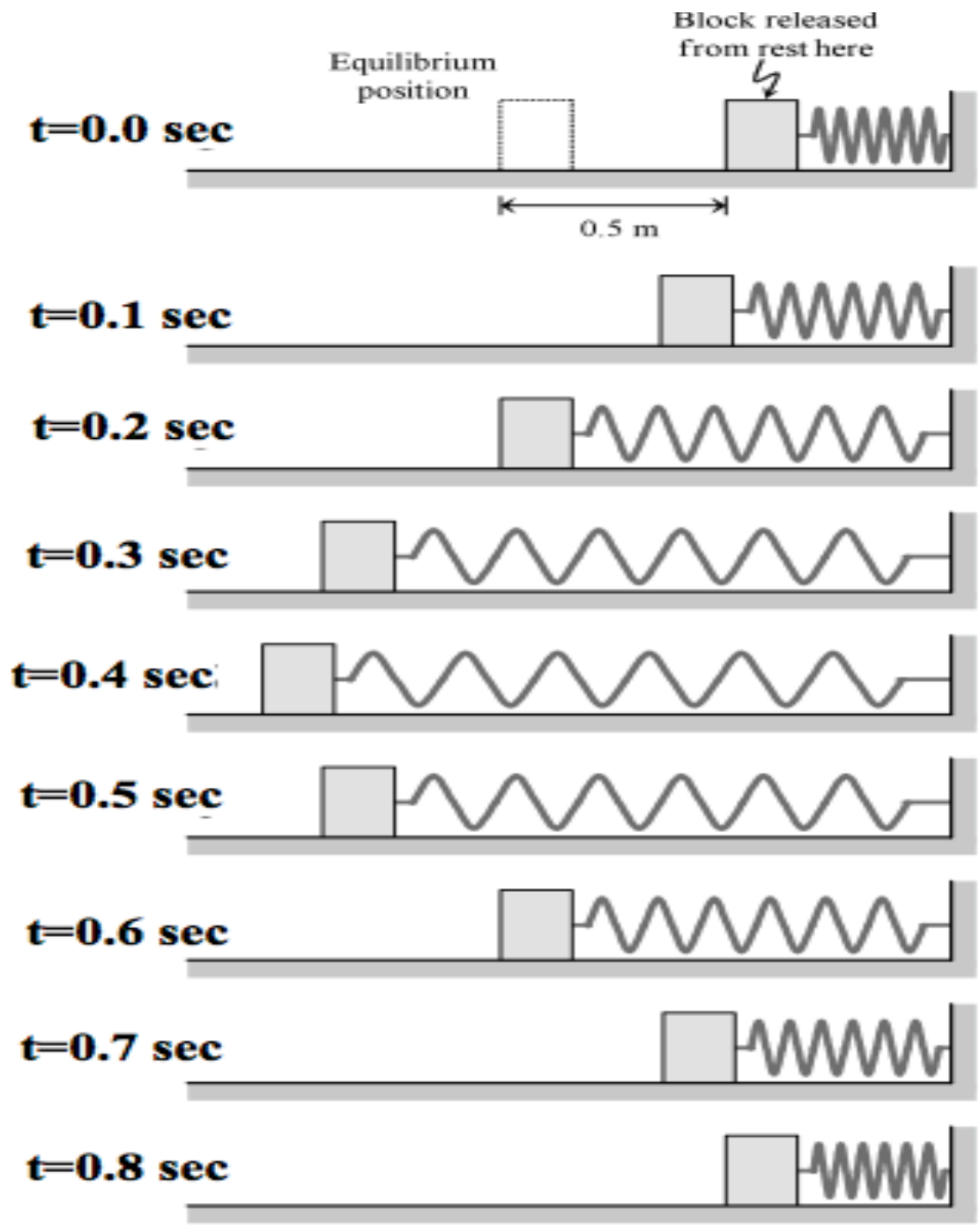


What is $(1+i)^2/(1-i)$

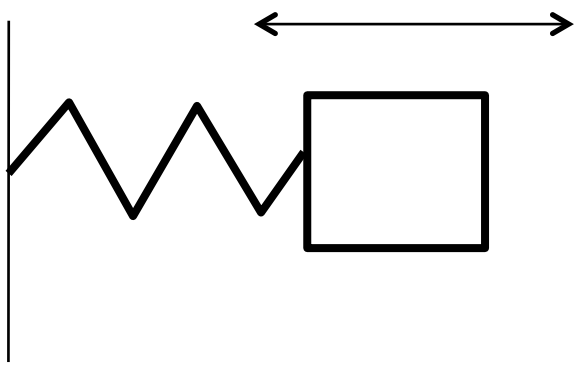


Based on the pictures what is the period of motion of the block?

A) .2 s
B) .4 s
C) .6 s
D) .8 s
E) None of these/ not enough info!

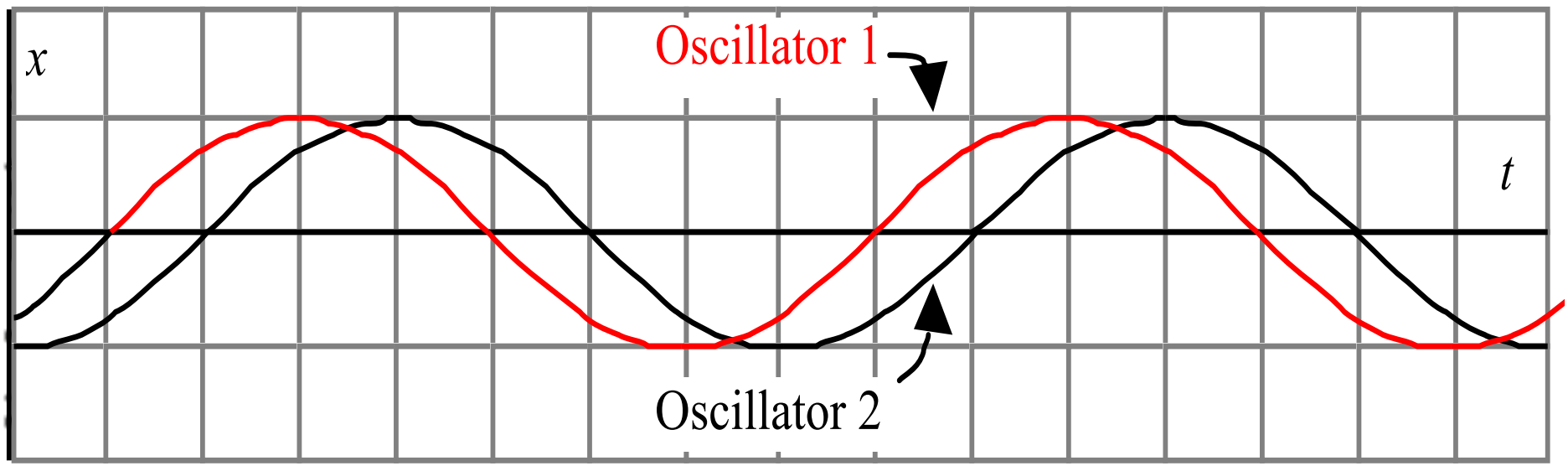


A mass m oscillates at the end of a spring (constant k)
It moves between x=.1 m to x=.5 m.
The block is at x=0.3 m at t=0 sec, moves out to x=0.5 m and returns to x=0.3 m at t=2 sec.



Write the motion in the form $x(t)=x_0+A\cos(\boldsymbol{\omega}t+\varphi)$, and find numerical values for x_0, A, $\boldsymbol{\omega}$, and φ

If you're done:
Write the motion in the form $x(t)=x'_0+A'\sin(\boldsymbol{\omega'}t+\varphi')$, and find numerical values for x'_0, A', $\boldsymbol{\omega'}$, and φ'



Oscillators have $x_i(t)=A_i\cos(\omega_i t+\varphi_i)$ (for i=1, 2)
Which parameters are *different*?

What is the difference between φ_1 and φ_2?
Which is slightly larger (more positive?)
A) φ_1 B) φ_2

What is the general solution to the ODE $\ddot{x}(t) = -\omega^2 x(t)$ where ω is some (known) constant?

A) $x(t) = A\cos\omega t + B\sin\omega t$

B) $x(t) = Ce^{i\omega t} + De^{-i\omega t}$

C) $x(t) = A\cos\omega t + B\sin\omega t + Ce^{i\omega t} + De^{-i\omega t}$

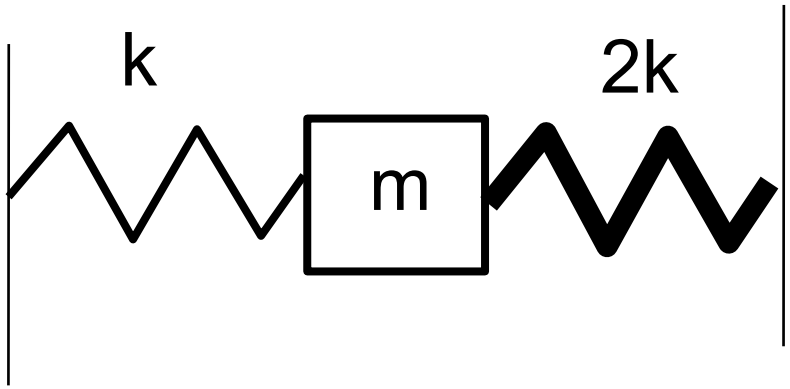
D) Exactly two of these are fully general!

E) All three of these are fine,

with choice C the most general

Is there any OTHER general form for this solution? How many are there? How many might there be?

Neglecting all damping, and considering just 1D motion, what is the angular frequency at which this mass will oscillate? (The mass is at the equilibrium position for both springs at the point shown)



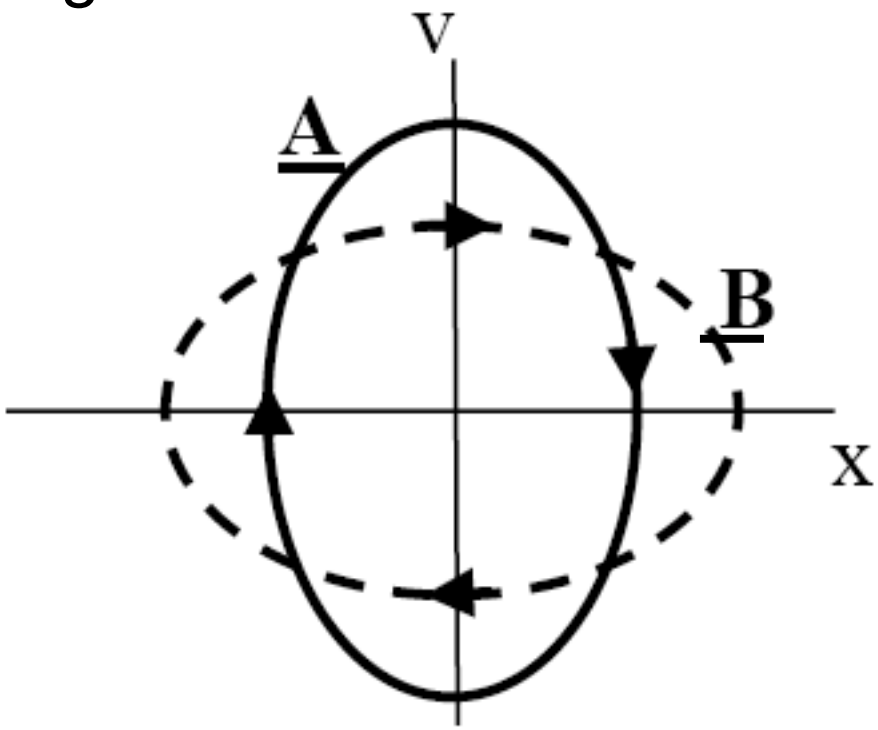
A) Sqrt[k/m]
B) Sqrt[1.5 k/m]
C) Sqrt[3k/m]
D) Sqrt[5k/m]
E) Something else!

Phase Space

If you finish early, click in on this question:

1) Phase paths A and B both describe a harmonic oscillator with the same mass m. Which path describes the system with a bigger spring constant k?

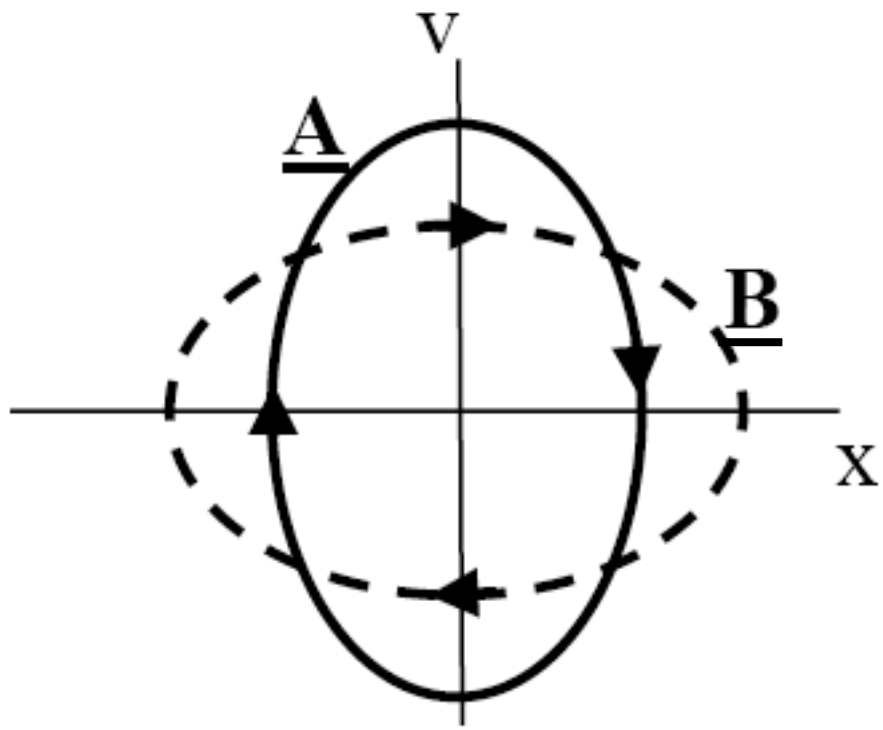
C) Other/undetermined…



2) How does a phase space diagram change, if you start it with a bigger initial stretch?

3) How does a phase space diagram change, if the phase "δ" in $x(t)=A\cos(\omega t-\delta)$ changes?

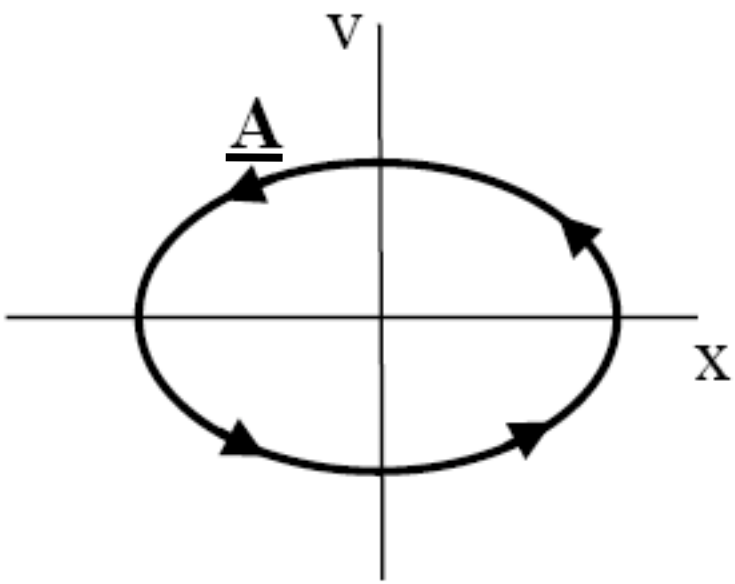
Phase paths A and B both describe a harmonic oscillator with the same mass m. Which path describes the system with a bigger spring constant k?

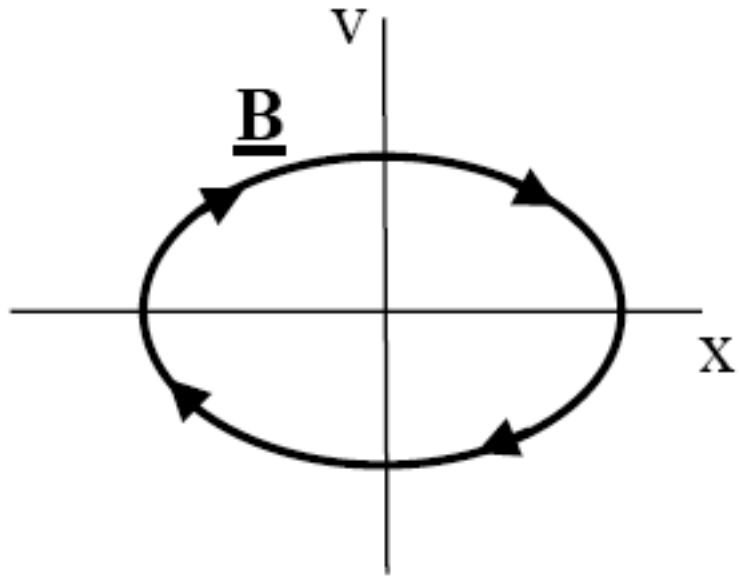


C) both are (or at least could be!) the same
D) Not enough info/???

Phase paths A & B below attempt to describe the mass-on-a-spring simple harmonic oscillator.

Which path is physically possible?





