Problem List

2.1 Terminal Speed of a Softball

Adapted from: Ambrose & Wittmann, *Intermediate Mechanics Tutorial*
Available at: `http://perlnet.umaine.edu/imt/VDF/HWVDF.pdf`

Given: Marino – Fall 2011

Consider a softball with diameter 10 cm and mass 200 g. Use the fact that the force of air resistance on a spherical object of diameter D can be approximated using coefficients $c_1 = (1.55{\times}10^{-4})D$ and $c_2 = 0.22D^2$ (all numerical values are in SI units).

(a) (1 point) Calculate the terminal speed of the softball taking into account both the linear and quadratic terms. Show all work.

(b) (0.5 points) The ratio of the quadratic and the linear terms of the force of air resistance can therefore be expressed as:

$$\frac{|c_2 v^2|}{|c_1 v|} = \frac{0.22D^2v^2}{(1.55{\times}10^{-4})Dv} = (1.4 \times 10^3)|v|D$$

Using the above ratio, for what range of speeds will (i) the linear term of air resistance dominate over the quadratic term? (ii) the quadratic term dominate over the linear term? If it were desired to approximate the effect of air resistance on a falling softball with either the linear term or the quadratic term (not both), which term would you keep? Explain your reasoning.

(c) (1 point) We have already seen in class an exact solution for velocity in the case where there is only linear or quadratic drag. So, based on your answer to part b, what is the approximate speed of a softball 2 seconds after it was released from rest?

(d) (1.5 points) Let's investigate how good this approximation is. If we keep both the linear and quadratic terms, we have a non-linear differential equation. In general we do not know how to solve this kind of differential equation, but we can solve it numerically. Use the `NDSolve` function in *Mathematica* to find a the numerical solution for the velocity at t=2 sec. Compare this to the approximate one that you found in part c. Was it a good approximation?

2.2 Stopping a Bugatti

Given: Marino – Fall 2011

The Bugatti Veyron Super Sport, the world's fastest and most expensive production car, has a maximum speed of 268 miles per hour[1]. The engine provides maximum forward force of 12,350 N on the 1900 kg car. Quadratic air resistance dominates, with a drag coefficient of .86 kg/m. The car starts from rest at one end of a 5 km long test track. Can the Bugatti reach a speed of 265 mph (very close to the maximum speed) before it runs out of road? (Watch your units on this one!)

[1] James May noted on the BBC show "Top Gear" that at this top speed "the tires will only last for about 15 minutes, but it's okay because the fuel runs out in 12 minutes."

2.3 PhET Simulation – Air Resistance

Given: Pollock – Spring 2011

In your textbook (section 2.4), Taylor solves for the case of a baseball being dropped from a high tower using quadratic air resistance, $F_D = -cv^2\hat{v}$. Lets look at the case of a ball being shot *up* at an initial speed v_0.

(a) Draw a free body diagram for a ball moving vertically upwards, subject to quadratic air drag. Write down a differential equation for this situation, and solve this differential equation for v(t). Make a rough sketch of v(t) vs. t, and briefly discuss any key features.

(b) Using your result from the previous part, find an expression for the time it takes to reach the top of the trajectory. (It will look simpler if you write it in terms of terminal velocity, which satisfies $v_t^2 = mg/c$)

(c) Now download the PhET simulation http://phet.colorado.edu/en/simulation/projectile-motion. On the top right, switch the object to "baseball." This sim uses quadratic drag: $F_D = -\frac{1}{2}c_0 A\rho_{air}v^2\hat{v}$, where c_0 is called the drag-coefficient, A is the cross-sectional area of the object being shot, and ρ_{air} is the density of air = 1.3 kg /m^3. (The sim shows you the value of c_0 and diameter it has picked for a baseball, on the right side of the simulation) Use your formula in part b for "time to top" with these numbers to deduce what numerical initial velocity v_0 you need to get the ball to reach the top of its trajectory at precisely t=3 sec. Now test it, you can input v_0 into the sim and fire the cannon. Aim the cannon at 90^o (or 88-89^o if it is easier to see the trajectory) and switch on air resistance. The little + and - glasses let you zoom in or out. Does the ball reach the top at t=3 sec? (It should!)

(d) When you fired the ball on the PhET sim, did it take longer for the ball to go from the ground to the top of the trajectory or from the top of the trajectory to the ground? Explain why this is the case.

