Problem List

1.1 Pulling a block up an incline

Given: Marino – Fall 2011

A 3-kg block is moved up a 37° incline (relative to horizontal) under the action of a constant *horizontal* force of 40 N. The coefficient of kinetic friction is 0.1 and the block is displaced 2 m along the incline.

(a) Calculate the work done by the 40 N force.

(b) Calculate the work done by friction.

(c) If the block is initially at rest, how long does it take the block to travel 2 m along the incline?

1.2 Pulling an object at an angle (with friction)

Given: Pollock – Spring 2011

You're at the airport, in a hurry to catch a plane, and the wheels on your suitcase (mass m) have completely frozen up, so they are skidding along the floor (with some small, constant coefficient of kinetic friction μ_K) Assuming you can pull (at any angle you want!) with some given, fixed arm-force $|\vec{F}_{\text{arm}}|$, at what angle from the horizontal should you pull to accelerate forward as fast as possible? After you find the angle - figure out a formula for this maximum possible forward acceleration for your suitcase. Your answers for both angle and acceleration should be in terms of *just* the given constants m, μ_K, and/or $|\vec{F}_{\text{arm}}|$ (and of course g).

(Explicitly check both your answers by considering units, and also examining the limit of $\mu_K \to 0$.)

Notes: Remember that we model kinetic friction as being simply proportional to the normal force, $|\vec{F}_{\text{arm}}| = \mu_K|\vec{N}|$. Of course in this problem you need to be careful, the normal force is NOT the weight of the suitcase - draw a free body diagram!)

Given: Pollock – Spring 2012

You're at the park; it's just finished snowing. You're trying to pull your kid sister on a sled (which together have mass m) as fast as possible. The sled is old, so there's some small, constant coefficient of kinetic friction μ_K between the sled bottom and the snow.

(a) Assuming you can pull (at any angle you want!) with some given, fixed arm-force $|\vec{F}_{\text{arm}}|$, at what angle from the horizontal should you pull to accelerate your sister and the sled forward as fast as possible?

(b) After you find the angle - figure out a formula for this maximum possible forward acceleration for your sister and the sled.

Your answers for both angle and acceleration should be in terms of *just* the given constants m, μ_K, and/or $|\vec{F}_{\text{arm}}|$ (and of course g).

(c) Explicitly check both your answers by considering units, and also examining the limit of $\mu_K \to 0$. Here is a much subtler question, which you might find challenging: can you still make sense of your answer in the limit of very *large* μ_K? Explain!

Notes: Remember that we model kinetic friction as being simply proportional to the normal force, $|\vec{F}_{\text{arm}}| = \mu_K|\vec{N}|$. Of course in this problem you need to be careful, the normal force is NOT the weight of the sled system - draw a free body diagram!)

1.3 Pushing a book; compressing a spring (with friction)

Given: Marino – Fall 2011

A librarian pushes a heavy textbook (with mass 2 kg) across a rough table (which has a coefficient of friction μ_k=0.2). If the book leaves his hand with speed 4 m/s and slides 2 m across the table towards a spring (with constant $k = 6$ N/m), what is the maximum compression of the spring? (Reminder: As the book compresses the spring, it still continues to experience the frictional force from the table.)

1.4 Time to slide up and back on a frictionless hill

Given: Pollock – Spring 2011

A particle is projected with velocity v_0 straight up a slope which makes an angle α with the horizontal. Find the time required to return to the starting point. (assume frictionless motion) Explicitly check the units of your final formula, and discuss the special cases $\alpha \to 0$ and $\alpha \to 90°$.

1.5 Kicking a soccer ball over two defenders

Adapted from: Thornton & Marion, *Classical Dynamics of Particles and Systems*, 2004.

Given: Pollock – Spring 2011

A soccer player kicks the ball with a velocity v_0 at an angle 45° (to reach maximum range). She want to kick it in such a way that it barely passes on top of two opposite team players of height h.

(a) Show that if the separation between the two opponents is $d \leq \frac{v_0}{g}\sqrt{v_0^2 - ?gh}$, the soccer player succeeds. (What is the numerical value of the ? in this formula?)

(b) Use Mathematica to plot d as a function of v_0, setting $h = 2m$. *Hint: If you want to plot a function* $f[x]$ *vs* x *with* $x_0 < x < x_1$ *with Mathematica you can use the instruction* $\text{Plot}[f[x], \{x, x_0, x_1\}]$. *Sqrt[] is the syntax for square roots. Note that Mathematica can't plot a function with symbolic coefficients (so you have to plug in the values for both g and h) Important: when you type a command into Mathematica, you must hit Shift-return, rather than just return, to have it evaluate what you typed!!.*

(c) Give a physical explanation of any major features in your plot.