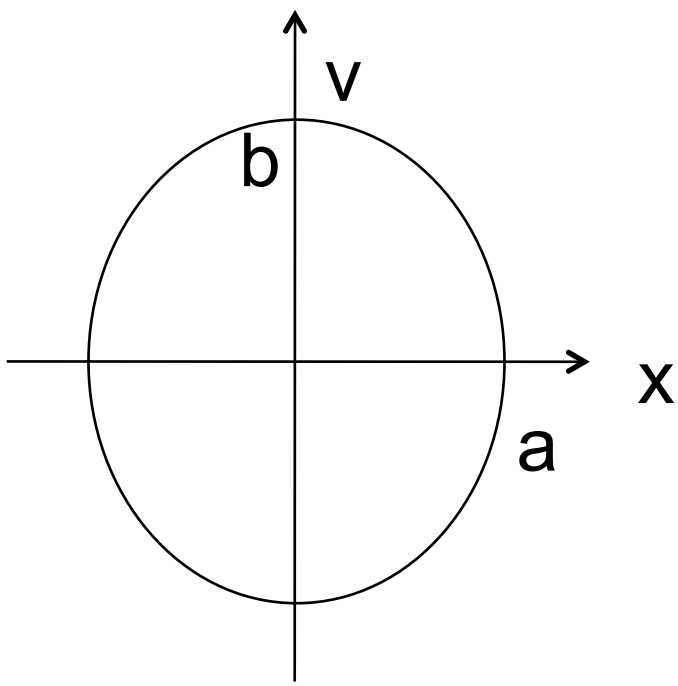
C) both are possible

D) neither is possible

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a = \sqrt{2E_0/k} = \text{max displacement}$$

$$b = \sqrt{2E_0/m} = \text{max speed}$$



$$b/a = \sqrt{k/m} = \omega$$

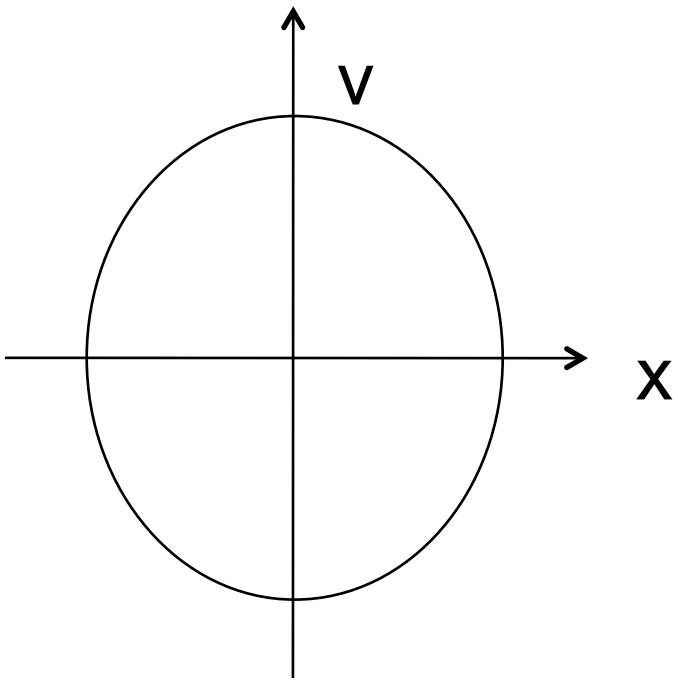
1) How does the phase space diagram change, if you start it with a bigger initial stretch?

2) How does the phase space diagram change, if the phase "δ" in x(t)=Acos(ωt-δ) changes?

For the phase space trajectory of a simple harmonic oscillator,
give a *physical* interpretation of the fact that:

- the trajectory crosses the horizontal *(x)* axis at right angles.

- the trajectory crosses the vertical *(v)* axes at right angles.

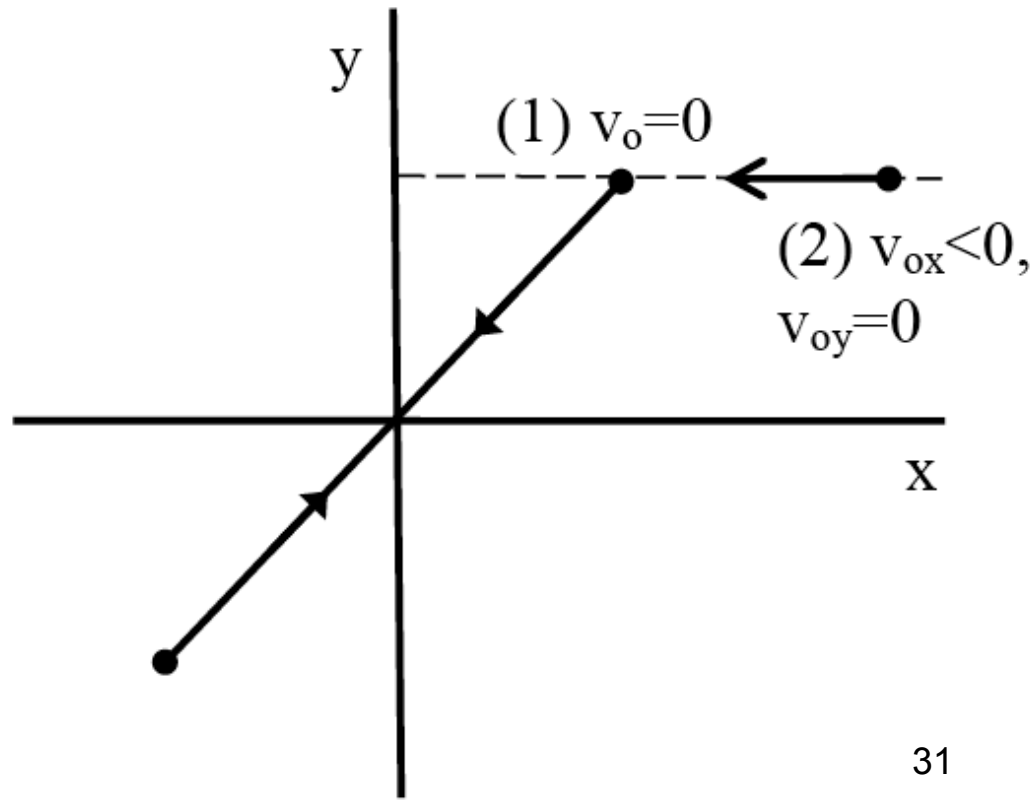


2D Harmonic motion

Compare the motion of a 2D harmonic oscillator with two different sets of initial conditions. In case (1) the particle is released from rest and oscillates along the path shown. In case (2) the particle starts with a larger x position and with a negative x component of the velocity.

What can you say about the amplitude of the x and y motion?

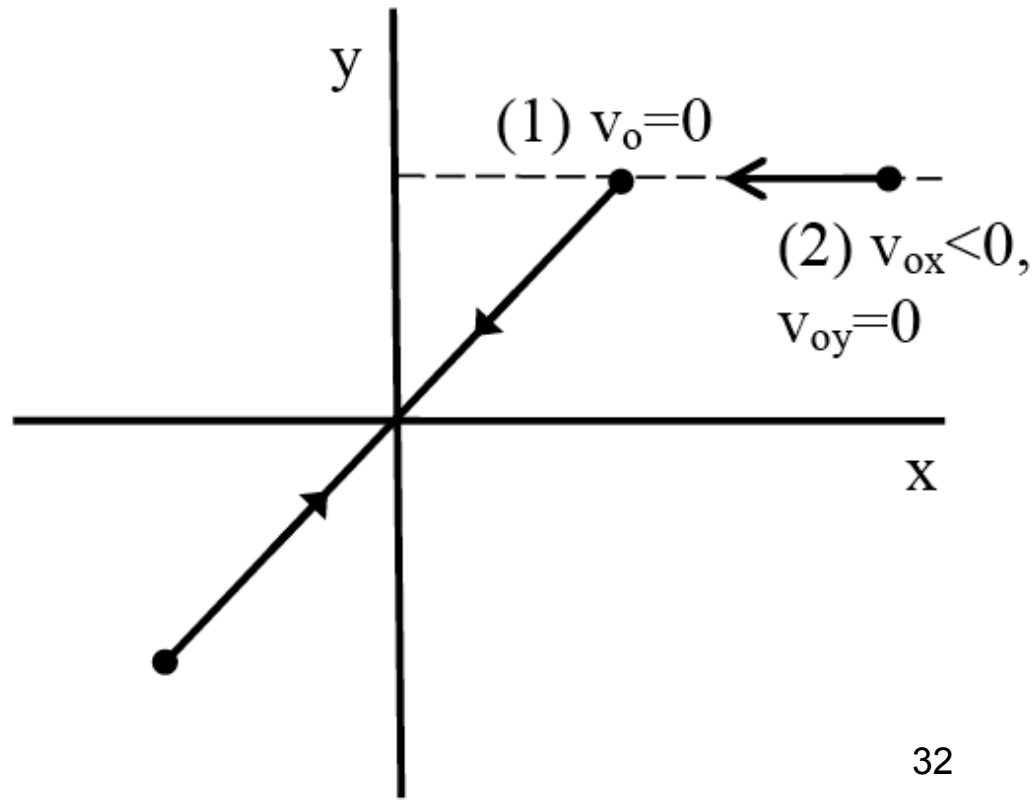
A) $A_{x1}>A_{x2}$, $A_{y1}>A_{y2}$
B) $A_{x1}<A_{x2}$, $A_{y1}=A_{y2}$
C) $A_{x1}=A_{x2}$, $A_{y1}>A_{y2}$
D) $A_{x1}<A_{x2}$, $A_{y1}<A_{y2}$
E) $A_{x1}=A_{x2}$, $A_{y1}=A_{y2}$



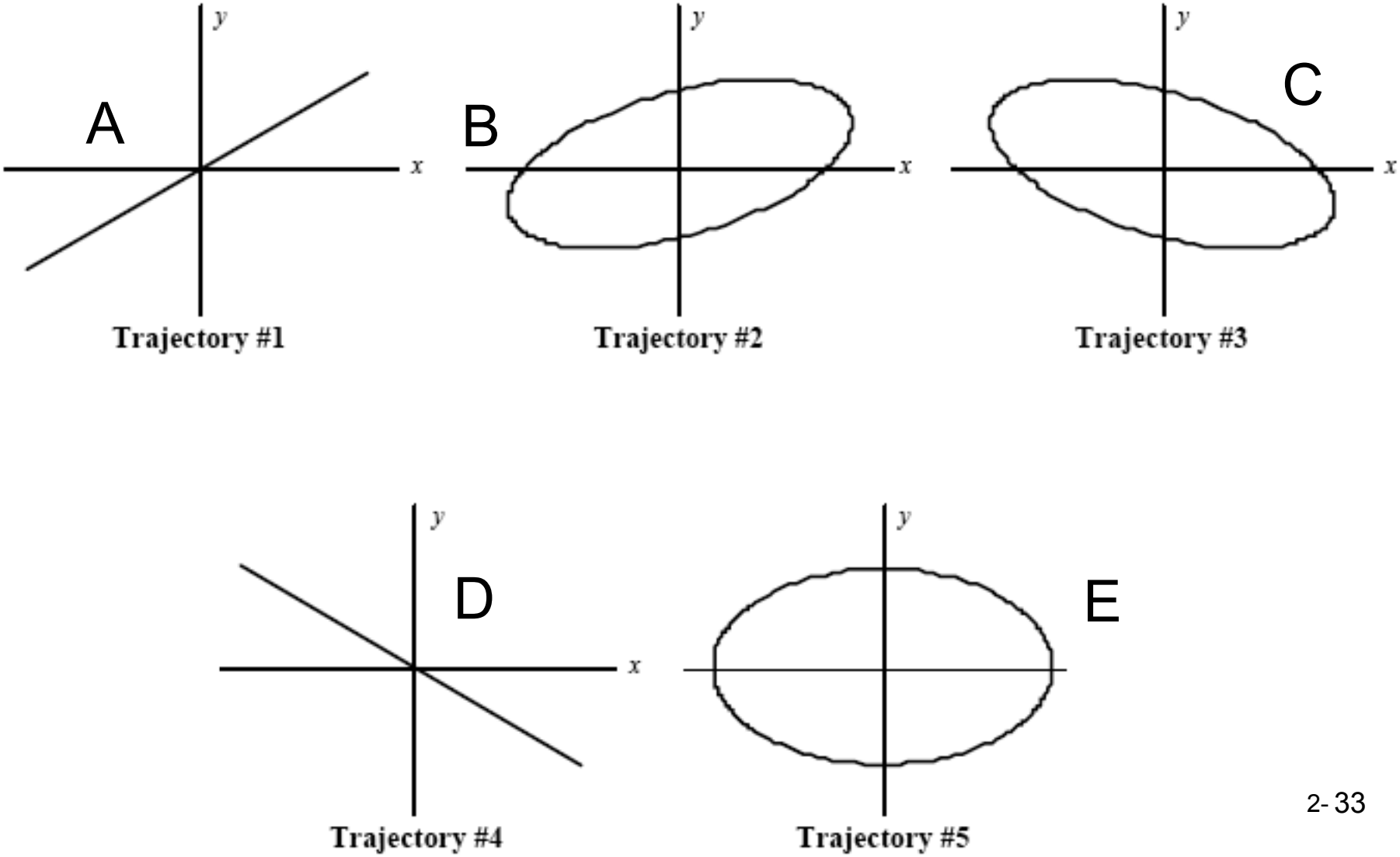
Compare the motion of a 2D harmonic oscillator with two different sets of initial conditions. In case (1) the particle is released from rest and oscillates along the path shown. In case (2) the particle starts with a larger x position and with a negative x component of the velocity.

What can you say about the frequency of the x and y motion?

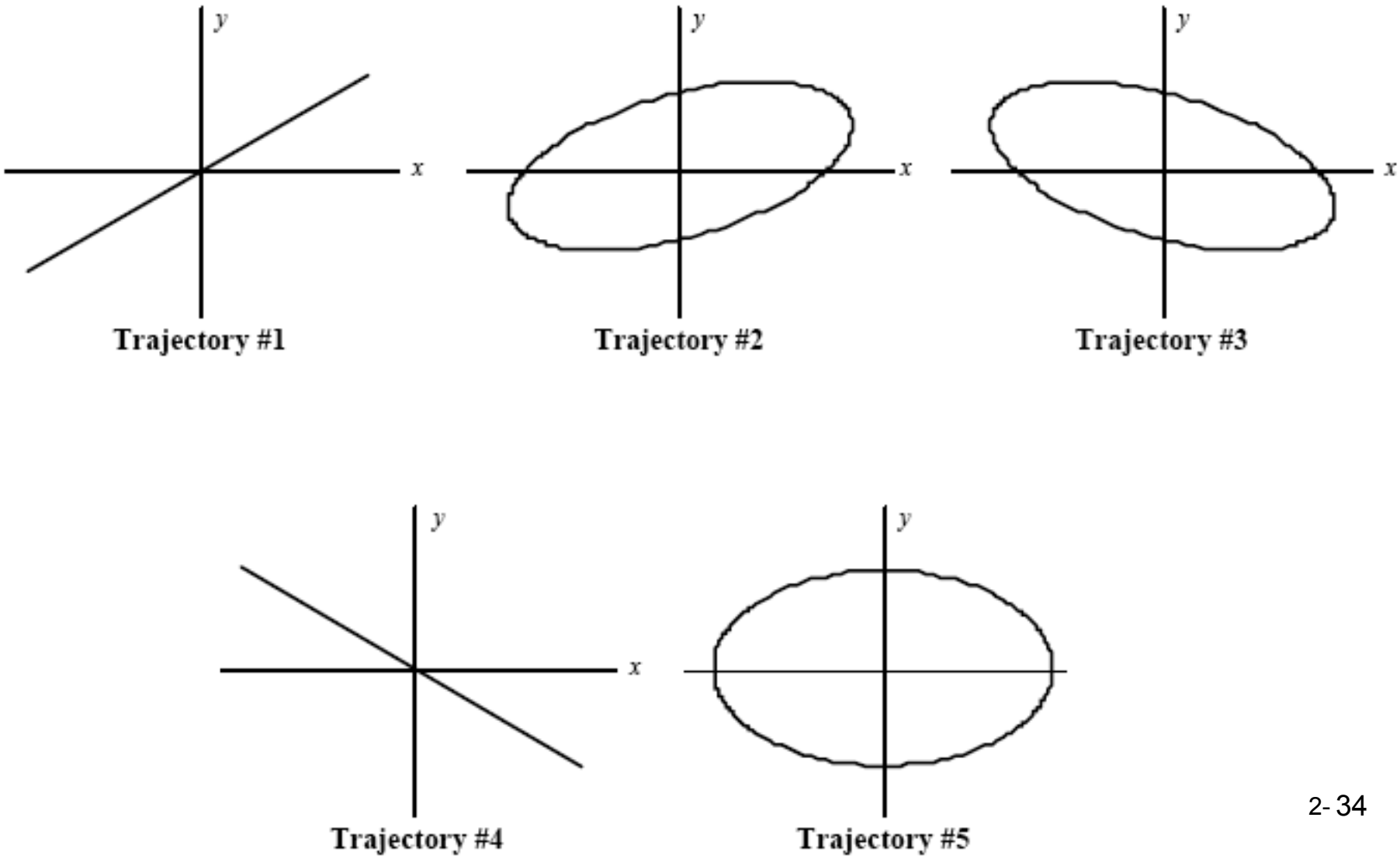
A) $\omega_{x1}>\omega_{x2}$, $\omega_{y1}>\omega_{y2}$
B) $\omega_{x1}<\omega_{x2}$, $\omega_{y1}=\omega_{y2}$
C) $\omega_{x1}=\omega_{x2}$, $\omega_{y1}>\omega_{y2}$
D) $\omega_{x1}<\omega_{x2}$, $\omega_{y1}<\omega_{y2}$
E) $\omega_{x1}=\omega_{x2}$, $\omega_{y1}=\omega_{y2}$



Which of the below trajectories most closely resembles case 2 in the last question, where $v_{y0}=0$ and $v_{x0}<0$ at the release point?

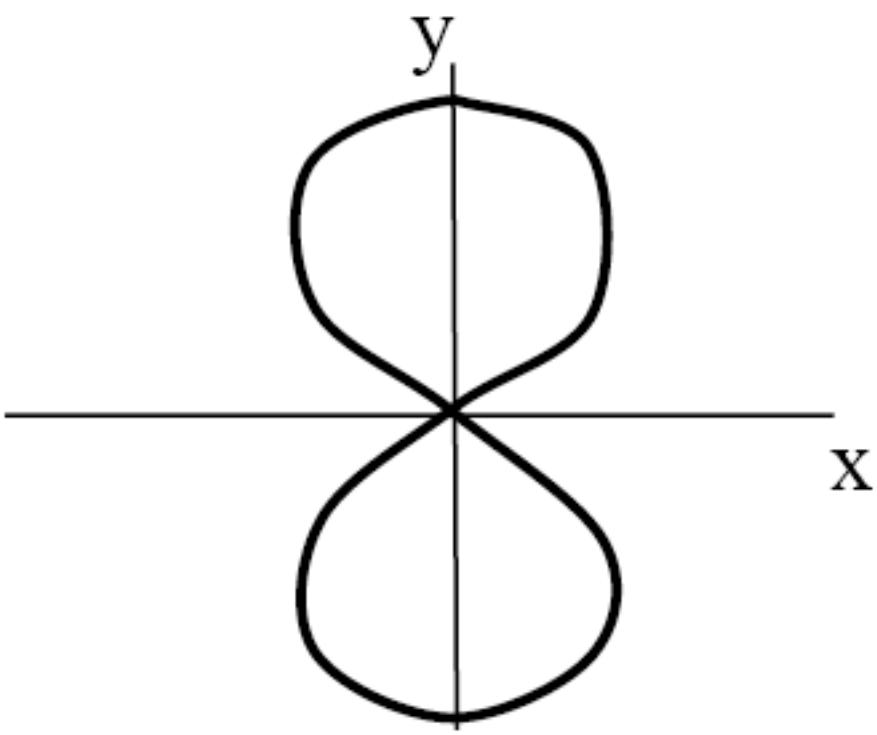


Shown below are several trajectories for a 2D oscillator. For which one is δ=0?