(e) Now let's look at another interesting feature of shooting an object up in the air. Start increasing the value of v_0 in the sim. Double it from what you had before, then increase it by 10, and then by 100. What is happening to the time to reach the top? Use your formal mathematical results from above to explain what is happening!

(f) Play with the PhET sim a little more and explore anything you are interested in. Write down one question that you have about what you notice while playing with the sim.

Given: Marino – Fall 2011

This problem is of interest for on ongoing study on the use and effectiveness of PhET computer simulations. If you consent to allow the researchers to have access to your homework solution to help improve the simulations, please sign the attached consent form and turn it in with your homework. Your solutions will be anonymous to the researchers and your decision on participation in this study will not impact your grade in this class.

In your textbook (section 2.4), Taylor solves for the case of a baseball being dropped from a high tower subject to quadratic air resistance, $F_D = -cv^2\hat{v}$. Let's now instead look at the case of a ball being shot *up* at an initial speed v_0.

(a) (0.5 points) Draw a free body diagram for a ball moving vertically upwards, subject to quadratic air drag. Write down a differential equation for this situation and solve this differential equation for $v(t)$. Make a rough sketch of $v(t)$ vs. t, and briefly discuss any key features.

(b) (0.5 points) Using your result from part a, find an expression for the time it takes to reach the top of the trajectory. (It will look simpler if you write it in terms of terminal velocity, which satisfies $v_t^2 = mg/c$.)

(c) (0.5 points) Now download the PhET simulation at: `http://phet.colorado.edu/en/simulation/projectile-motion`. On the top right, switch the object to baseball. This sim uses quadratic drag: $F_D = -\frac{1}{2}c_0 A\rho_{air}v^2\hat{v}$, where c_0 is the drag coefficient, A is the cross-sectional area of the object being shot, and ρ_{air} is the denstiy of air $= 1.3 \quad kg/m^3$. (The sim shows you the value of c_0.) By experimenting with the sim, what initial velocity makes the ball reach the top at approximately 3 sec?

(d) (0.5 points) Now use your formula in part b for "time to top" to deduce what numerical initial velocity, v_0, you need to get the ball to reach the top of its trajectory at precisely $t = 3$ sec. How does your calculated value compare to your "experimental" value?

(e) (0.5 points) When playing with the PhET sim, does it seem to take longer for a ball to go from the ground to the top of a trajectory or from the top of the trajectory to the ground? Explain why this is the case.

(f) (0.5 points) Again, playing with the sim, write down the initial velocity that makes the ball reach the top of its trajectory at 4 sec, then 5 sec, then 6 sec, and so on. What do you notice happening? Make a plot of t_{top} vs. v_0 and explain in words how this relates to what you see on the sim.

Extra Credit: Play with the PhET sim a little more and explore anything you are interested in. Write down one question that you have about something you notice when playing with the sim.

2.4 Check your friend's solution – Air Resistance

Given: Marino – Fall 2011

An object is released from rest far above the ground. The object experiences a force due to the air (drag) that is proportional to the object's speed $\vec{F} = -b\vec{v}$. In this problem let's define the y-axis to point *down*.

(a) (0.5 points) Your friend, who is also taking Classical Mechanics, has calculated $v_y(t)$. He/she knows that the terminal velocity of the object is 40 m/s, and has found a solution, $v_y(t) = 40(1 - e^{-t/2})$, but isn't confident that it is right. In physics we often want to check our answers by picking situations where you independently know what the answer should be without using your formula, and then checking that your proposed formula gives these answers. In this case, you know $v_y(t)$ for $t = 0$ and $t \to \infty$ from your basic understanding of physics. Check your friend's solution at these two points. How is his/her equation looking so far?

(b) (0.5 points) Given that the drag force on this object is $\vec{F} = -b\vec{v}$, the drag force is not very significant for small times. What is the formula for $v_y(t)$ in the case of no drag?

(c) (1 point) Given the above, check your friend's solution by finding an approximate for that is valid for small t. If you find that it is incorrect, can you suggest to your friend what term(s) in the equation he/she might want to double check?

2.5 John Taylor's range approximation

Given: Pollock – Spring 2011

(a) In Taylor's book, he simplifies Eqn 2.42 for the range of a projectile by assuming that $R^2 \sim R_{vac}^2$. Show that if instead you use the quadratic equation to solve for R directly in Eqn 2.42, and make the appropriate Taylor series expansion, that you obtain the expression given in Equation 2.44.