Given: Pollock – Spring 2012

A soccer player kicks the ball with a velocity v_0 at an angle 45° (to reach maximum range). She wants to kick it in such a way that it barely passes on top of two opposite team players of height h.

(a) Show that if the separation between the two opponents is $d \leq \frac{v_0}{g}\sqrt{v_0^2 - ?gh}$, the soccer player succeeds. (What is the numerical value of the ? in this formula?)

(b) Use a computational tool of your choice (Mathematica or Python) to plot d as a function of v_0, setting $h = 2.0\ m$. To do this, write the equation as a function rather than "hard-coding" the equation into the plot function.
Additional help on writing functions in Mathematica is available here: `http://youtu.be/1A4f91yMVhA`

(c) Give a physical explanation of any major features in your plot.

(d) What are some benefits of writing functions to plot over "hard-coding" plots?

1.6 Dropping hay from an airplane

Given: Pollock – Spring 2012

In the great blizzard of '89, a rancher had to drop hay bales from an airplane to feed her cattle. Suppose the plane flew horizontally at a steady 120 km/hr, and dropped the bales from 60 m above the flat range. (Neglect air resistance in the first two parts of this question)

(a) If she wanted the bales of hay to land 20 m behind the cattle (so as to not hit them!), where should she push the bales out of the plane? (Clearly define a coordinate system so we understand your answer)

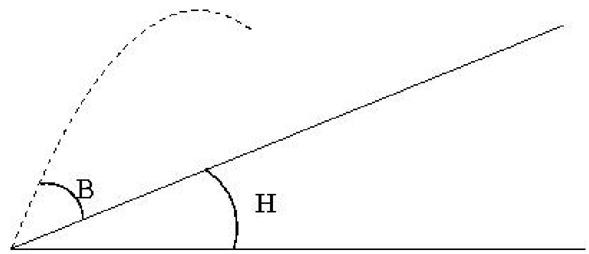
(b) What is the largest time error she could make while pushing the bales out of the plane, to ensure not hitting the cattle?

(c) If we did NOT neglect air resistance for the dropping hay, describe qualitatively (no detailed calculations, please!) what would change about your answers to both previous questions.

1.7 Projectile shot up a hill

Given: Marino – Fall 2011

A slingshot launches an angry bird[1] up in the air with a speed v_o at an angle of B (with respect to the ramp) up a hill of with an incline of H above the horizontal.



(a) Show that distance *along the hill* where the bird lands can be expressed as $\frac{2v_0^2 \sin(B)\cos(B+H)}{g\cos^2(H)}$. (Hint: use tilted coordinates where one axis is along the slope of the hill, and some angle addition identities might be helpful here.)

(b) For a given H, at what angle should the bird be launched to achieve the maximum distance?

(c) Show that the expression for the maximum distance is $d_{max} = \frac{v_o^2}{g(1+sin(H))}$

(d) For $H = 20°$, use *Mathematica* to make a plot of the expression you obtained in part a from B=0 to 90°. Attach the plot to this assignment, and verify that the maximum distance value occurs at the angle given by your answer to part b.

Given: Pollock – Spring 2011

An inclined plane is tilted at an angle α above the horizontal. A ball is launched (initial velocity v_0) into the air directly up the inclined plane, launched at an *additional* angle β above the plane's surface.

(a) Draw the situation here, and roughly sketch a predicted path of the ball for some initial conditions that you choose.

(b) (*Hint: before you start to work on this question, read the whole thing and then think carefully about what coordinate system would be easiest here.*) Find the ball's position as a a function of time (until it hits the plane). Show that the ball lands a distance $R = 2v_0^2 sin\beta cos(\beta + \alpha)/(gcos^2\alpha)$ from its launch point. Show that for given v_0 and α, the maximum possible range up the inclined plane (i.e. the maximum value of the distance between landing and launch point) is $R_{max} = v_0^2/[g(1 + sin\alpha)]$.

(c) Think of two specific combinations of α, β, and v_0 for which you can easily predict the outcome (either the ball's position as a function of time, or the range/maximum range) and use these combinations to check your calculations in the previous step (or to check that the final formulas we gave you are OK)

[1] Apparently the bird is so upset over some stolen eggs that it can't fly.