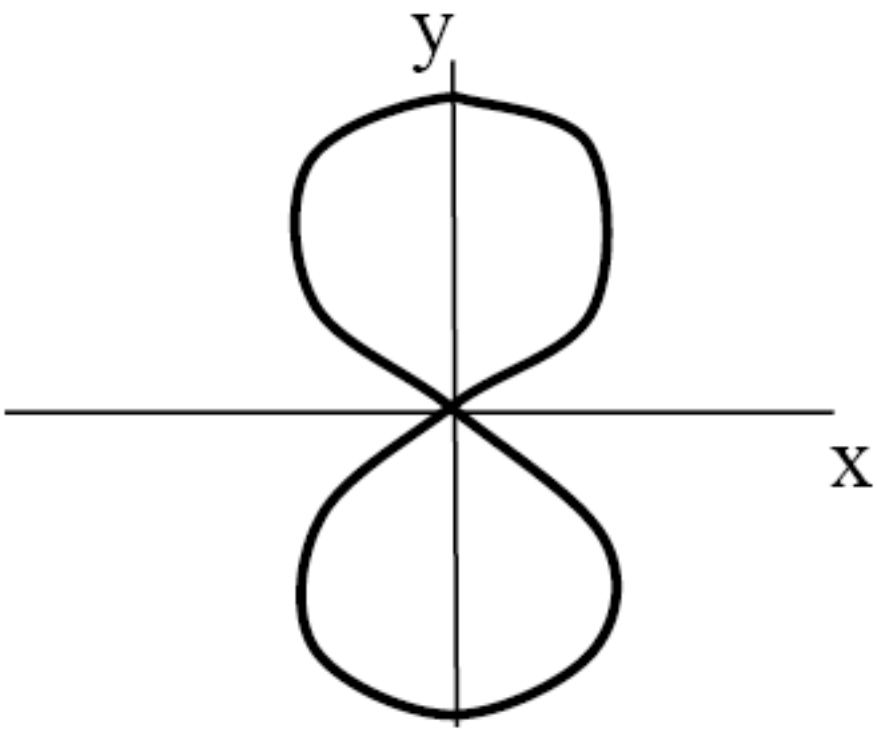
A 2D oscillator traces out the following path in the xy-plane.
What can you say about the frequencies of the x and y motion?

A) $\omega_x = 4\omega_y$
B) $\omega_x = 2\omega_y$
C) $\omega_x = \omega_y$
D) $\omega_x = 0.5\ \omega_y$
E) $\omega_x = 0.25\ \omega_y$



A 2D oscillator traces out the following path in the xy-plane.
What can you say about the Amplitudes of the x and y motion?

A) $A_x > A_y$
B) $A_x \approx A_y$
C) $A_x < A_y$



Consider a super ball which bounces up and down on super concrete. After the ball is dropped from an initial height h, it bounces with no dissipation and executes an infinite number of bounces back to height h.

Is the motion of the ball in z simple harmonic motion?

A) yes
B) no
C) ???

Damped oscillations

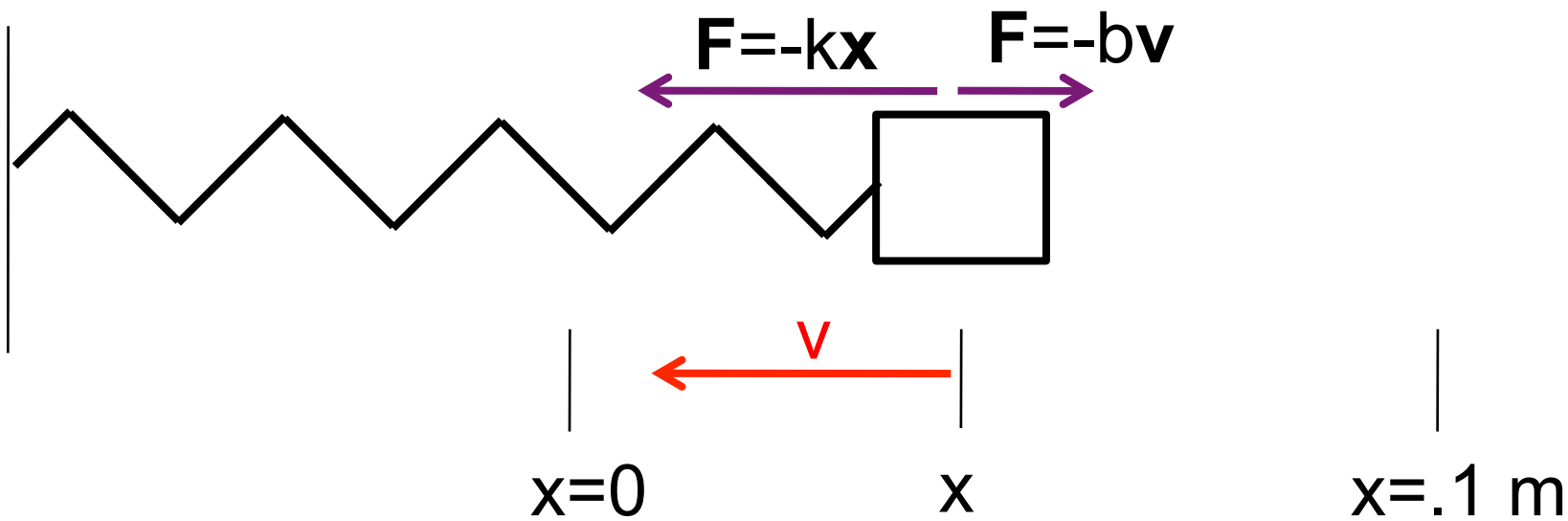
An oscillator is released from rest at x=+.1 m, and undergoes ideal SHM.

If a small damping term is added, how does $|F_{net}|$ *differ* from the ideal situation when the mass is on its way from +.1 m to the origin?

A) Slightly *larger* than the undamped case.
B) Slightly *smaller* than the undamped case
C) The *same* as the undamped case
D) It varies (and thus none of the above is correct)
E) The answer depends on how big the damping is.

Hint: **Draw a free body diagram**!

the mass is on its way from +.1 m to the origin...



The ODE for damped simple harmonic motion is:

$$\frac{d^2y}{dt^2} + 2\beta\frac{dy}{dt} + \alpha y = 0$$

What are the signs of the constants α and β?

A) α,β>0
B) α,β<0
C) α<0, β>0
D) α>0, β<0
E) Depends!

A mass on a spring has a small damping term added. What happens to the period of oscillation?

A) Slightly *larger* than the undamped case.
B) Slightly *smaller* than the undamped case
C) The *same* as the undamped case

What are the roots of the auxiliary equation $D^2 + D - 2 = 0$?

A) 1 and 2
B) 1 and -2
C) -1 and 2
D) -1 and -2
E) Other/not sure...

What are the roots of the auxiliary equation for $y''(t) + y(t) = 0$?

A) 1 and -1
B) 1 and 0
C) just 1
D) just i
E) i and -i

What are the roots of the auxiliary equation $y''(t) + \omega^2 y(t) = 0$?

A) ω and $-\omega$
B) ω and 0
C) just ω
D) just $i\omega$
E) $i\omega$ and $-i\omega$

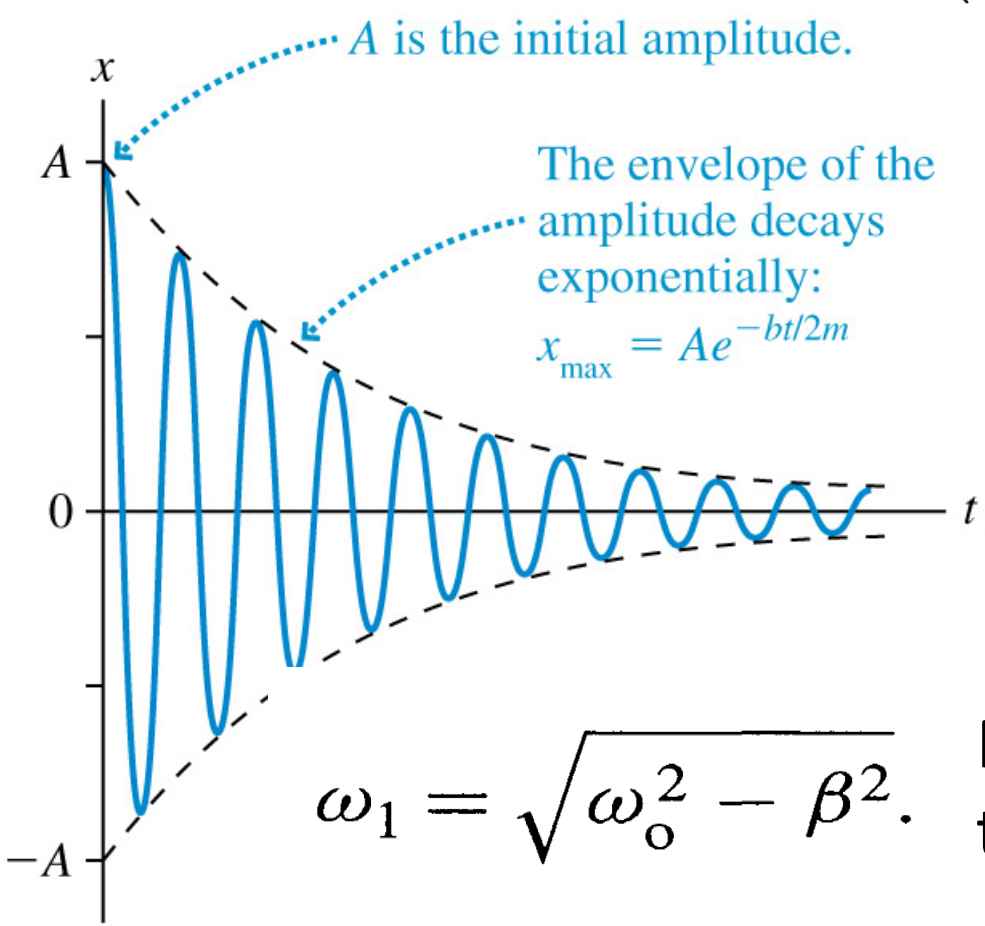
Are $e^{i\omega t}$ and $\cos(\omega t)$ linearly independent?

A) yes
B) no
C) It depends on omega
D) ???

Underdamped Oscillations

An *underdamped oscillation* *with* $b<w_0$:

$$x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta).$$



$$\beta < \omega_o$$

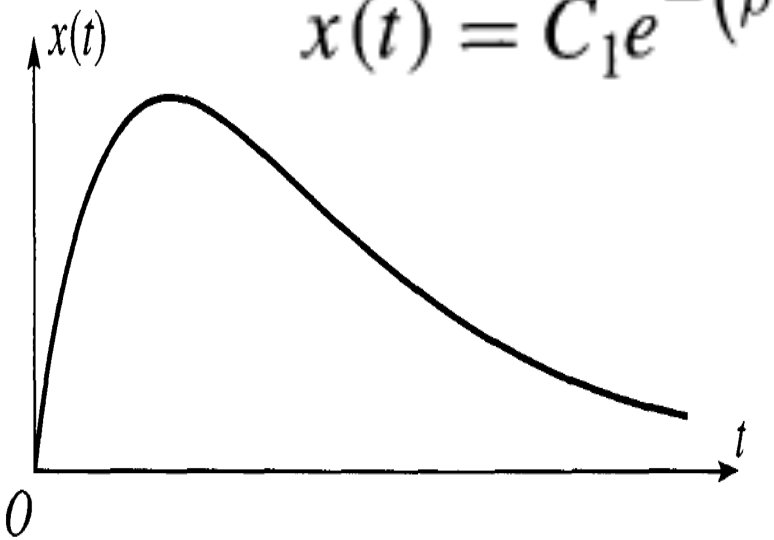
$$\omega_1 = \sqrt{\omega_o^2 - \beta^2}.$$

Note that damping ***reduces*** the oscillation frequency.

Overdamped Oscillations

An *overdamped oscillation* *with* $b > w_0$:

$$x(t) = C_1 e^{-\left(\beta - \sqrt{\beta^2 - \omega_o^2}\right)t} + C_2 e^{-\left(\beta + \sqrt{\beta^2 - \omega_o^2}\right)t}.$$



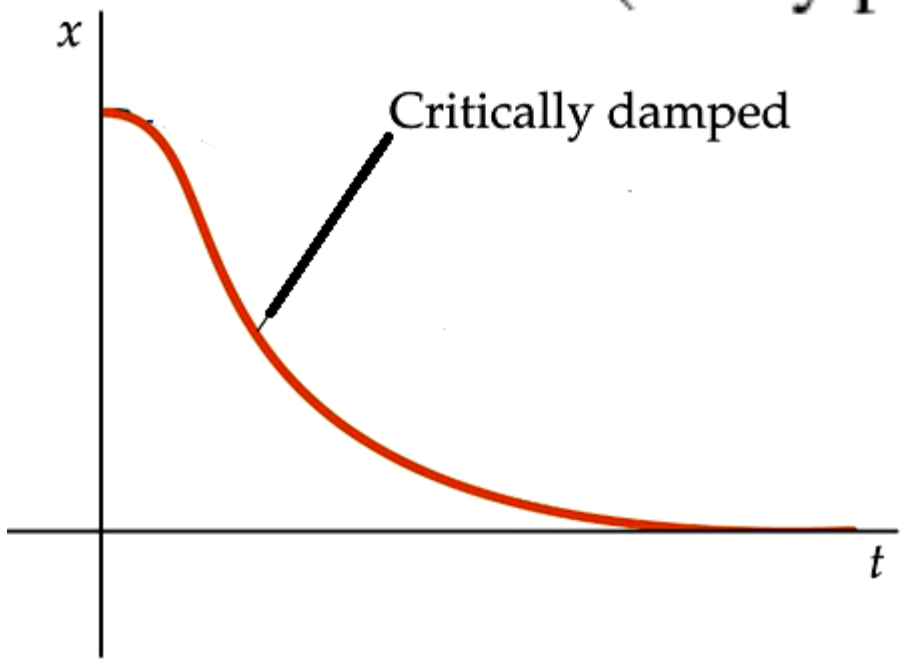
$$(\text{decay parameter}) = \beta - \sqrt{\beta^2 - \omega_o^2}$$

Critically damped Oscillations

A critically damped *oscillation* *with* $b = w_0$:

$$x(t) = C_1 e^{-\beta t} + C_2 t\, e^{-\beta t}.$$

$$(\text{decay parameter}) = \beta = \omega_0$$



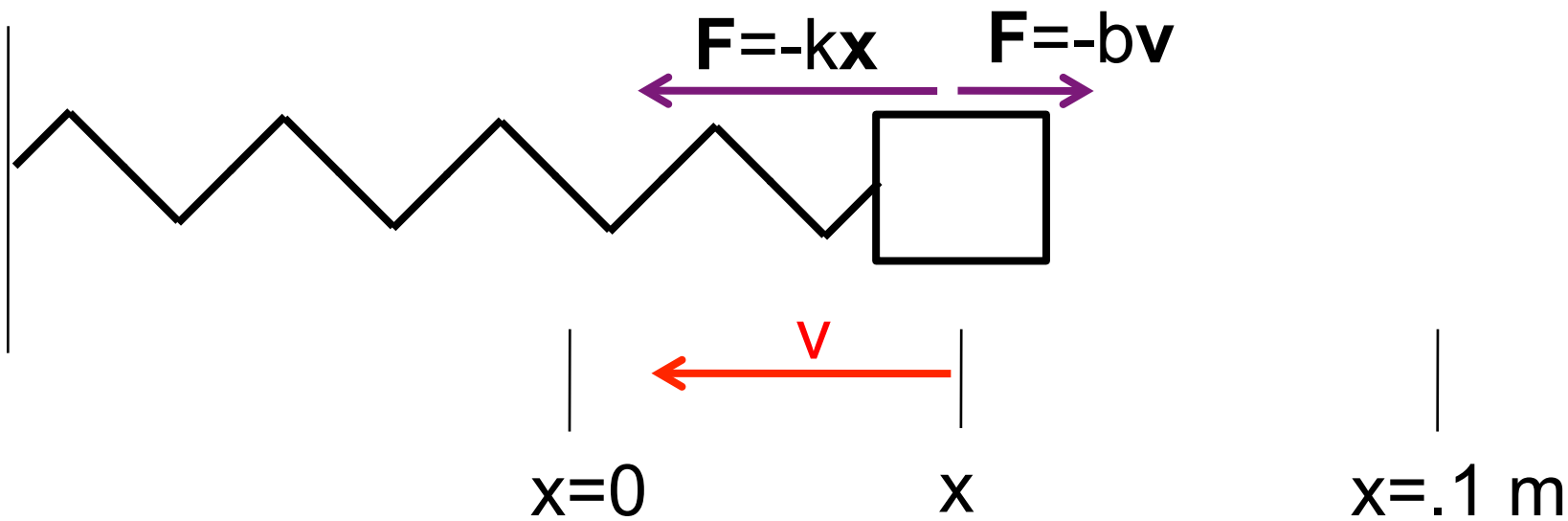
A mass on a spring has a small damping term added. When it passes through x=0, which is correct?

A) The mass is instantaneously speeding up
B) The mass is instantaneously slowing down
C) The mass is at a maximum speed (and is thus neither speeding up nor slowing down
D) The answer depends on which WAY it is passing through the origin.

An oscillator has a small damping term added.
We release it from rest.
Where do you think v_{max} first occurs?

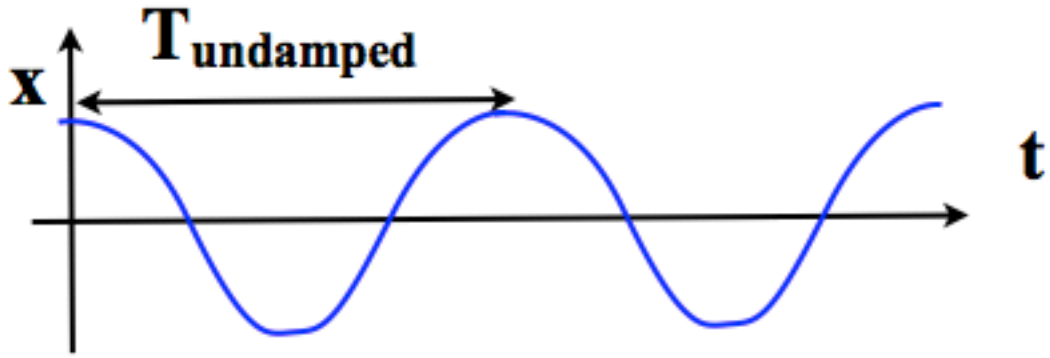
A) Just BEFORE reaching x=0
B) Just AFTER reaching x=0
C) Precisely when x=0
D) ???

HINT: At the instant it passes through x=0, is it speeding up, slowing down, or at a max speed? Does this help?