(b) If you use your result from the previous part to compute R when $v_{y0} = v_{ter}$, what do you get? Does this make sense? (Briefly, discuss)

Given: Marino – Fall 2011

In Taylor's book, he simplifies Eqn 2.42 for the range of a projectile by assuming that $R^2 \simeq R_{vac}^2$. Show that if you use the quadratic equation to solve for R in Eqn 2.42, and make the appropriate Taylor series expansion that you obtain the expression given in Equation 2.44.

2.6 Taylor expansions of the Gamma factor

Given: Pollock – Spring 2011

Recall from special relativity that for a particle moving at a relativistic speed, v, the energy $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{(1-v^2/c^2)}}$. Find the first two terms of the Taylor series expansion of the energy, in the non-relativistic limit $v << c$. What is the second term? Does this make sense?

Given: Marino – Fall 2011

Recall from special relativity that for a particle moving at a relativistic speed, v, the energy E=γmc^2, where $\gamma = \frac{1}{\sqrt{(1-\frac{v^2}{c^2})}}$.

(a) (0.75 point) Assuming that $v/c << 1$, find the first two terms of the series expansion of the energy by doing a Taylor Series expansion of $\frac{1}{\sqrt{1-x}}$ around $x = 0$.

(b) (0.5 points) What is the second term? Does this make sense for a particle with v<<c?

(c) (0.75 point) Now instead do a Taylor Series expansion of $\frac{1}{\sqrt{x}}$ around $x = 1$. How does this compare to your answer from part a?

(d) (0.5 points) Could you also have taken the first 2 terms of the Taylor Series expansion of $\frac{1}{\sqrt{x}}$ around $x = 0$ to obtain this expression for the case where $v/c << 1$? Why or why not?

Given: Pollock – Spring 2012

Recall from special relativity that for a particle moving at a relativistic speed, v, the energy $E = \gamma mc^2$, where $\gamma = \frac{c}{\sqrt{c^2 - v^2}}$.

(a) In the non-relativistic limit, find the first two nonzero terms in the series expansion of the energy by using the Taylor expansion of $\frac{1}{\sqrt{1-x}}$. What is the non-relativistic limit? What is x in this case? About what point are you expanding? What is the second term? When Einstein did what you just did (100 years ago), I think he must have smiled a big smile at that second term - why?

(b) How can an expansion of $\frac{1}{\sqrt{x}}$ be made to agree with your answers to part a? Comment. *Hint: you won't be expanding around $x = 0$ this time! About what point should you expand?*

(c) I claim that an expansion of $\frac{1}{\sqrt{x}}$ about $x = 0$ makes little sense. Why? Interpret this limit physically.

2.7 Playing ball with Todd Helton (Colorado Rockies)

Given: Marino – Fall 2011

Consider a baseball with diameter 8 cm and mass 150 g. Use the fact quadratic drag dominates for a baseball, has a magnitude of $F(v) = 0.157\rho D^2 v^2$, where D is the diameter of the ball and ρ is the air density (all numerical values are in SI units). For this problem, we will ignore any effects due to spin on the ball. As Taylor points out on page 62, the range of a projectile subjected to quadratic drag cannot be solved analytically. But it can be evaluated numerically, using a program like *Mathematica*.

(a) (1 point) Todd Helton recently hit a home run during a Rockies baseball game in Denver at Coors Field, where the density of air is 1.07 kg/m^3. Assuming that the ball had an initial speed of 112 mph (50 m/s) and left the bat at an angle of 35° with respect to the horizontal, how far did the ball go? (You may assume that it starts and lands at the same height.) Hint: Use NDSolve in Mathematica to solve a system of equations. You will have 2 equations to solve simultaneously, the x and y force equations, and will need to specify the initial x and y positions and the initial x and y velocities. You can then plot the solutions for y(t) and x(t) and see what the x distance is when y=0.

(b) (0.5 points) Had Mr. Helton hit the same ball at Dodger Stadium in Los Angeles, where the density of air is 1.3 kg/m^3, how far would it have gone? In its first decade of operation, players at Coors Field hit an above-average number of home runs. Explain why this makes sense.