1.8 Parametrized particle trajectory (Cartesian)

Given: Marino – Fall 2011

Point P sits on the rim of wheel of radius ρ that is rolling on the ground in the positive x direction. Its position vector can be described by the sum of a vector v_1 for circular motion about the center of the wheel plus a vector v_2 that describes the horizontal motion of the center of the wheel along the line $y = \rho$ at constant velocity. In other words, its position at time t is given by

$$\vec{r} = \vec{v_1} + \vec{v_2} = (\rho \sin(\omega t)\hat{x} + \rho \cos(\omega t)\hat{y}) + (\rho\omega t\hat{x} + \rho\hat{y})$$

(a) Find an expression for the velocity of Point P.

(b) Are there any times when the velocity is 0? Where is Point P on the wheel at these times (on the top, on the front edge, etc)? Does this make sense?

Given: Pollock – Spring 2011

Trajectory of a particle. A particle moves in a two-dimensional orbit defined by

$$x(t) = \rho_0[2 - \sin(\omega t)] \tag{1}$$
$$y(t) = \rho_0[1 - \cos(\omega t)] \tag{2}$$

(a) Sketch the trajectory. Find the velocity and acceleration (as vectors, and also their magnitudes), and draw the corresponding velocity and acceleration vectors along various points of your trajectory. Discuss the results physically - can you relate your finding to what you know from previous courses? Finally: what would you have to change if you want the motion go the other way around?

(b) Prove (in general, not just for the above situation) that if velocity, $\vec{v}(t)$, of any particle has constant magnitude, then its acceleration is orthogonal to $\vec{v}(t)$. Is this result valid/relevant for the trajectory discussed in part a?
Hint There's a nice trick here - consider the time derivative of $|\vec{v}(t)|^2 = \vec{v} \cdot \vec{v}$.

Note: What you have proven in part b is quite general, and very useful. It explains why, e.g., $d\vec{\hat{r}}/dt$ *must point in the* $\vec{\hat{\phi}}$ *direction in polar coordinates. Do you see why?*

Given: Pollock – Spring 2012

Trajectory of a particle. A particle moves in a two-dimensional orbit defined by

$$x(t) = \rho_0[1 + \cos(\omega t)] \tag{3}$$
$$y(t) = \rho_0[2 + \sin(\omega t)] \tag{4}$$

(a) Sketch the trajectory. Find the velocity and acceleration (as vectors, and also their magnitudes), and draw the corresponding velocity and acceleration vectors along various points of your trajectory. Discuss the results physically - can you relate your finding to what you know from previous courses? Finally: what would you have to change if you want the motion to go the other way around?

(b) Plot the trajectory using your favorite computational tool. For this plot, set $\rho_0 = 1$ and $\omega = 2\pi$. This is a called a "parametric plot".

(c) Prove (in general, not just for the above situation) that if velocity, $\vec{v}(t)$, of any particle has constant magnitude, then its acceleration is orthogonal to $\vec{v}(t)$. Is this result valid/relevant for the trajectory discussed in part a?
Hint There's a nice trick here - consider the time derivative of $|\vec{v}(t)|^2 = \vec{v} \cdot \vec{v}$.

Note: What you have proven in part c is quite general, and very useful. It explains why, e.g., $d\vec{\hat{r}}/dt$ *must point in the* $\vec{\hat{\phi}}$ *direction in polar coordinates. Do you see why?*

1.9 Parametrized particle trajectories (Plane-polar)

Given: Marino – Fall 2011

A honeybee exits its hive and flies in spiral path. Its position $\mathbf{r}$ at time t is given in polar coordinates by

$$r = be^{kt} \qquad\qquad \phi = ct$$

where b, c, and k are constants.

(a) Use the expressions that we found for $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ in polar coordinates to find the velocity and acceleration vectors for the bee. Are the velocity and acceleration vectors orthogonal to each other?

(b) Show that the angle between the velocity and acceleration vector remains constant as the bee moves away.

1.10 Kinematics in spherical polar & cylindrical coordinates

Given: Pollock – Spring 2011

In this problem we want to generalize the analysis that you did in class for the motion of a particle in polar coordinates to spherical coordinates. The three unit vectors: $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ which describe spherical coordinates can be written as:

$$\hat{r} = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k}, \quad (5)$$
$$\hat{\theta} = \cos\theta\cos\phi\hat{i} + \cos\theta\sin\phi\hat{j} - \sin\theta\hat{k}, \quad (6)$$
$$\hat{\phi} = -\sin\phi\hat{i} + \cos\phi\hat{j}. \quad (7)$$

(a) Let's investigate that the definitions given in Eqs.(5-7) make sense. First, define in your own words what orthonormal vectors are, and then check to see if these vectors $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are orthonormal. Then, convince yourself (and the grader) that e.g. at least the x-component of $\hat{r}$ is correct, using a simple geometric picture. (Taylor's Fig 4.16, or Boas Fig 4.5 should help)

(b) If the particle is constrained to move with $\phi = 0$, state in simple words what this means in terms of the particle motion. Sketch the three spherical unit vectors at some point $\phi = 0$ and $r = R$ for some particular (nonzero) angle θ of your choice.