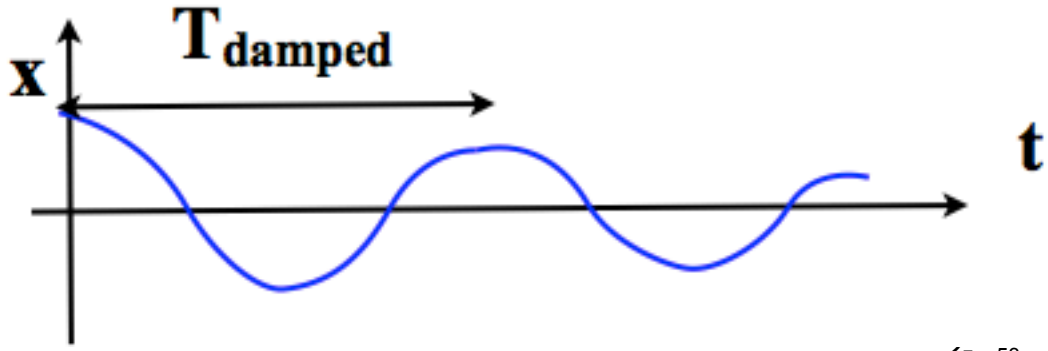

F=-kx
F=-bv
v
x=0
x
x=.1 m

For a damped oscillator, how does the the period between successive maxima compare to the undamped case?
(Assume k and m are the same)

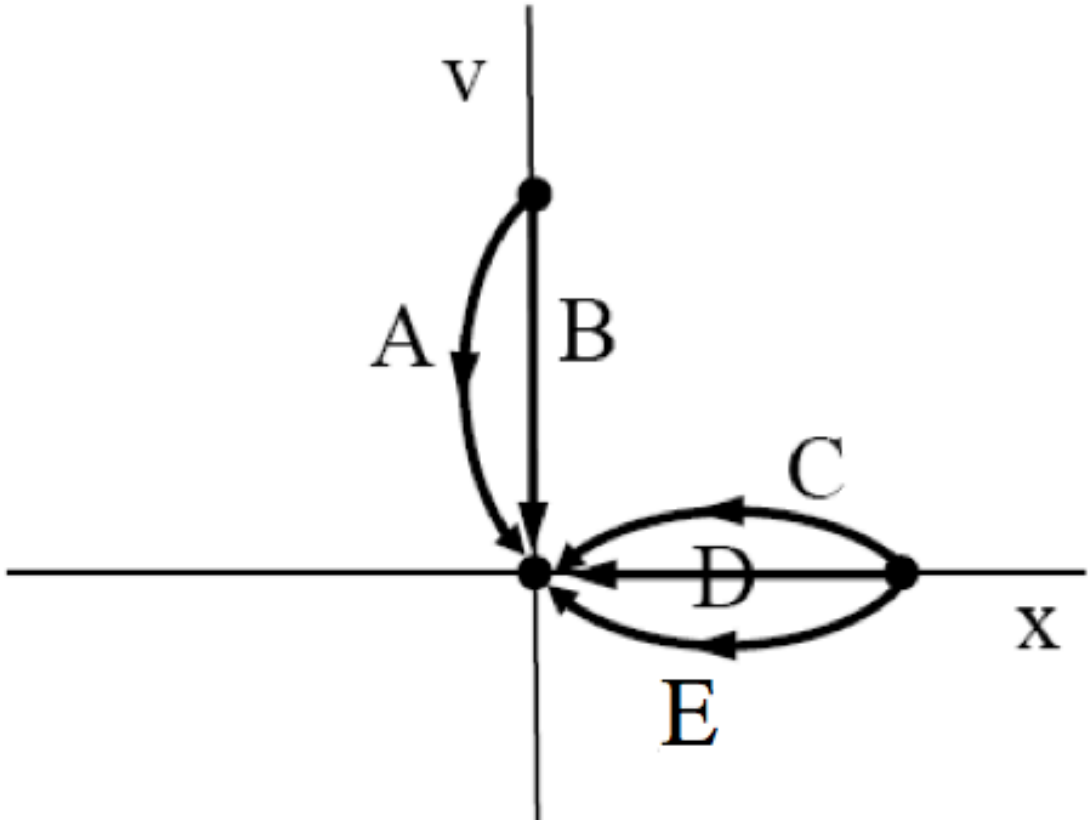
A) same
B) damped is bigger
C) undamped is bigger



Not necessarily on the same horizontal scale!



Which phase path below best describes overdamped motion for a harmonic oscillator released from rest?



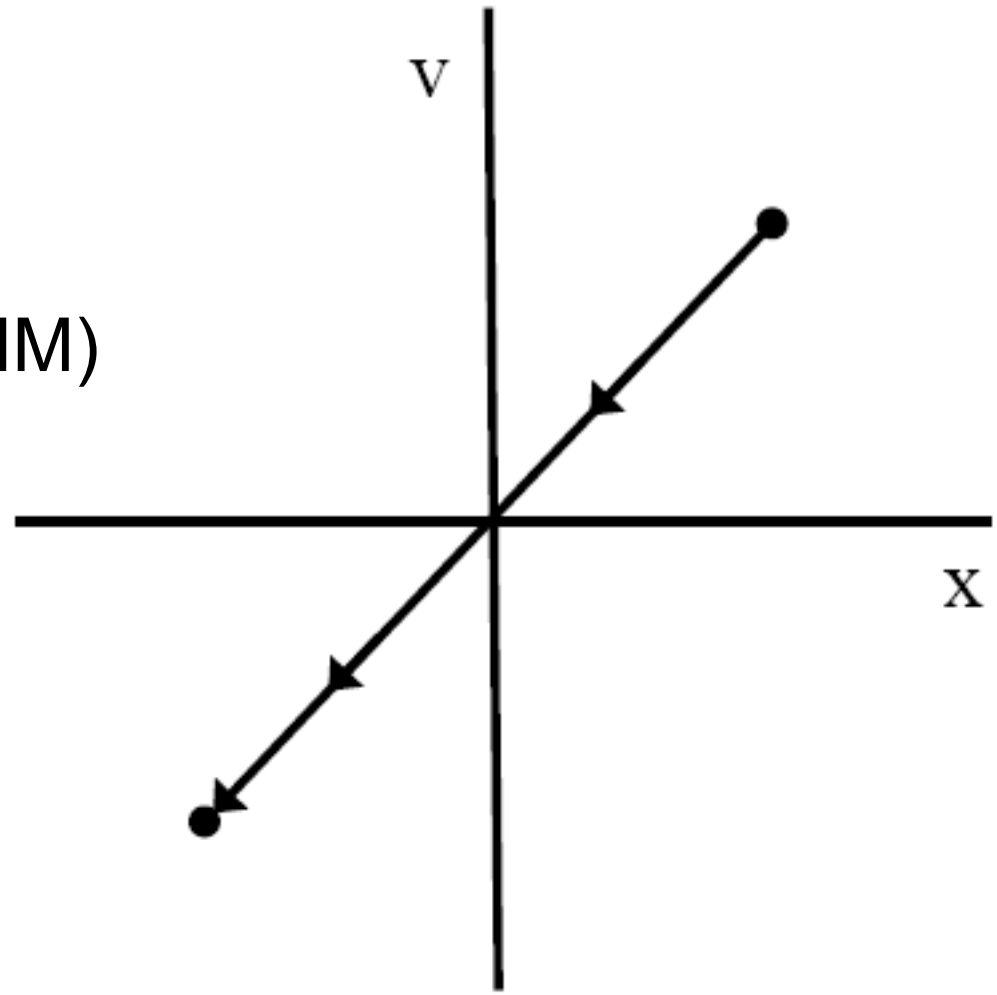
Challenge question: How does your answer change if the oscillator is “critically damped”?

What kind of damping behavior should the shock absorbers in your car have, for the most comfortable ride?

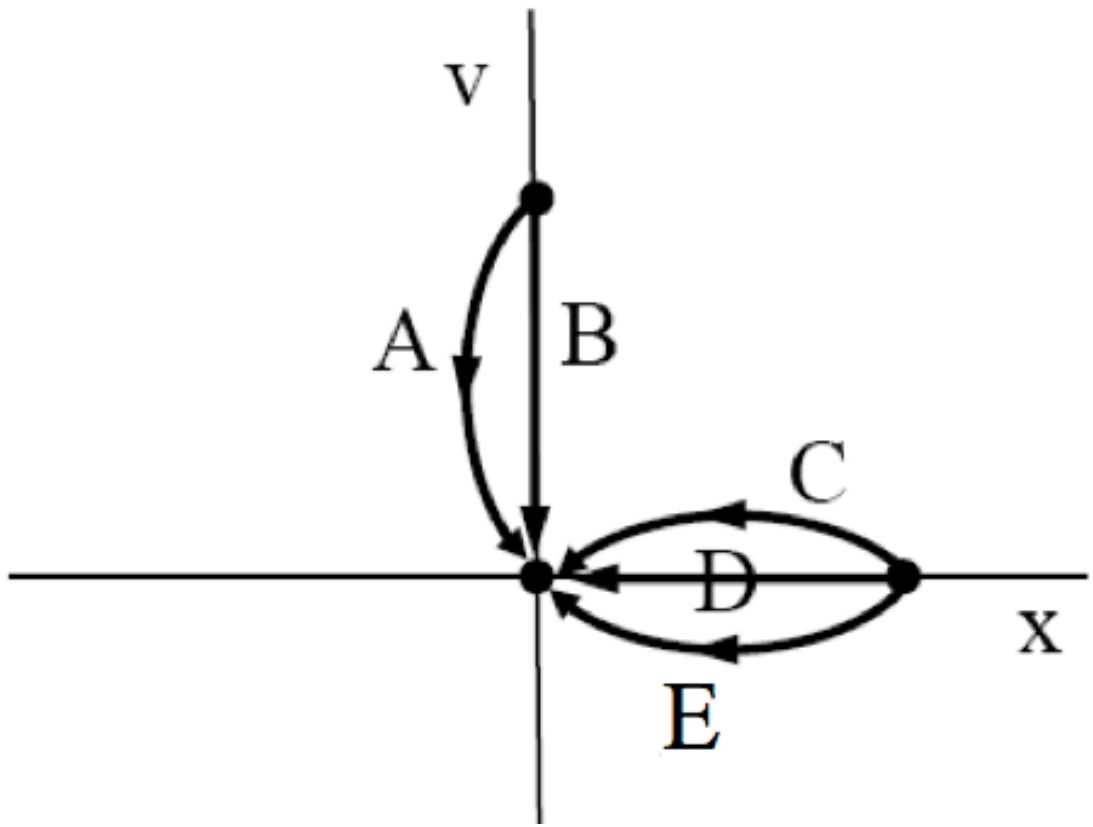
A) No damping is best
B) under-damping
C) critical damping
D) over-damping

What kind of oscillator motion does this phase space diagram describe?

A) overdamped
B) underdamped
C) critically damped
D) undamped (ideal SHM)
E) ??? (not possible?)



Which phase path below best describes overdamped motion for a harmonic oscillator released from rest?



Challenge question: How does your answer change if the oscillator is “critically damped”?

Important concepts

Angular frequency $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

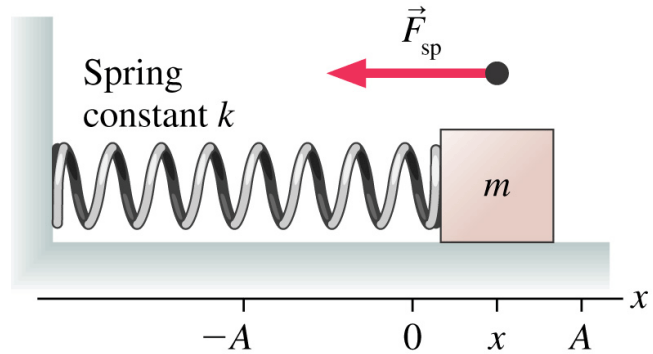
Frequency: $f = \frac{1}{T}$

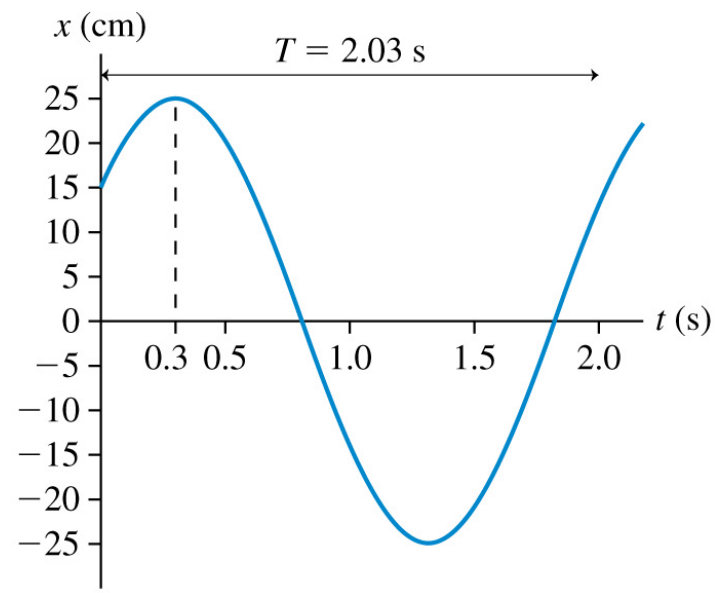
Position: $x(t) = A\cos(\omega t + \delta)$

Velocity: $v(t) = -A\omega\sin(\omega t + \delta)$

Acceleration: $a(t) = -\omega^2 x(t)$

Energy: $E = \frac{mv^2}{2} + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$





No friction implies conservation of mechanical energy

Damped Oscillations

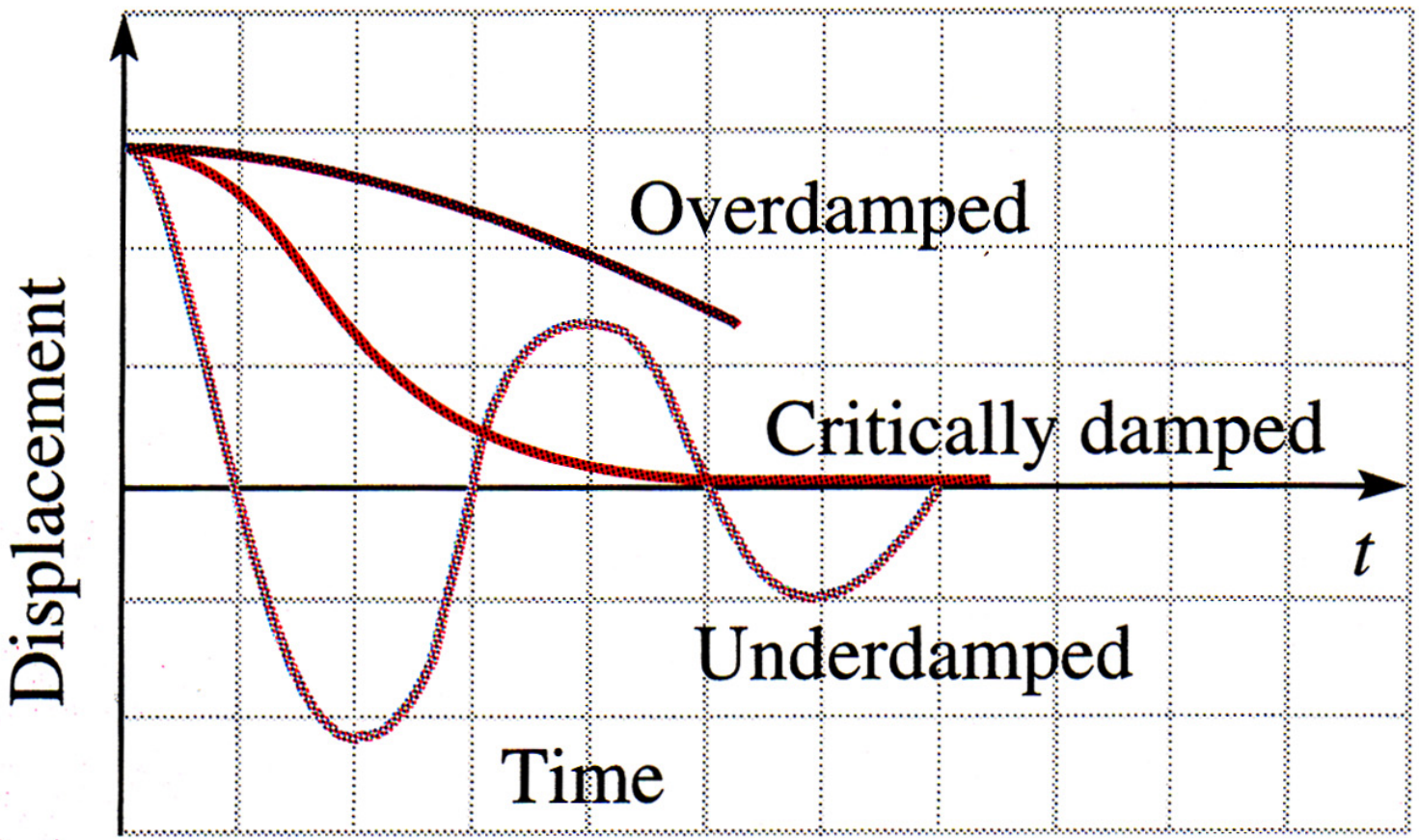
$$F_{drag} = -bv$$

$$2\beta = b/m$$

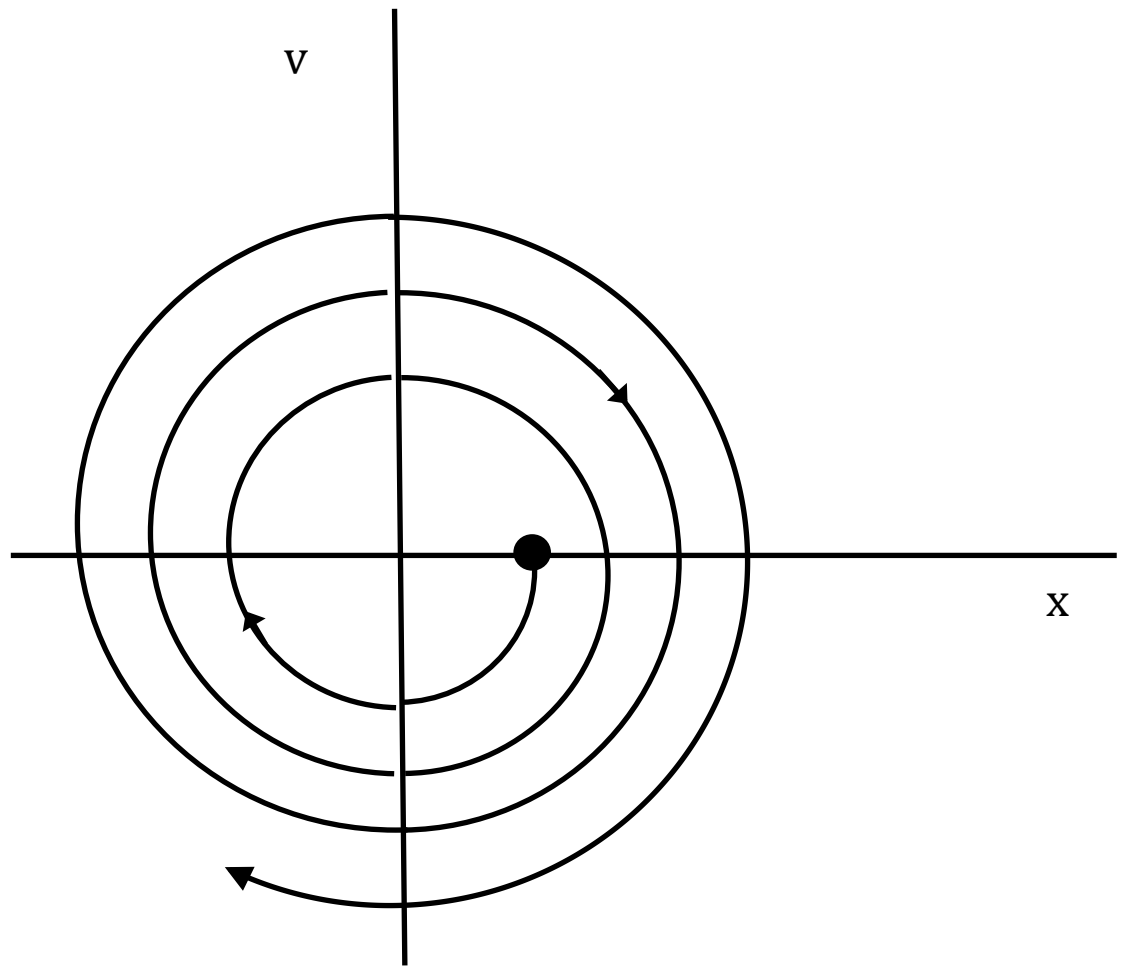
$$F = m\ddot{x} = -kx - b\dot{x}$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^{\,2} x = 0$$