2.8 Making sense of the Lotka-Volterra equations

Given: Pollock – Fall 2011

The pair of coupled ODEs

$$\frac{dx(t)}{dt} = Ax(t) - Bx(t)y(t) \tag{1}$$

$$\frac{dy(t)}{dt} = -Cy(t) + Dx(t)y(t) \tag{2}$$

is referred to as the Lotka-Volterra equation and is supposed to represent the evolution of the populations of a predator and its prey as a function of time, A, B, C, D are positive constants.

(a) Which of the variables, $x(t)$ or $y(t)$, represents the predator? Which represents the prey? What reasons do you have for your choice?

(b) What do the parameters A, B, C, and D represent? Why do you say so?

(c) Are there values for the populations that would lead to a stable solution for both groups?

You might be wondering why we are asking you to think about an ecology question in Phys 2210. This question is really just about developing intuitions about ODEs. Physics is full of ODEs! You will need such intuitions in almost every branch of physics.

2.9 Romeo and Juliet

Adapted from: Strogatz, *Nonlinear Dynamics and Chaos*, 2001.

Given: Pollock – Spring 2012

Romeo and Juliet are in love, but in our version of this story, the characters are a little different than tradition holds. Let R(t) = Romeo's feelings for Juliet at time t. Large positive values means deep love, large negative values means strong hatred. Similarly, Juliet's feelings for Romeo will be characterized by J(t). Now let's consider a "model for the lovers" governed by the general linear coupled differential equations,

$$\frac{dR(t)}{dt} = aR(t) + bJ(t) \tag{3}$$

$$\frac{dJ(t)}{dt} = cR(t) + dJ(t) \tag{4}$$

If a>0 and b=0, I might describe Romeo as "in love with his love". His emotional state has nothing to do with Juliet's feelings at all; he just falls more in love the more in love he is. . . a very narcissistic individual!

(a) Let's assume none of the parameters are zero. Consider *all four* possible permutations of signs of a and b, and for each, describe the emotional character of Romeo. (For instance, one of the combinations might be called a "emotionally distant jerk", which one? But maybe you have a better or totally different description/interpretation of this same combination!)

(b) Suppose that Juliet has the exact same *emotional character* as Romeo. (I'm not saying J(t)=R(t), but they respond to each other in analogous ways) What would this say about parameters c and d in terms of a and b? In this case are there values for the parameters a and b that would lead to equilibrium (i.e. all time derivatives vanish)? Is there any equilibrium solution for J(t) and R(t) besides "complete mutual indifference", i.e. R=J=0?

You might be wondering why we are asking you to think about such a whimsical question like this in Phys 2210. This question is really just about developing intuitions about ODEs. Physics is full of ODEs! You will need such intuitions in almost every branch of physics. We got the question from Strogatz' "Nonlinear Dynamics and Chaos" text.

2.10 Applying the general solution of first-order differential equations

Given: Pollock – Spring 2011

Boas has a very useful formula for solving *any* linear first-order differential equation: Eq. 3.9 page 401. We casually sketched the derivation of this formula in class, but you can check now how useful it is. For example, consider an electric circuit containing a resistor in series with an inductor L and a source of emf $V(t) = V_0 e^{i\omega t}$. You may or may not have treated "RL" circuits in Phys 1120, but the resulting equation for the current is a pretty straightforward ODE:

$$L\frac{dI}{dt} + RI = V(t) = V_0 e^{i\omega t} \tag{5}$$

Find the current $I(t)$ as a function of time, assuming $I(0) = 0$. *(Note: Be careful - don't mix up the symbol I in I(t) (current) with the symbol I that Boas defines! By the way, in practice, because we started with a complex voltage, you would take the real part of the answer to figure out physical currents, but no need to bother with that for now)*

2.11 Determining the equation of motion – Gravity and Air Drag

Adapted from: Ambrose & Wittmann, *Intermediate Mechanics Tutorial*
Available at: `http://perlnet.umaine.edu/imt/VDF/HWVDF.pdf`

Given: Pollock – Spring 2011

Consider a ball that moves vertically under the influences of both gravity and air resistance. For the purposes of this problem, take vertically upward as the positive direction. For each equation of motion below, determine whether that equation applies to (i) a situation in which the ball moves upward, (ii) a situation in which the ball moves downward, (iii) either of these, or (iv) neither of these. Explain your reasoning for each case.