(c) Show that the velocity of any particle in spherical coordinates is given by:

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\hat{\phi} \quad (8)$$

(d) Use the previous part to compute the angular momentum $\vec{L} = \vec{r} \times \vec{p}$ in terms of $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$, and then show that $L_z = mr^2\dot{\phi}\sin^2\theta$.

Hint! Just think about these three unit vectors - can you very quickly and intuitively argue what the cross product of $\hat{r}$ is with each of the spherical unit vectors without doing any calculations at all? Just be careful of signs.

Extra credit (up to 4 bonus points, but won't count off if you don't do it) If a particle is restricted to move on the surface of a sphere, $r(t) = R$, and if angular momentum (in particular L_z) is conserved (meaning, L_z doesn't change with time) *and assuming the particle starts its motion somewhere on the z axis*, what can you say about $\dot{\phi}$ at later times?

Given: Pollock – Spring 2012

Let's generalize the analysis you did in class (for the motion of a particle in polar coordinates) to *spherical* coordinates. The three unit vectors: $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ which describe spherical coordinates can be written as:

$$\hat{r} = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k}, \quad (9)$$
$$\hat{\theta} = \cos\theta\cos\phi\hat{i} + \cos\theta\sin\phi\hat{j} - \sin\theta\hat{k}, \quad (10)$$
$$\hat{\phi} = -\sin\phi\hat{i} + \cos\phi\hat{j}. \quad (11)$$

(a) First, investigate that the definitions given in Eqs.(9-11) make sense. Define in your own words what orthonormal vectors are, and then check to see if these vectors $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are indeed orthonormal. Then, convince yourself (and the grader) that e.g. at least the x-component of $\hat{r}$ is correct, using a simple geometric picture. (Taylor's Fig 4.16, or Boas Fig 4.5 should help)

(b) If we constrain the particle to move with $\phi = 0$, state in simple words what this means in terms of the particle motion. Sketch the three spherical unit vectors at some point $\phi = 0$ and $r = R$ for some particular (nonzero) angle θ of your choice.

(c) Show that the velocity of any particle in spherical coordinates is given by:

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\hat{\phi} \quad (12)$$

(d) Let's switch to *cylindrical* polar coordinates. Find expressions for the cylindrical unit vectors ($\hat{\rho}, \hat{\phi}$, and $\hat{z}$) in terms of Cartesian coordinates ($\hat{\imath}, \hat{\jmath}, \hat{k}$) Then, take a derivative with respect to time to find $d\hat{\rho}/dt$ in cylindrical coordinates.

1.11 Motion of a ball between two cylinders

Given: Pollock – Spring 2011

Imagine two concentric cylinders, centered on the vertical z axis, with radii $R \pm \epsilon$, where ϵ is very small. A small *frictionless* object of radius ϵ is inserted between the two cylinders, so that it can be considered a point mass that can move freely at a fixed distance from the vertical axis. At time $t = 0$ the puck is released at height h with a purely angular initial velocity ω_0.

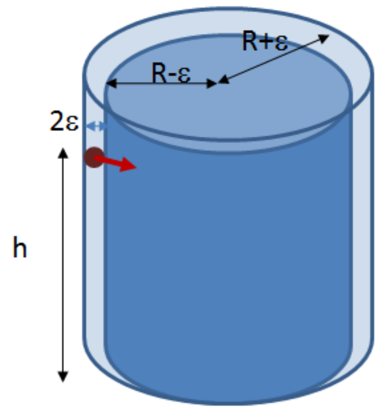


Figure 1:

(a) Write down Newton's second law first in terms of cartesian coordinates and then in terms of cylindrical polar coordinates. Which form would be easiest to use to solve for the motion of the ball described above? Why? (*If you need a reminder about what cylindrical polar coordinates are, problem 1.47 on p. 40 of Taylor might help. If you need a reminder about Newton's law in other coordinate systems, you might look back to Taylor's discussion that led up to Eq 1.48*)

(b) Describe and/or sketch a prediction of what the motion of the ball will be for $t > 0$. You can assume that h and ω are positive, and that the ball is subject to the force of gravity (on earth) which points in the $-\hat{z}$ direction.

(c) Use the equations you wrote in part (a) to solve for the motion of the ball. Does it match what you predicted? If not, explain how to reconcile any differences.