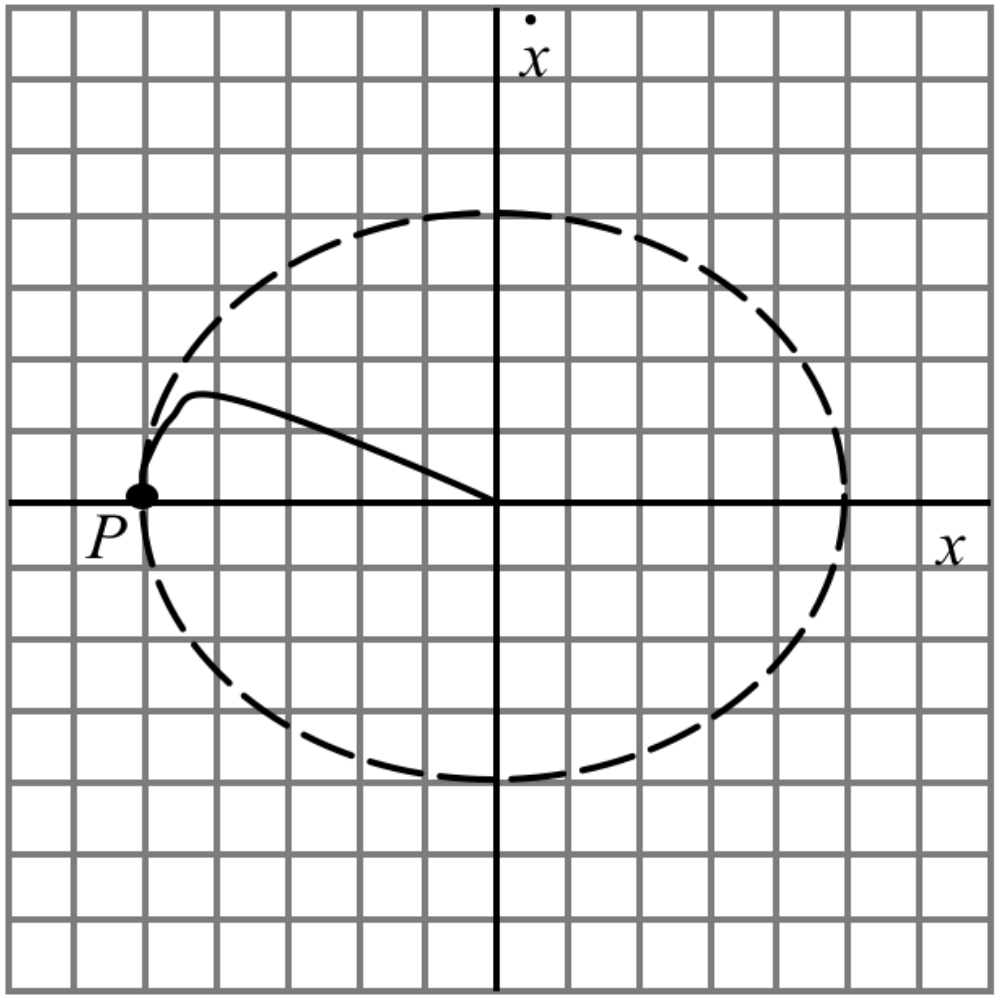
Damped oscillations



What kind of motion does this phase path describe?



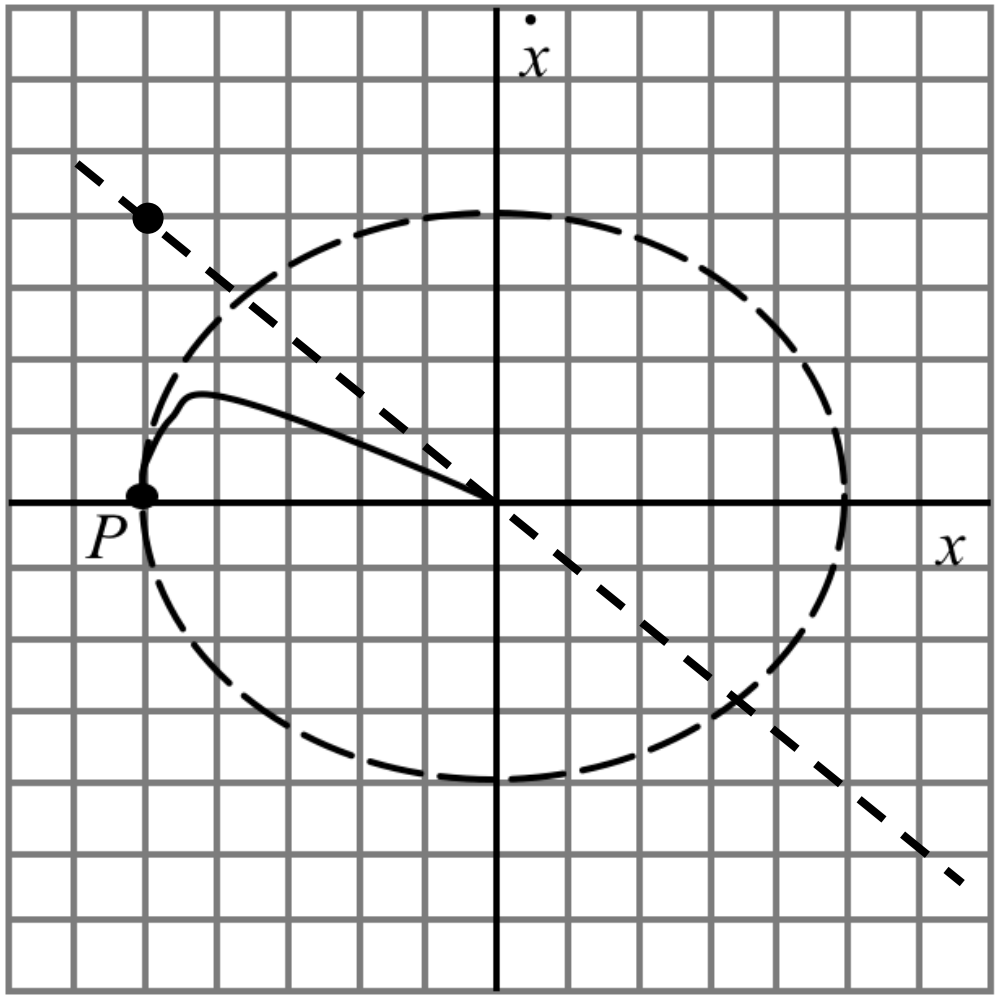
A) overdamped
B) underdamped
C) critically damped
D) impossible to tell
E) This motion is impossible



This phase space plot (the solid line that starts at P and ends at the origin) represents what system?

A) undamped B) under-damped C) critically damped

D) over-damped E) Not enough info to decide!!

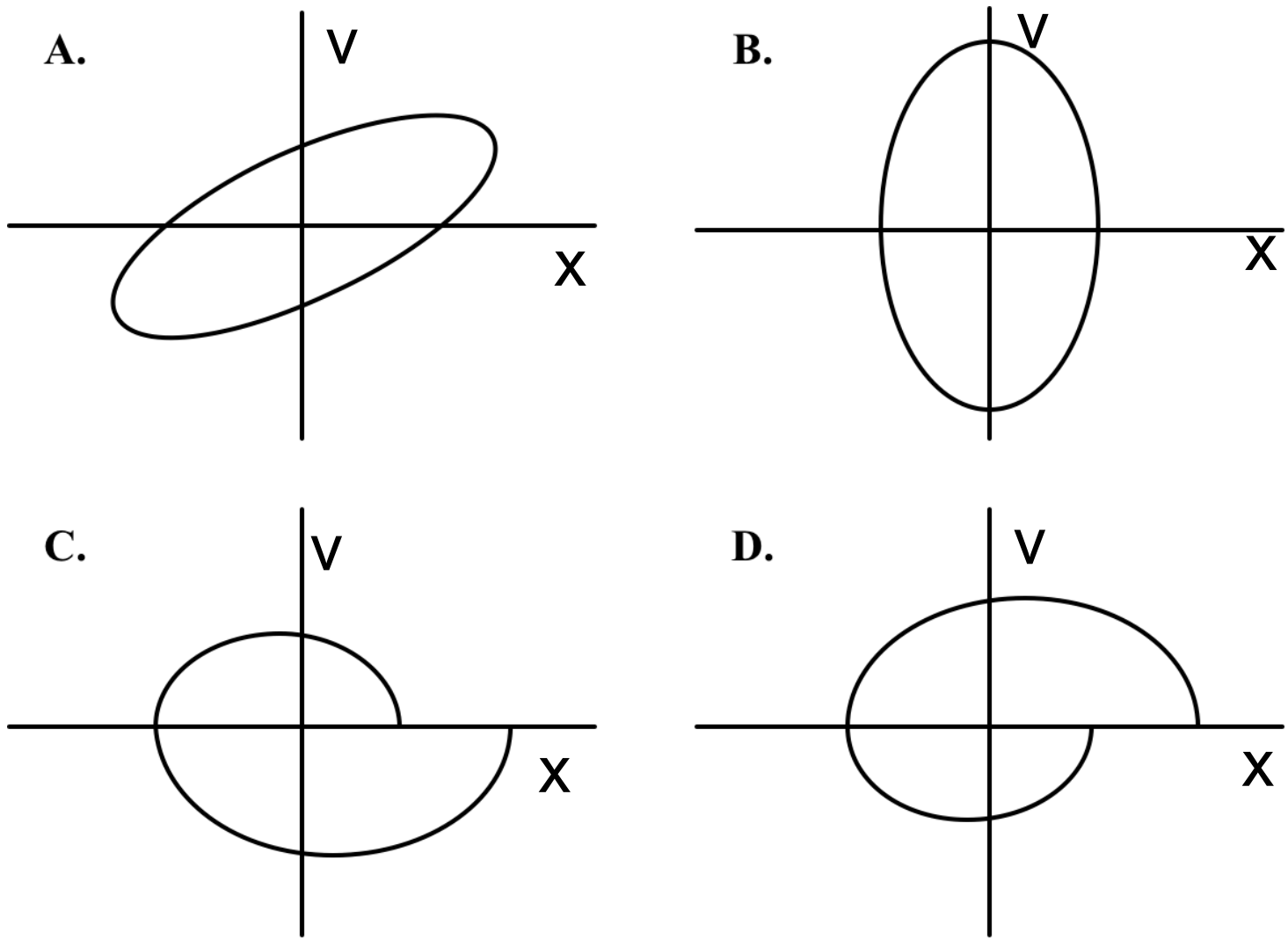


This phase space plot (the solid line that starts at P and ends at the origin) represents what system?

A) undamped B) under-damped C) critically damped

D) over-damped E) Not enough info to decide!!

Which could be the phase space diagram for one full period of an underdamped 1D oscillator?



E) None of these, or MORE than 1!!

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

Given the differential equation for an RLC circuit, which quantity is analagous to the damping term in a mechanical oscillator?

A) R, resistance
B) L, inductance
C) C, capacitance
D) Q, charge
E) t, time

Challenge question: What do the other two electrical quantities “correspond to” in the mechanical system?

http://vnatsci.ltu.edu/s_schneider/physlets/main/osc_damped_driven.shtml

How was Midterm 2 for you?

A) Too hard and/or long - no fair!

B) Hard, but fair

C) Long, but fair

D) Seemed reasonable.

E) I had such a nice break, I can’t remember that far back…

The density of a spherical object is $\rho(r)=c/r^2$ (out to radius R, then 0 beyond that)

This means the total mass of this object is M =

$$A)\quad \rho V = \left(\frac{c}{r^2}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{4\pi c}{3} r$$

$$B)\quad \rho V = \left(\frac{c}{R^2}\right)\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi c}{3} R$$

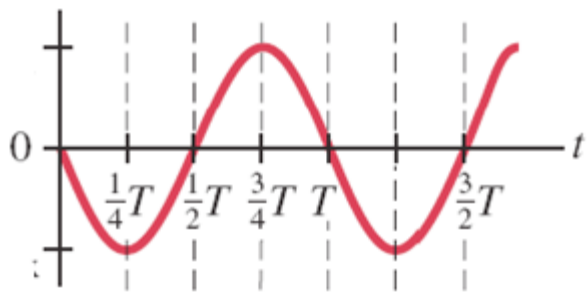
$$C)\quad \rho V = \left(\frac{c}{r^2}\right)\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi c R^3}{3r^2}$$

$D)$ None of these is correct!

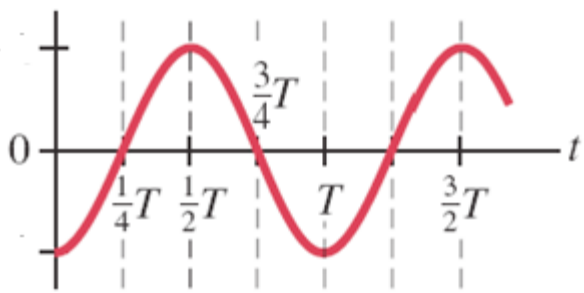
Quick Review after the break

If a spring is pulled and released from rest, the plots shown below correspond to:

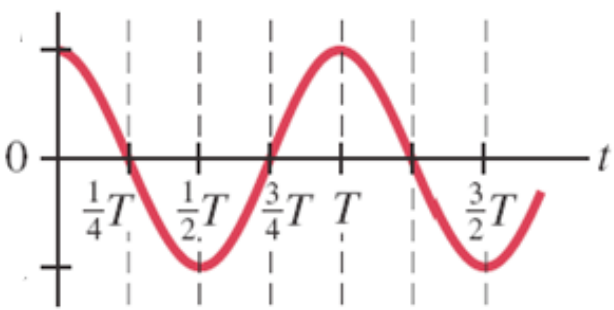
I



II



III



A) I: velocity, II:position, III:acceleration

B) I: position, II:velocity, III:acceleration

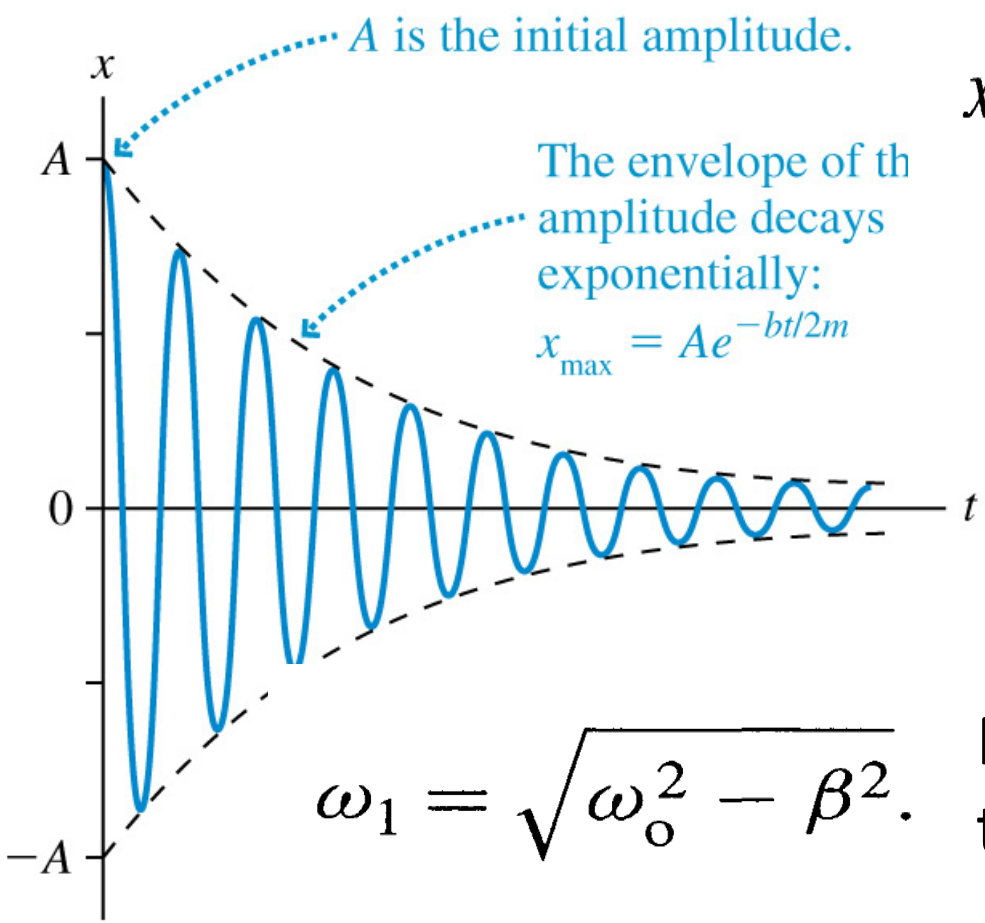
C) I: acceleration, II:velocity, III:position

D) I: velocity, II:acceleration, III:position

E) I do not have enough information!