a) $m(dv_y/dt) = -mg + c_1 v_y$,

b) $m(dv_y/dt) = -mg - c_1 v_y$

c) $m(dv_y/dt) = -mg + c_2 v_y^2$,

d) $m(dv_y/dt) = -mg - c_2 v_y^2$

2.12 Time of flight with linear drag

Given: Pollock – Spring 2011

Consider a grain of pollen which is ejected from the anther of a plant vertically up with initial speed v_0. Assume the air resistance on the grain of pollen can be modeled to a very good approximation by a linear drag. Measuring the position of the grain, y, upward from the point of release:

(a) Find the time for the grain to reach its highest point and its position y_{max} at that time. (Note that velocity $v_y(t)$ and position $y(t)$ have been derived in the lecture notes, and the text. Of course, if you want to rederive them yourself, that's probably a very good idea, but not required for credit)

(b) Show that as the drag coefficient approaches zero your answer in part b reduces to the well know freshman physics result $y_{max} = \frac{v_o^2}{2g}$. **Hint:** If the drag coefficient is very small the terminal velocity is very big so v_0/v_{ter} is very small. Use the expansion $\ln(1+\epsilon) \sim \epsilon - \epsilon^2/2$ for $\epsilon \ll 1$.

Given: Pollock – Spring 2012

You're hanging out by a campfire after acing the first Phys 2210 exam. You notice a tiny piece of fly ash that is ejected from the fire vertically upward with initial speed v_0. Since physicists tend to love to think about casual observations like this more formally, you decide to try to predict if the ash will reach tree-level. Let's measure the position of the fly ash, y from the point of release, once again taking vertically upward as the positive direction.

(a) Clearly the physics of "flying ash" could be very complicated! But, let's start with the *simplest possible* model and go from there. If you consider ash to be a point particle of mass m acting only under gravity, what is the equation of motion? What is the maximum height (in terms of v_0 and g)?

(b) The next simplest model might be to also include simple air drag. Taylor discusses the choice between "linear" and "quadratic" drag. Knowing that ash particles are extremely tiny, and not particularly fast moving to start with, if you had to choose just ONE of these, which should it be? Briefly (qualitatively is all we want) justify your answer.

(c) Now, using this model, find the time for the fly ash to reach its highest point and its position y_{max} at that time. *(Note that velocity $v_y(t)$ and position $y(t)$ are derived in the lecture notes, and the text. Of course, it's always a good idea to rederive them yourself, but not required for credit. However, you should be careful, because in the text, Taylor defines +y to be the other way, so watch your minus signs! I suggest using notation from question 1 to define symbols. No numbers here, do it all symbolically.)*

(d) Show that as the drag coefficient approaches zero your answer in part c reduces to the well known freshman physics result you got in part a. **Hint:** If the drag coefficient is small the terminal velocity is big, so v_0/v_{ter} is very small. Use the Taylor expansion $\ln(1+\epsilon) \approx \epsilon - \epsilon^2/2 + \dots$ for $\epsilon \ll 1$.

2.13 Particle sliding down a plane with air resistance

Given: Pollock – Spring 2011

A particle of mass m slides down an inclined plane under the influence of gravity. If the motion is resisted by a force $f = kmv^2$, show that the time required to move a distance d after starting from rest is

$$t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{A \sin\theta}} \tag{6}$$

where θ is the angle of inclination of the plane. What is A in this formula in terms of k and g?(12 pts).

Hint: To solve this problem you will want to know how the position of the particle depends on time. Taylor's section on "vertical motion with quadratic drag" (p. 60-61) should be extremely helpful on this problem. The math/formulas are not identical, but closely related.

Given: Pollock – Spring 2012

A grain of pollen (mass m) slides down a smooth, flat, tilted solar panel (tilted at θ from horizontal) under the influence of gravity.

(a) If the motion is resisted by a linear drag force $f = kmv$ find an expression for how far down the plane the grain slides in time t, assuming it is released from rest. **Hint:** Draw a free body diagram to start!

(b) Show that as the drag coefficient becomes very small, your answer in part a reduces to a simple freshman physics result (which is what?) Comment briefly on the similarities and differences between this problem and question 2. **Hint:** Here, I claim the useful Taylor expansion you'll want will be $e^x \approx 1 + x + x^2/2 + \ldots$, for $x \ll 1$ And yes, you'll really need to go all the way out to that $x^2/2$ term!

For you to think about. You do not have to write this up for credit, but we'll revisit this idea again this term, it's very important! The power x in e^x must be unitless. "Drag coefficient" has units. So it can not be exactly correct to say the "drag coefficient becomes small", without answering "small compared to what"? What exactly is it that is really the "small thing, x" in this case?

2.14 Finding the range numerically (Air Resistance)

Given: Pollock – Spring 2011

Mathematica problem: Consider a ball thrown at an angle θ above the horizontal ground with an initial speed v_0 in a medium with linear drag. For numerical solutions, Mathematica doesn't deal well with unknown symbols (!), so let's consider a particular case where $v_0 = v_{ter}$ and $v_{ter}^2/g = 1$. We know that in vacuum the maximum range is at $\theta = \pi/4$. Let's try to estimate the maximum range (and angle) when we include air resistance:

(a) Plot Eq.(2.37) in Taylor for different values of θ and try by inspection to figure out the angle at which the range is maximum. (With the assumptions above, most unit-full quantities become "one", but do be careful of the fact that e.g. $v_{0,x}$ is $v_0 \cos(\theta)$, not v_0), etc.

(b) UseFindRoot to find the range when $\theta = \pi/4$. (Syntax: FindRoot[f[x]==0,{x,0.5}] finds a numerical solution for f[x]=0, starting to look near the point x=0.5)

(c) Repeat for different values of θ (homing in on a small range near the angle you estimated in part a). Continue until you know the maximum range and the corresponding angle to two significant figures. Connect to point a) and compare with the ideal vacuum values. Does your answer seem reasonable? Briefly, explain.

Extra credit Check what happens in the limit $v_0 \gg v_{ter}$ (say $v_0 = 100v_{ter}$) and $v_0 \ll v_{ter}$ (say $v_0 = 0.01v_{ter}$). Discuss if your results make sense. (+4 points)

Note: When you turn in your homework, judiciously use "print selection" on a minimalist subset of your MMA notebook so we can see your final results, and your discussion, without our having to read through multiple pages of preliminary plots and calculations.

Given: Pollock – Spring 2012

Consider a ball thrown at an angle θ above the horizontal ground with an initial speed v_0 in a medium with linear drag. For numerical solutions, computational tools don't deal well with unknown symbols (!), so let's consider a particular case where $v_0 = v_{ter}$ and $v_{ter}^2/g = 1$. We know that in vacuum the maximum range is at $\theta = \pi/4$. Let's try to estimate the maximum range (and angle) when we include air resistance:

(a) Plot Eq.(2.37) in Taylor for different values of θ and try by inspection to figure out the angle at which the range is maximum. (With the assumptions above, most unit-full quantities become "one", but do be careful of the fact that e.g. $v_{0,x}$ is $v_0 \cos(\theta)$, not v_0), etc. Briefly comment on real-world implications!

(b) Use a root finder to find the range when $\theta = \pi/4$. Any root finder will require you to make a guess for the root. This is what is called the "neighborhood of the root" or "bracketing the root". A screencast showing how to use Mathematica to find roots is available here: `http://youtu.be/673IQ6Z-6Yc`

(c) Repeat for different values of θ (homing in on a small range near the angle you estimated in part a). Continue until you know the maximum range and the corresponding angle to two significant figures. Connect to point a) and compare with the ideal vacuum values. Briefly, discuss.

Extra credit Check what happens in the limit $v_0 \gg v_{ter}$ (say $v_0 = 100v_{ter}$) and $v_0 \ll v_{ter}$ (say $v_0 = 0.01v_{ter}$). Discuss if your results make sense. (+4 points)

Note: Judiciously use "print selection" on a minimalist subset of your MMA notebook so we can see your results, and discussion, without having to wade through pages of preliminary plots and calculations.

2.15 A car braking – Friction & Air Resistance

Given: Pollock – Spring 2011

Consider a sports car which is braking hard. There are two significant resistive forces acting on it, a quadratic (cv^2) air drag, and a constant (μmg) frictional force. When you write Newton's law, if you are interested in finding v(x) (rather than v(t)), there is a nice trick, known as the "v dv/dx rule", which uses the chain rule to rewrite $\dot{v} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$.

(a) Write down the equation of motion for $\dot{v} = f(v)$ and use the "v dv/dx" rule to solve the equation of motion directly for v(x), and show that the distance the car needs for a full stop is:

$$x_{max} = \frac{A^2}{2\mu g} \ln\left(\frac{A^2 + v_0^2}{A^2}\right) \tag{7}$$

here μ is the friction coefficient. What is the constant A in this case (in terms of given parameters in the differential equation?)