Underdamped Oscillations

An *underdamped oscillation* *with* $\beta<\omega_0$:



$$x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta).$$

$$\beta < \omega_o$$

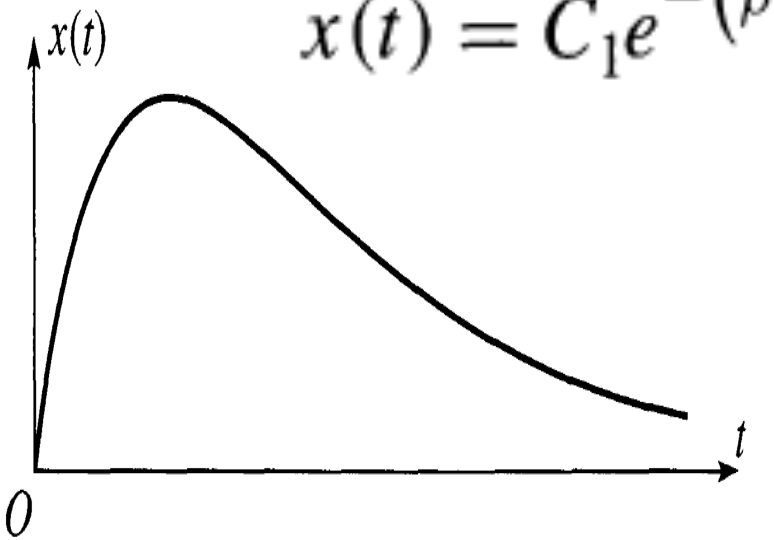
$$\omega_1 = \sqrt{\omega_o^2 - \beta^2}.$$

Note that damping ***reduces*** the oscillation frequency.

Overdamped Oscillations

An *overdamped oscillation* *with* $\beta > \omega_0$:

$$x(t) = C_1 e^{-\left(\beta - \sqrt{\beta^2 - \omega_o^2}\right)t} + C_2 e^{-\left(\beta + \sqrt{\beta^2 - \omega_o^2}\right)t}.$$



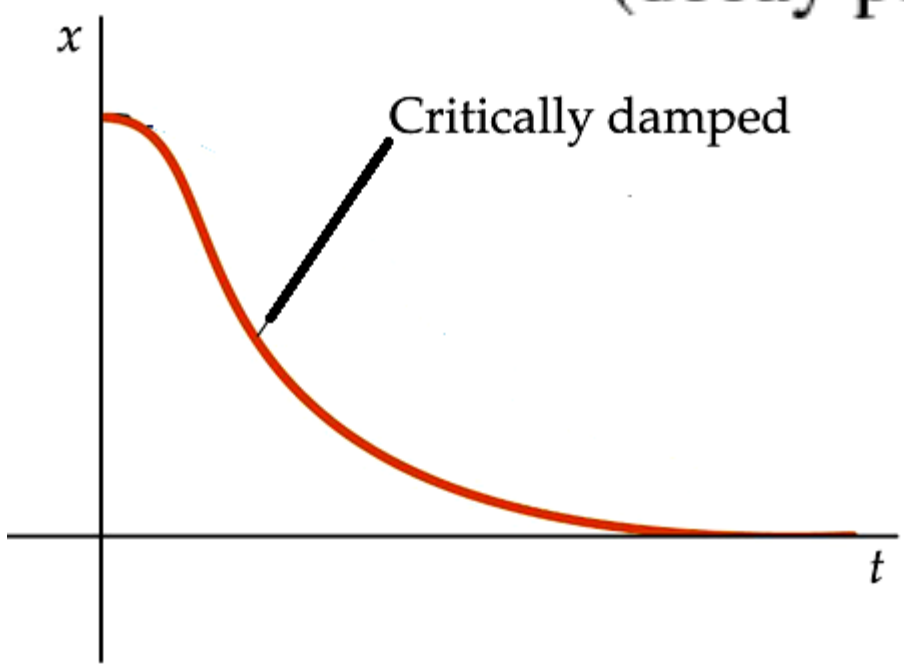
$$(\text{decay parameter}) = \beta - \sqrt{\beta^2 - \omega_o^2}$$

Critically damped Oscillations

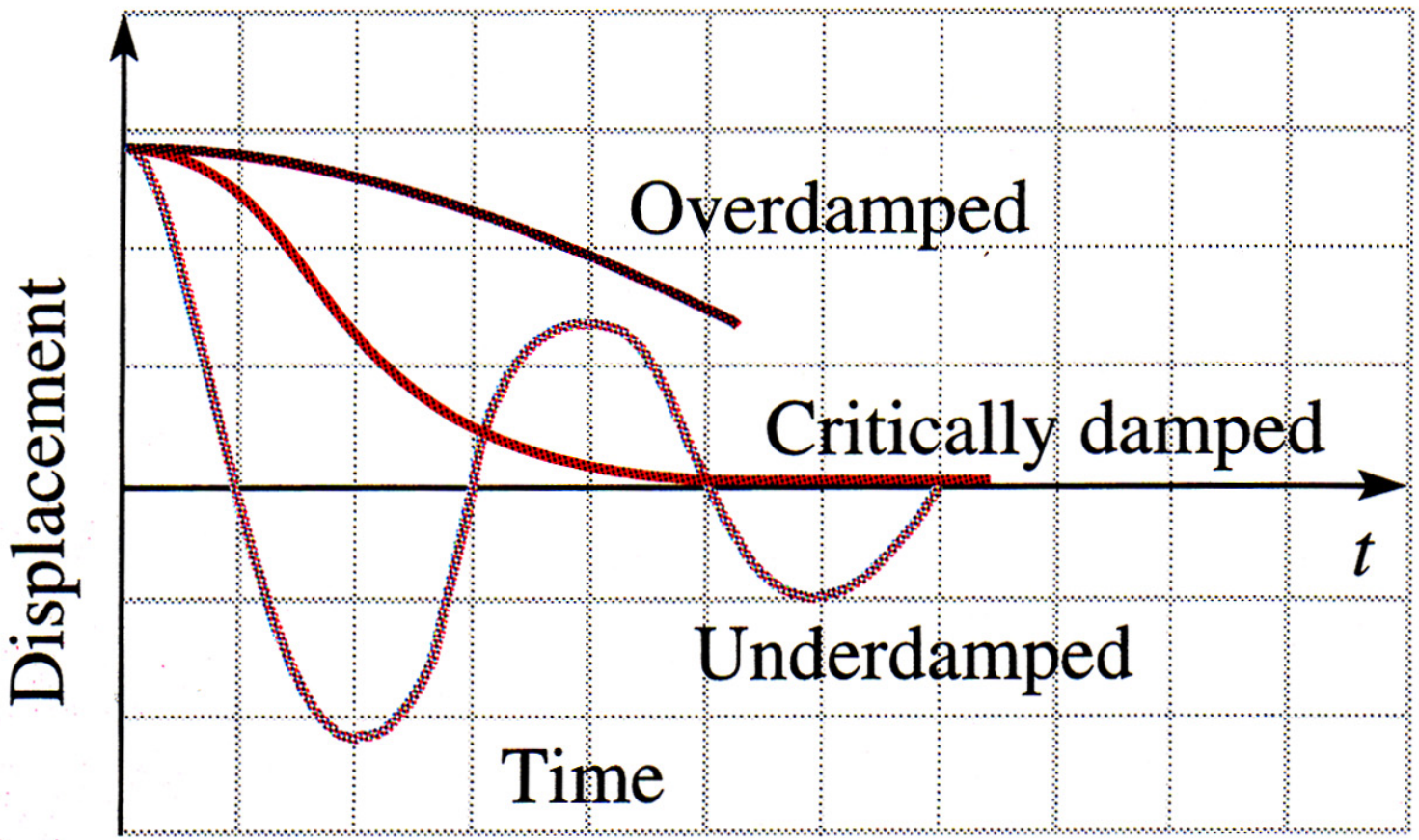
A critically damped *oscillation* *with* $\beta = \omega_0$:

$$x(t) = C_1 e^{-\beta t} + C_2 t\, e^{-\beta t}.$$

$$(\text{decay parameter}) = \beta = \omega_0$$



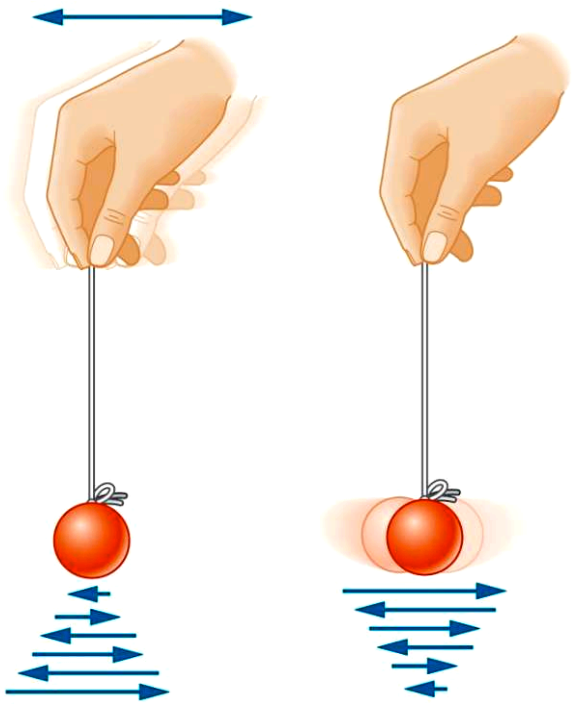
Damped oscillations



Driven oscillations

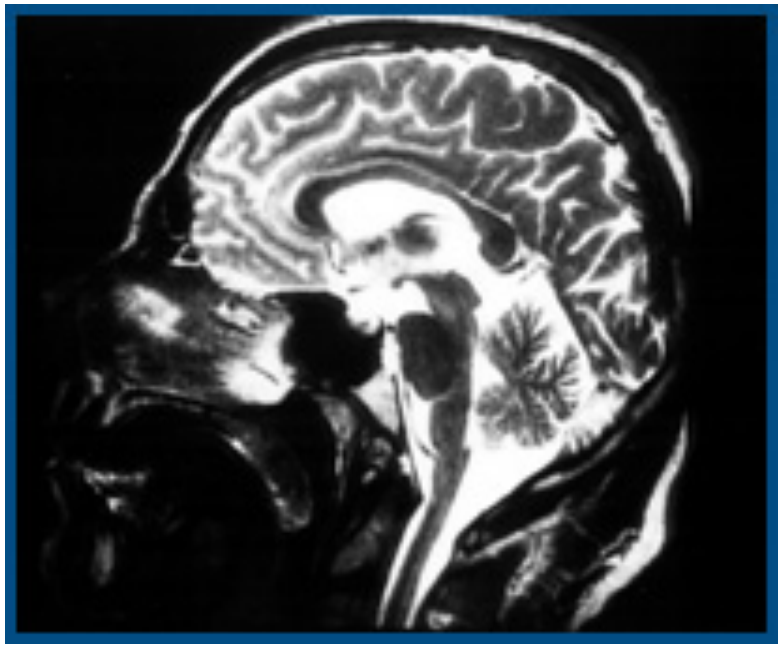
This class

Driven Oscillations & Resonance



Driven oscillators and Resonance:

Emission & absorption of light
Lasers
Tuning of radio and television sets
Mobile phones
Microwave communications
Machine, building and bridge design
Musical instruments
Medicine
- nuclear magnetic resonance
  magnetic resonance imaging
- x-rays
- hearing



Nuclear magnetic Resonance Scan

What is a particular solution to the equation $y''+4y'-12y=5$?

A) $y=e^{5t}$
B) $y=-5/12$
C) $y=-a\ 5/12$
D) $y= a\ e^{2t} +b\ e^{-6t} +5$
E) Something else

General solution is thus $y= a\ e^{2t} +b\ e^{-6t} - 5/12$

What might you try for a particular solution to

$$y''+4y'+6y=3e^{2t} \text{ ?}$$

How about $= 4e^{2it}$?

How about $= f_0\cos(2t)$?

Consider the following equation

$$x''+16x=9\sin(5t),\quad x(0)=0,\ x'(0)=0$$

The solution is given by

A. $x(t)=c_1\cos(4t)+c_2\sin(4t)-\sin(5t)$

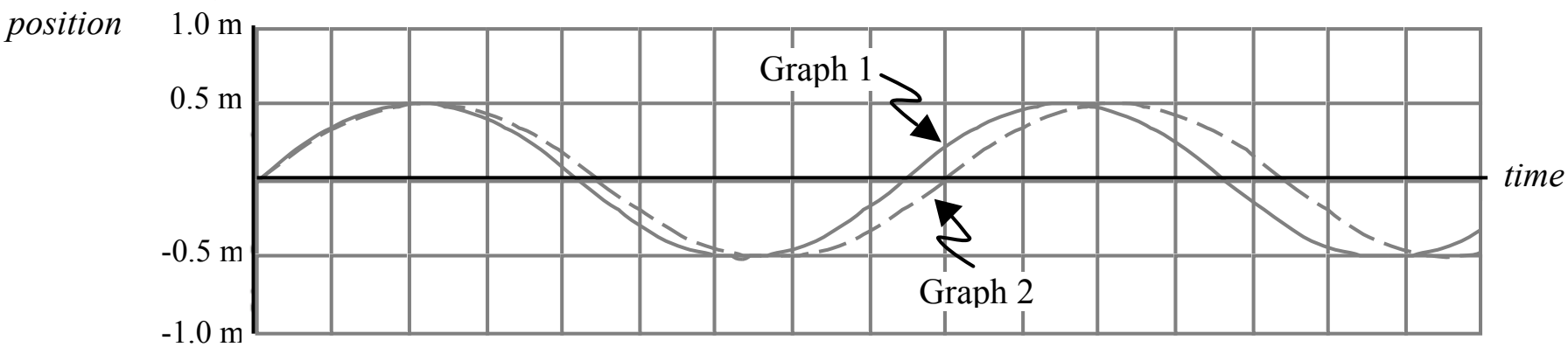
B. $x(t)=5/4\sin(4t)-\sin(5t)$

C. $x(t)=\cos(4t)-\cos(5t)$

D. I do not know

A) Done with page 1
B) Done with page 2
C) Done with page 3
D) Done with the puzzle below

Puzzle: Taylor p. 189 says $\omega_r = \sqrt{\omega_0^2 - 2\beta^2}$
Suppose one oscillator is ideal, with amplitude 0.5 m. Another is lightly damped, and driven, so that by t=0 a (**resonant**) steady-state amplitude of 0.5 m is reached. Which graph is which?

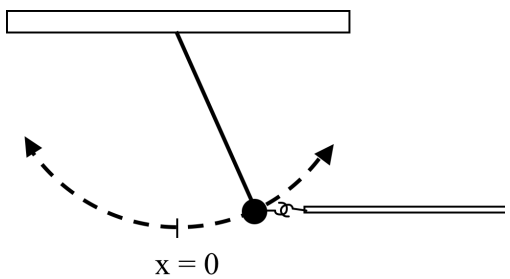


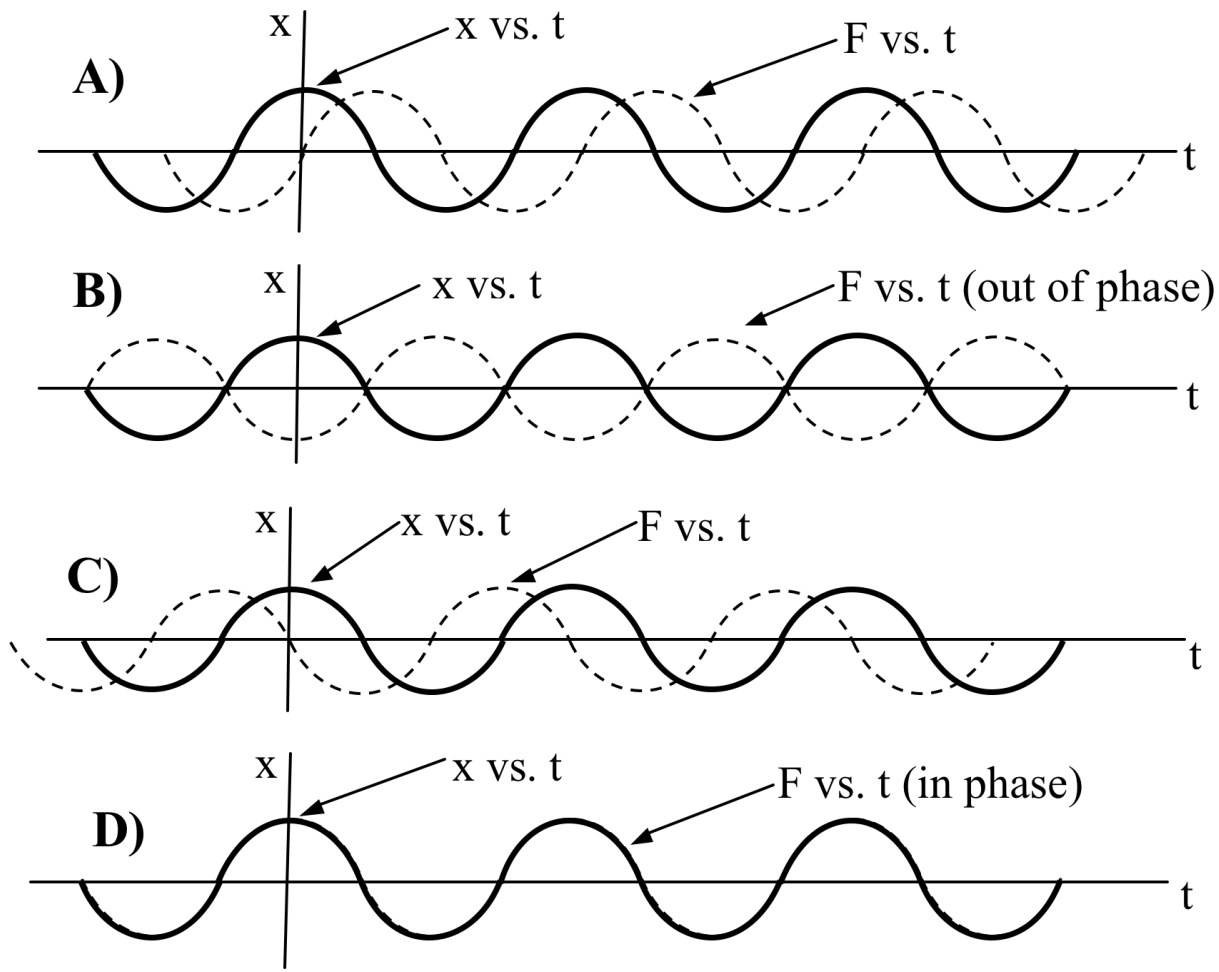
What is the *most general* form of the solution of the ODE $u''+4u=e^t$?

A) $u=C_1e^{2t} + C_2e^{-2t} + C_3e^t$
B) $u=A\cos(2t-\delta) + C_3e^t$
C) $u=C_1e^{2t} + C_2e^{-2t} + (1/5)e^t$
D) $u=A\cos(2t-\delta) + (1/5)e^t$
E) Something else!???

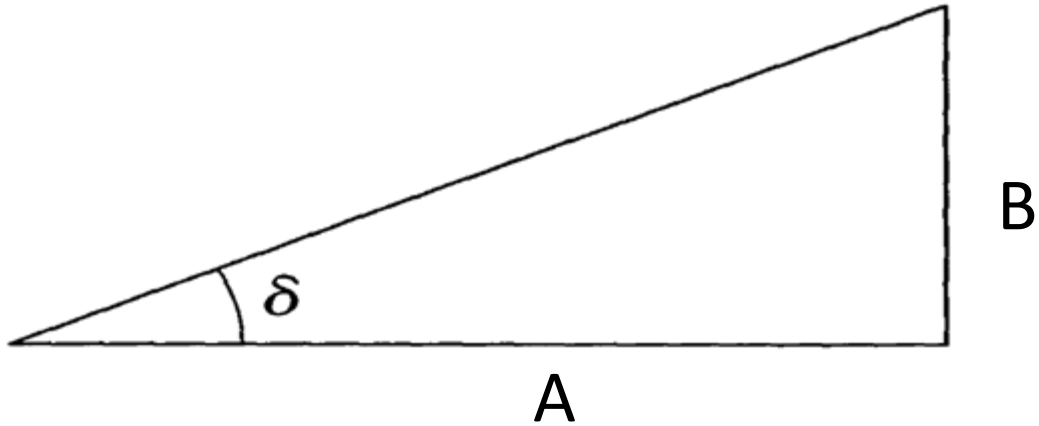
A person is pushing and pulling on a pendulum with a long rod. Rightward is the positive direction for both the force on the pendulum and the position x of the pendulum mass

To maximize the amplitude of the pendulum, how should the phase of the driving force be related to the phase of the position of the pendulum?





The phase angle d in this triangle is



A. $\delta = \arctan\left(\frac{B}{A}\right)$ B. $\delta = arc\cot\left(\frac{A}{B}\right)$

C. $\delta = \arctan\left(\frac{A}{B}\right)$ D. More than one is correct

D. None of these are correct

Consider the general solution for a damped, driven oscillator:

Which term dominates for large t?

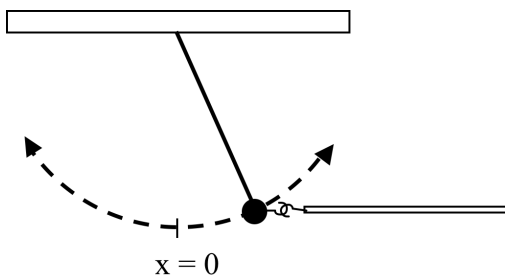
$$x(t) = \underbrace{C_1 e^{-\beta t} e^{+\sqrt{\beta^2-\omega_0^2}\,t}}_{\text{term A}} + \underbrace{C_2 e^{-\beta t} e^{-\sqrt{\beta^2-\omega_0^2}\,t}}_{\text{term B}} + \underbrace{A\cos(\omega t - \delta)}_{\text{term C}}$$

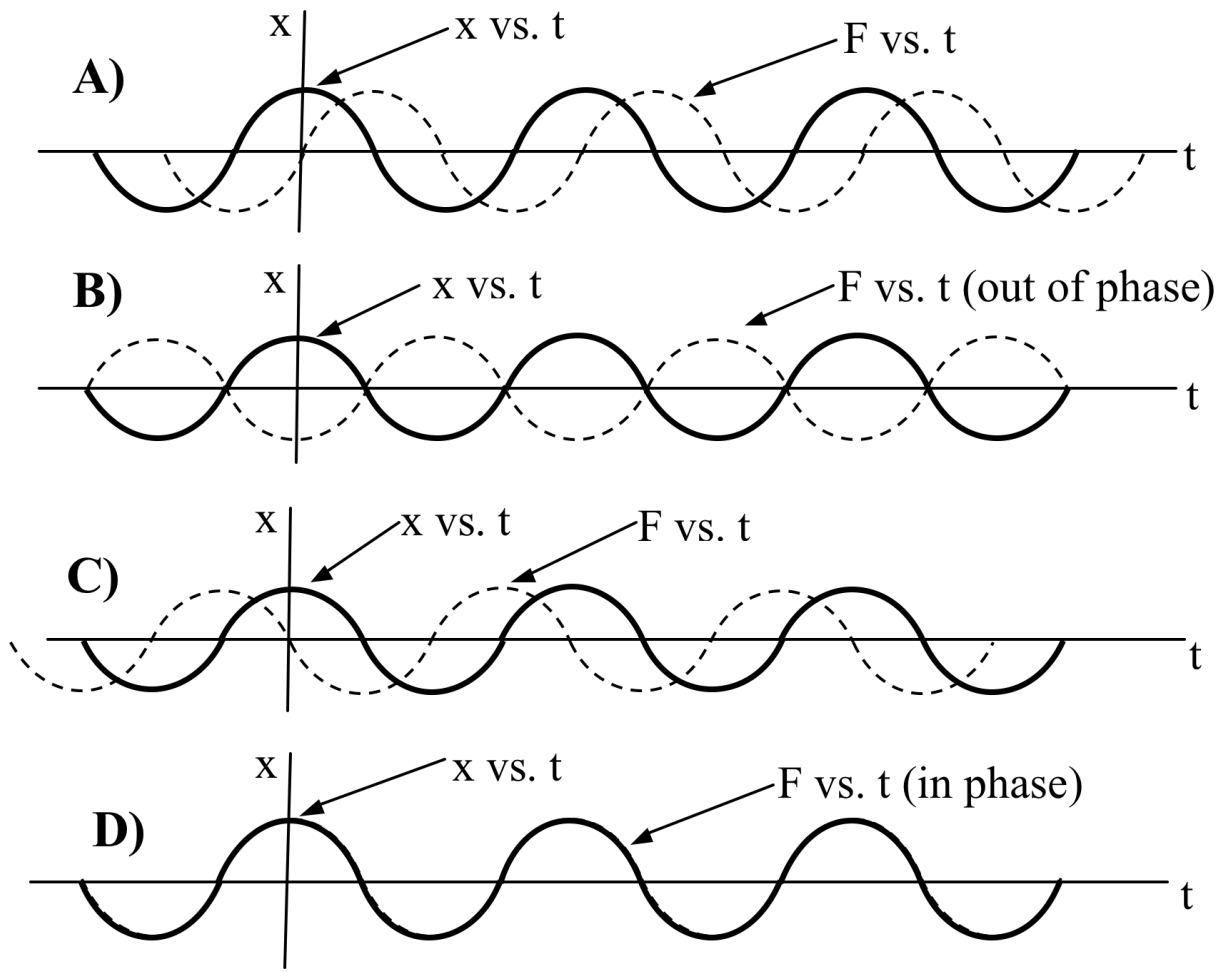
D) Depends on the particular values of the constants
E) More than one of these!

Challenge questions: Which term(s) matters most at small t?
Which term "goes away" first?

A person is pushing and pulling on a pendulum with a long rod. Rightward is the positive direction for both the force on the pendulum and the position x of the pendulum mass

To maximize the amplitude of the pendulum, how should the phase of the driving force be related to the phase of the position of the pendulum?





Consider the amplitude

$$A^2 = \frac{f_0^{\,2}}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$

In the limit as ω goes to infinity, A

A. Goes to zero
B. Approaches a nonzero constant
C. Goes to infinity
D. I don’t know!

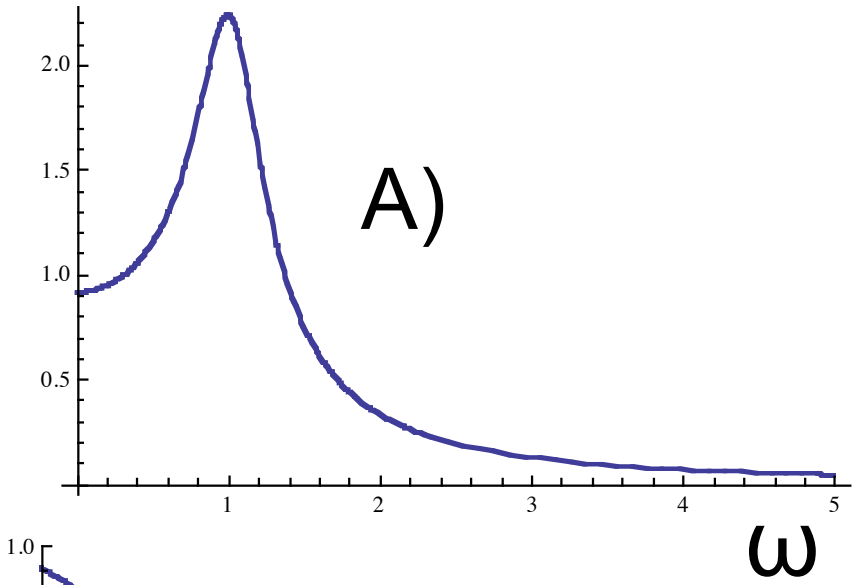
Consider the amplitude

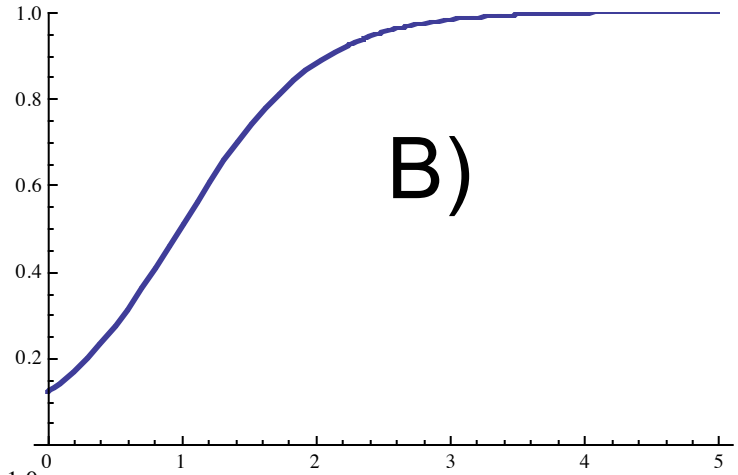
$$A^2 = \frac{f_0^{\,2}}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$

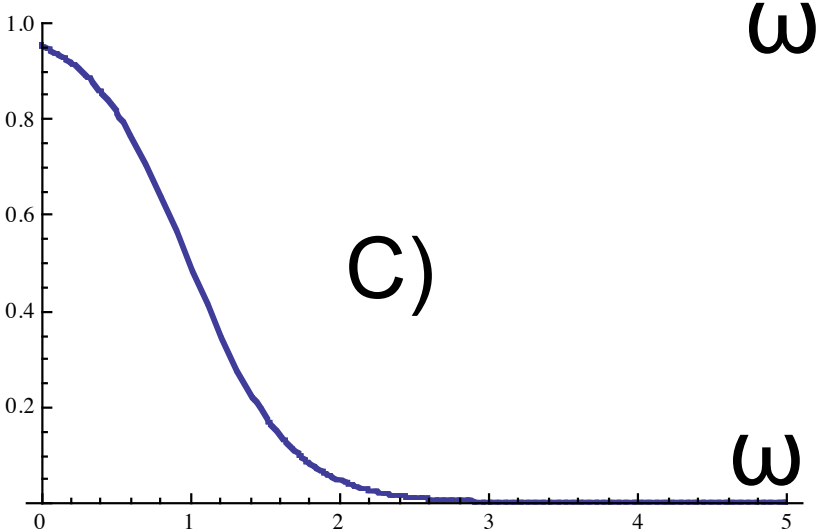
In the limit of no damping, A

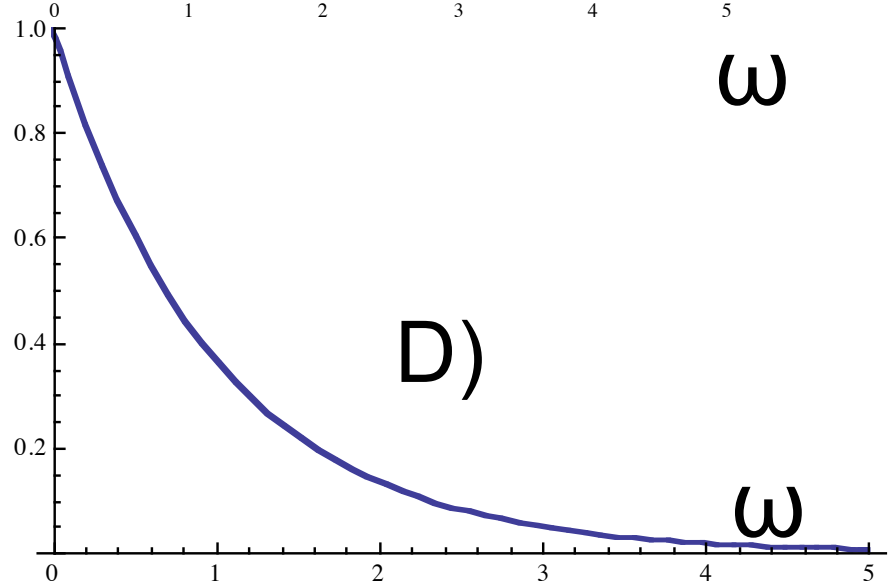
A. Goes to zero
B. Approaches a constant
C. Goes to infinity
D. I don’t know!

What is the shape of $A = \dfrac{1}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2}}$



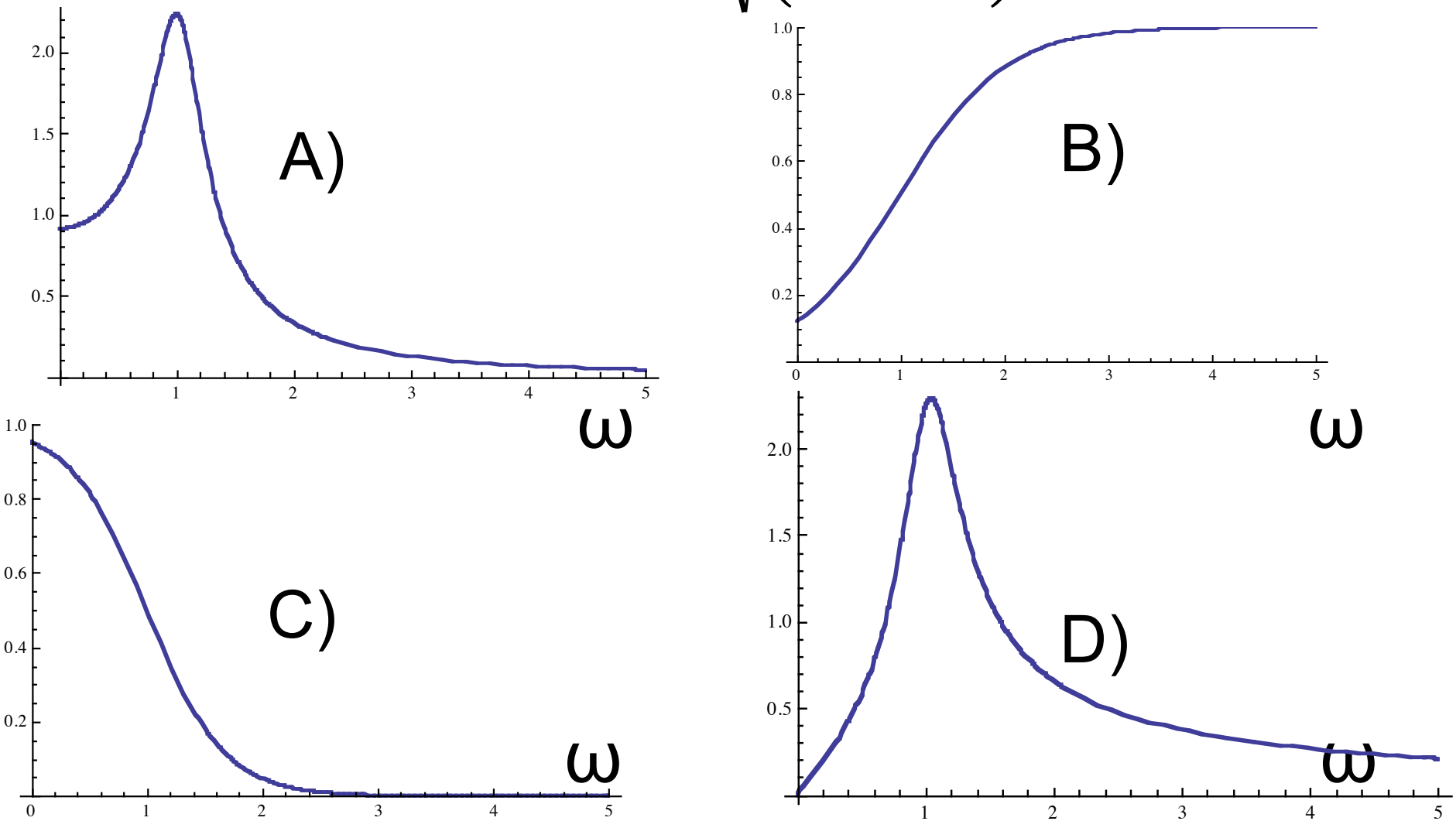






E) None of these looks at all correct?!

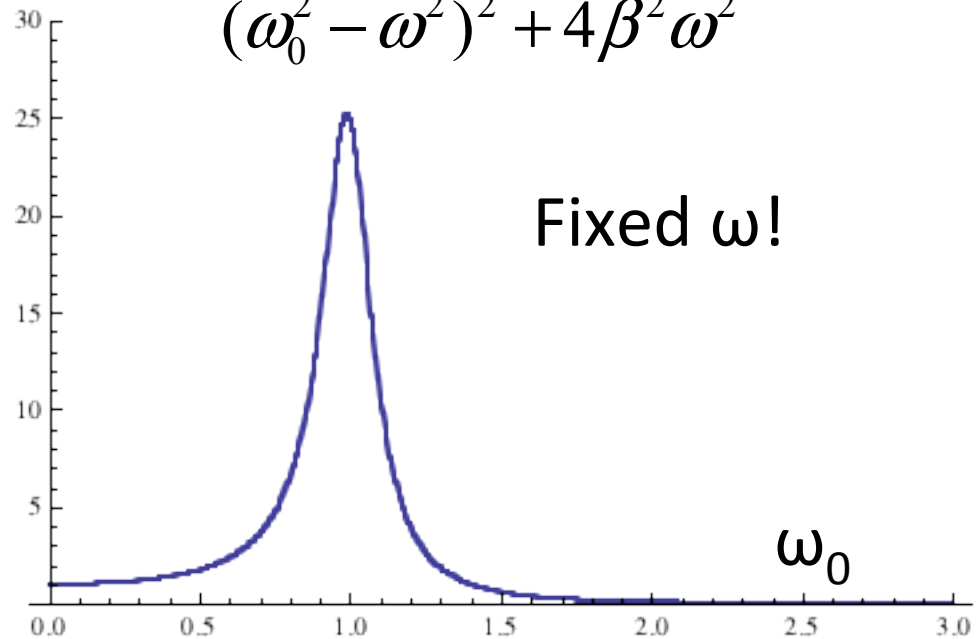
What is the shape of $A = \dfrac{1}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2}}$



E) None of these looks at all correct?!

If you have a damped, driven oscillator, and you increase damping, β, (leaving everything else fixed) what happens to the curve shown?

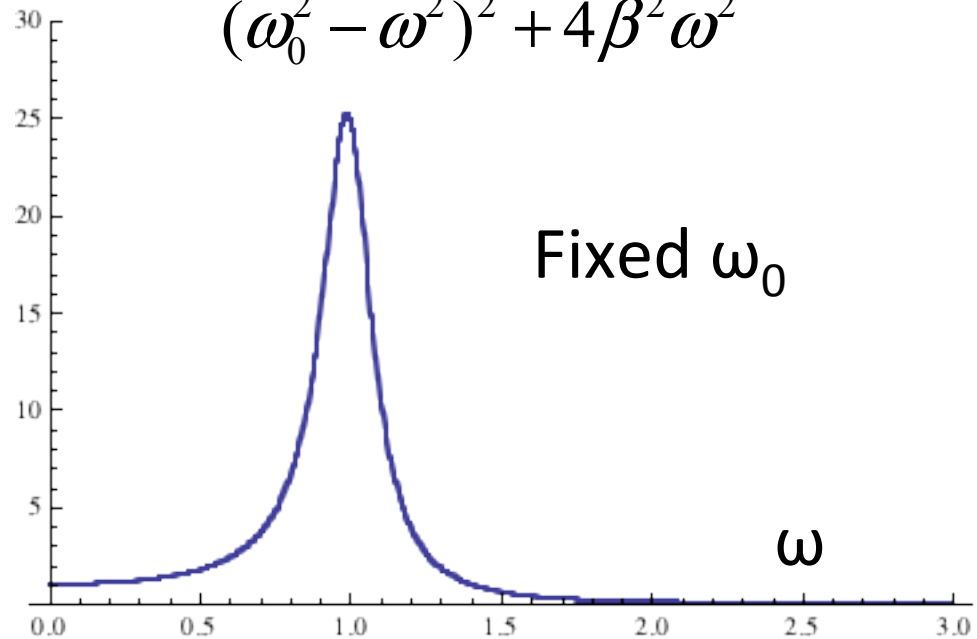
$$A^2 = \frac{f_0^{\,2}}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$



A) It shifts to the LEFT, and the max value increases.
B) It shifts to the LEFT, and the max value decreases.
C) It shifts to the RIGHT, and the max value increases.
D) It shifts to the RIGHT, and the max value decreases.
E) Other/not sure/???

If you have a damped, driven oscillator, and you increase damping, β, (leaving everything else fixed) what happens to the curve shown?

$$A^2 = \frac{f_0^{\,2}}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$



A) It shifts to the LEFT, and the max value increases.
B) It shifts to the LEFT, and the max value decreases.
C) It shifts to the RIGHT, and the max value increases.
D) It shifts to the RIGHT, and the max value decreases.
E) Other/not sure/???

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

Given the differential equation for an RLC circuit, which quantity is analagous to the inertial (mass) term in a mechanical oscillator?

A) R, resistance
B) L, inductance
C) C, capacitance

Challenge question: What are the other two quantities analagous to ?

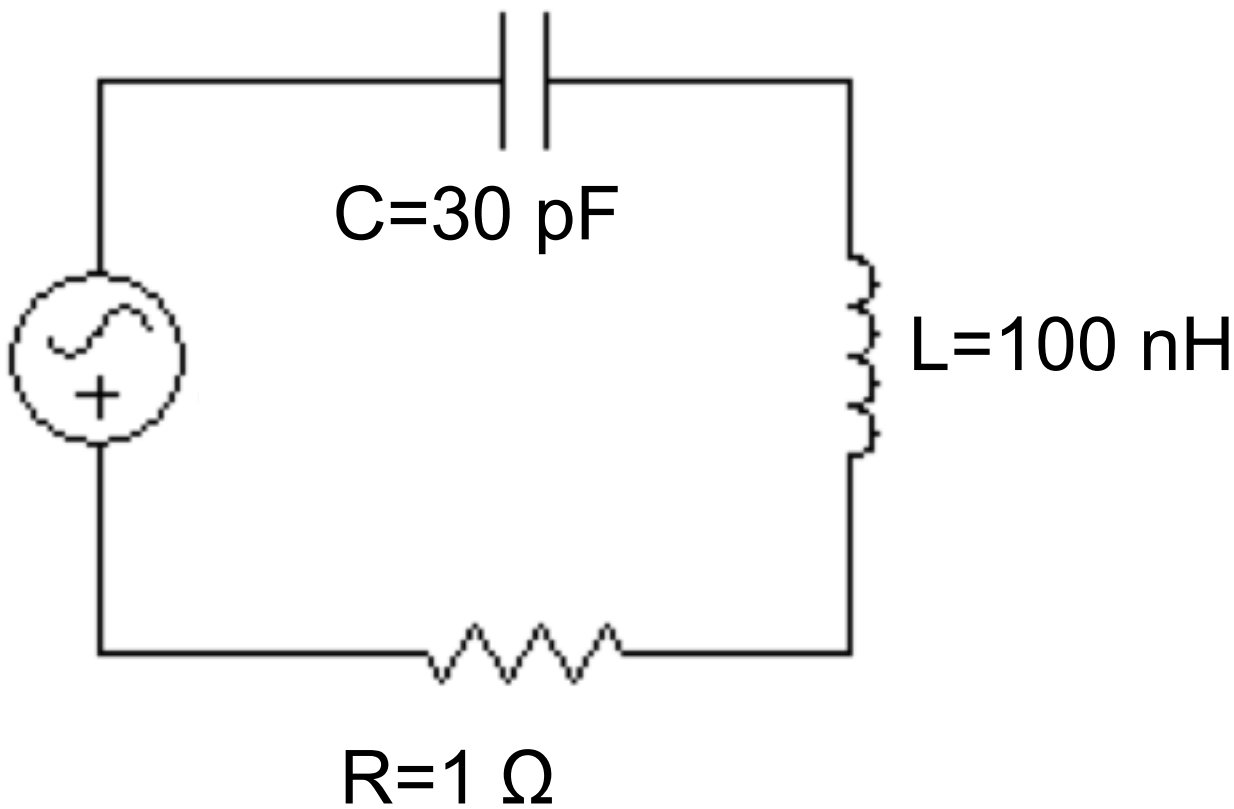
$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

What is ω_0?

A) C
B) 1/C
C) 1/Sqrt[C]
D) 1/LC
E) 1/Sqrt[LC]

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=a_0e^{i\omega t}$$

If we attach an AC voltage source to the circuit shown below, for what frequency do we get the maximum charge flowing through the circuit?



Our particular solution is : $x_p(t) = Ce^{i\omega t}$

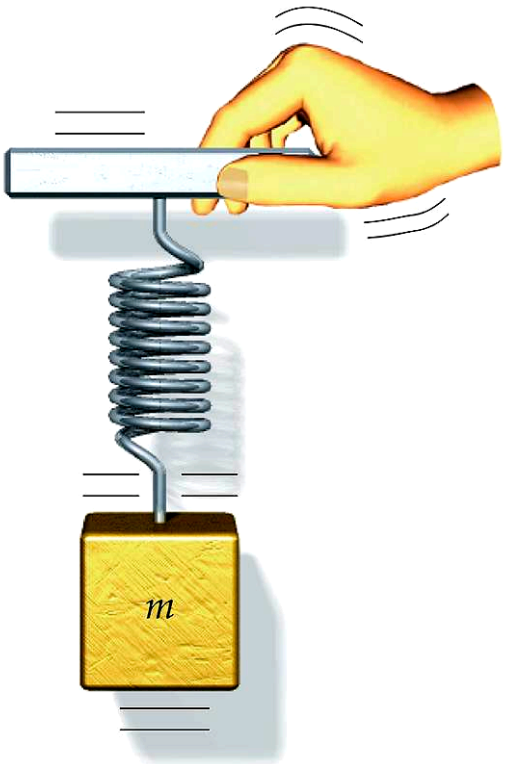
with $C = \dfrac{f_0}{(\omega_0^2 - \omega^2) + 2\beta\omega i} \quad = Ae^{-i\delta}$

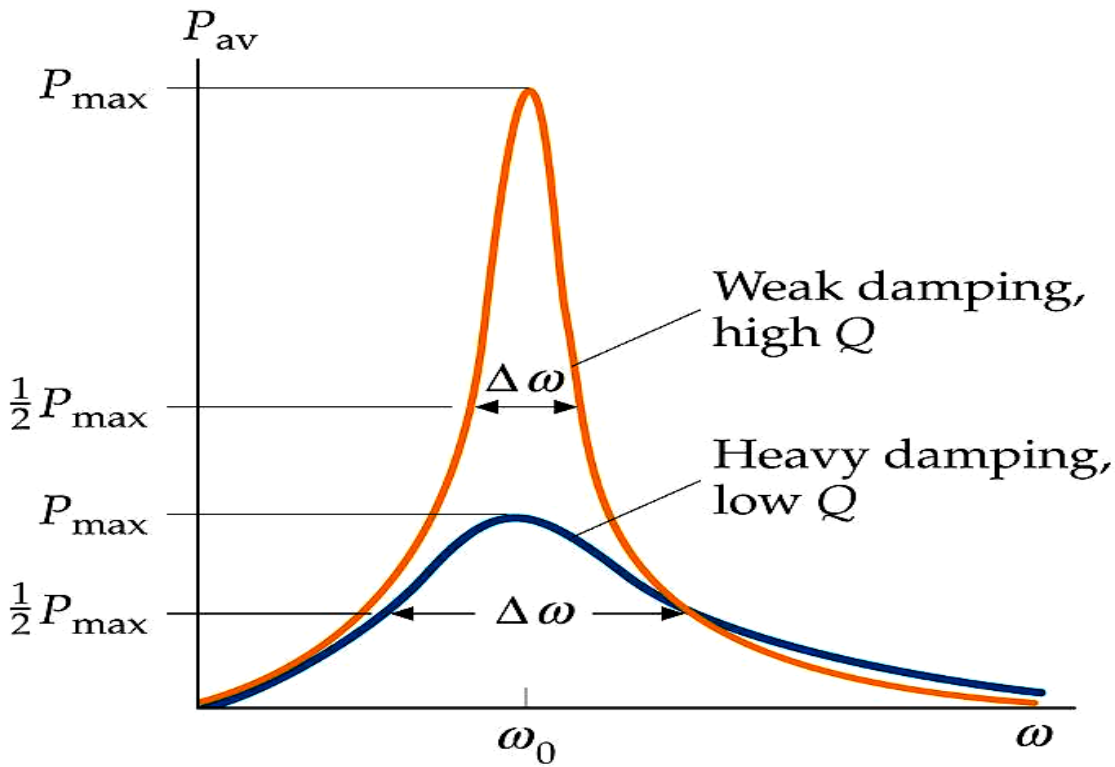
So $\delta = \tan^{-1} \dfrac{2\beta\omega}{(\omega_0^2 - \omega^2)}$

At resonance, this means the phase between x and F is:

A) 0
B) 90°
C) 180°
D) Infinite
E) Undefined/???

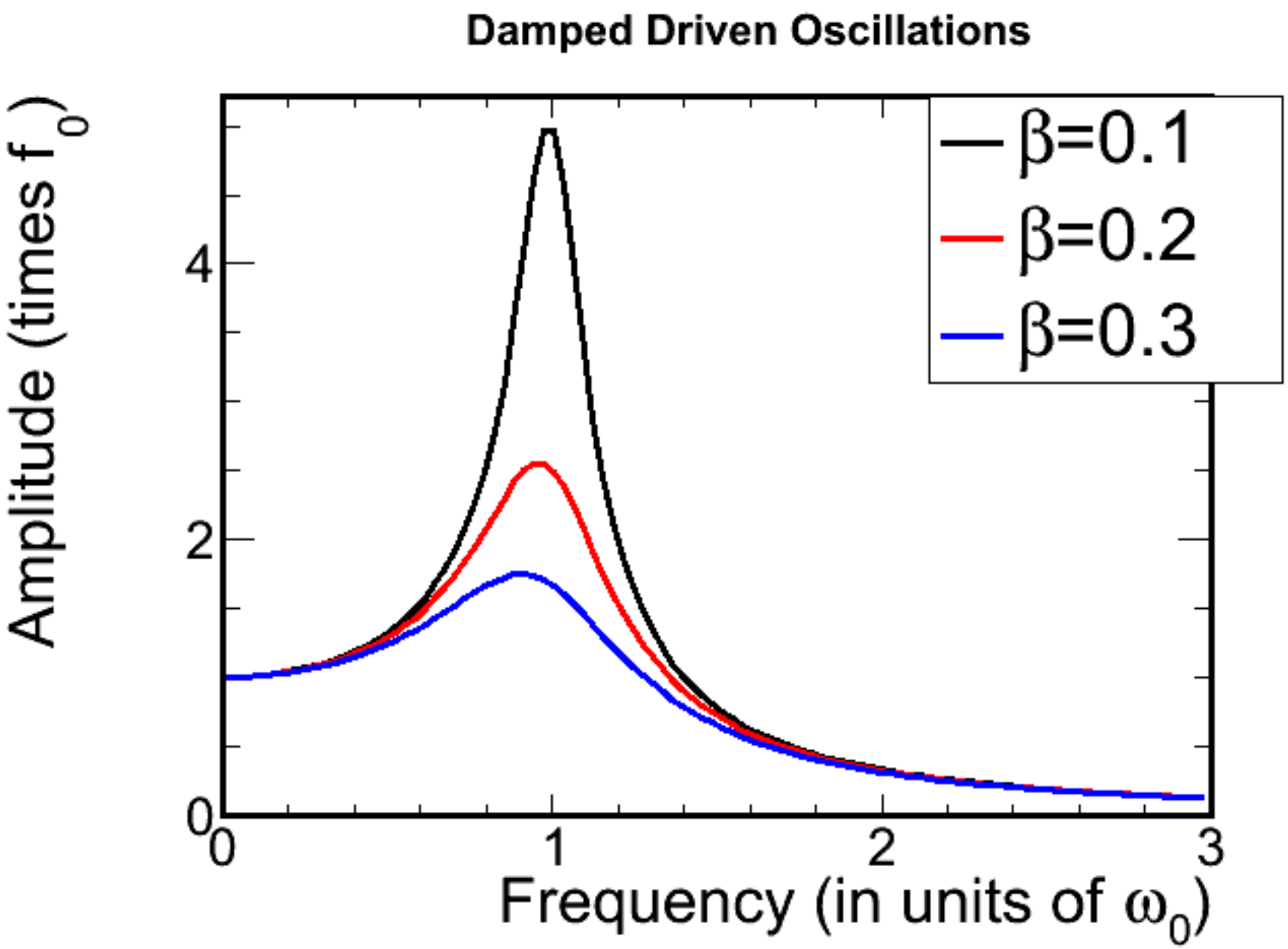
Driven Oscillations & Resonance



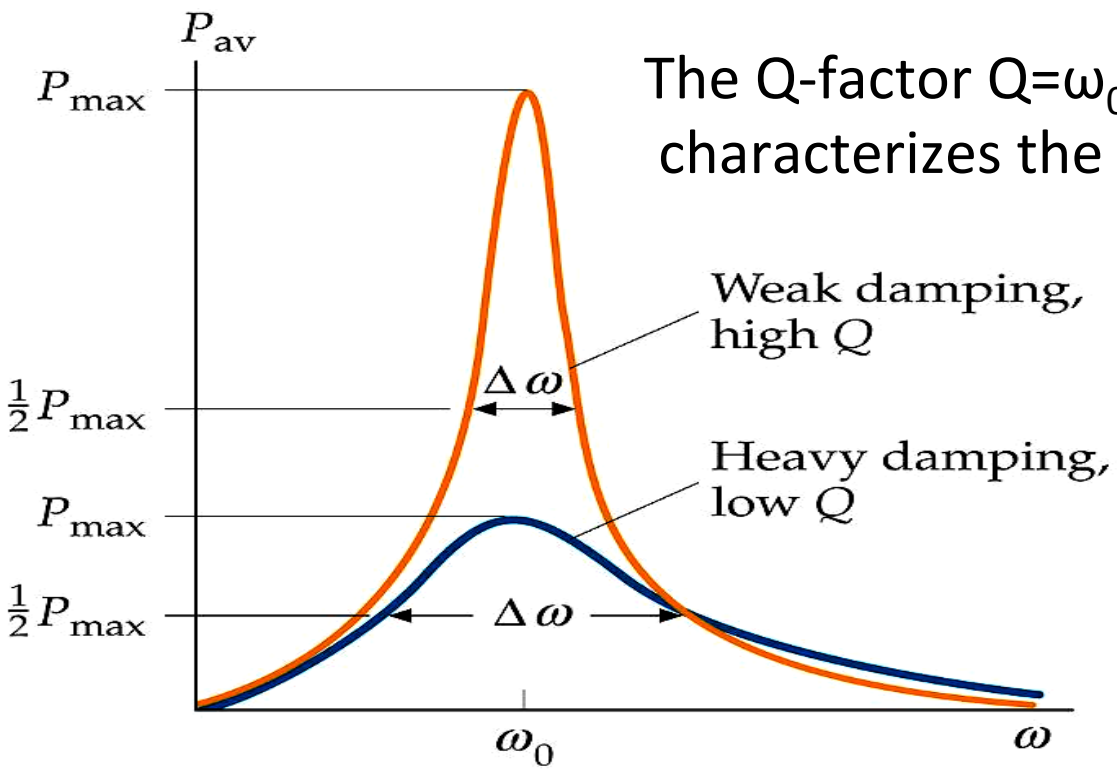


The Q-factor characterizes the sharpness of the peak

$Q=\omega_0/2\beta$ (proportional to τ/T)


Damped Driven Oscillations
Amplitude (times f_0)
Frequency (in units of ω_0)
0
2
4
0
1
2
3
β=0.1
β=0.2
β=0.3

Driven Oscillations & Resonance



The Q-factor $Q=\omega_0/2\beta$ (proportional to τ/T) characterizes the sharpness of the peak

What is the (very) approximate Q of the simple harmonic oscillator shown in front of class?

A) much less than 1 B) of order 1 C) Much greater than 1

What is the approximate Q of the simple harmonic oscillator shown in class?

A) 1
B) 100
C) 10000
D) .01
E) .0001

Can you break a wine glass with a human voice if the person sings at precisely the resonance frequency of the glass?

A) Sure. I could do it
B) A highly trained opera singer might be able to do it.
C) No. Humans can't possibly sing loudly enough or precisely enough at the right frequency. This is just an urban legend.

Myth Busters video http://dsc.discovery.com/videos/mythbusters-adam-savage-on-breaking-glass.html