(b) The SSC Ultimate Aero TT, one of the world's fastest production cars (at a cool $600,000+), has a maximum speed of 412 Km per hour. The engine provides maximum forward force of 11260 N on the 1200 kg car. For this car, c =0.86 kg/m. On a race track, the car exits a turn with a speed of v_0 = 300 km/hr. As soon as the driver enters the straight track after the turn, she realizes there is another car blocking the track 2 km away. The driver slams on the brakes. Assuming a friction coefficient $\mu = 0.7$, use your above result to compute the distance required for the Aero TT to stop. Compare this distance to the corresponding distance in the absence of air drag (i.e. just kinetic road friction). (Briefly, discuss)

(c) If the blocking car was removed when the car had slowed to a velocity of 200 Km/h and then the driver starts accelerating again with constant maximum force, what is the velocity that the Aero TT reaches after it travels a distance of 2 Km from the point it started re-accelerating? (Briefly, discuss)

(d) Make a rough sketch (by hand, not with Mathematica!) of the car's position x(t) described by the "story" of parts b and c (i.e., starting from when the driver entered the straight track, and ending at the final position after part c.) Comment briefly on interesting features of your graph (e.g., signs of slope, signs of concavity, interesting points...)

Given: Pollock – Spring 2012

A car is braking hard. There are two significant resistive forces acting on it, a quadratic (cv^2) air drag, and a constant (μmg) frictional force. If you are interested in finding v(x) (rather than v(t)), there is a commonly used method known as the "v dv/dx rule", which uses the chain rule to rewrite $\dot{v} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$.

(a) Write down the equation of motion for $\dot{v} = f(v)$ and use the "v dv/dx" rule to solve the equation of motion directly for v(x), and show that the distance the car needs for a full stop is:

$$x_{max} = \frac{A^2}{2\mu g} \ln\left(\frac{A^2 + v_0^2}{A^2}\right) \tag{8}$$

here μ is the friction coefficient. What is the constant A in this case (in terms of given parameters in the differential equation?)

(b) My car has mass $m \approx 1600$ kg, and (I estimate) $c \approx 0.6$ kg/m. I'm driving down I-70 towards Moab, traveling 75 mph, when I see a stopped car on the highway 300 m front of me. I slam on the brakes! Assuming a friction coefficient of $\mu = 0.7$, find my stopping distance (do I crash, or am I safe?), and then compare it to what it would have been in the absence of any air drag (i.e. just the road friction alone). Briefly, comment. If I'd been driving a sports car, traveling at twice this speed (but with all other parameters the same) would air drag have made a more significant difference? Again, comment!

(c) Car engines are complicated, they are not simple "constant force" devices, but I estimate that at highway speeds my car might apply a fairly steady forward force against the road of about 1500 N. If the stopped car managed to pull off to the side of the highway when I had slowed to 30 mph and I immediately started re-accelerating with this constant maximum forward force, how much highway distance would I need to get my car back up to 75 mph? (Include the quadratic air resistance, of course!) *Hint: the v dv/dx rule may be useful again*

(d) Make a rough sketch (by hand, not with Mathematica!) of my car's velocity $v(t)$ described by the "story" of parts b and c. *Note: that's v(t), not v(x). Don't calculate it, just sketch what it should look like.* Comment briefly on interesting features of your graph (e.g., signs of slope, signs of concavity, interesting points...)

2.16 Golfing on the moon

Given: Pollock – Spring 2011

Find the regulation size (and mass, while you're at it) for a golfball.

(a) The drag coefficient c_D for a golfball is (very roughly) 0.3. (Note: the c in cv^2 is given by $\frac{1}{2}c_D\rho D^2$) Calculate the numerical value of the quadratic drag constant "c" for a golf ball traveling in air, and write down the equations of motion required to solve for x(t) and y(t) of a golfball with (just) quadratic drag.

(b) In class we did a MMA Tutorial using NDSolve. (The code/worksheet is still available on our course calendar for Tues Jan 25) Modify your code so that you have *quadratic* drag. For simplicity and concreteness, set the angle to 45 degrees, and pick an initial speed of 80 mi/hr (converted to SI metric, of course) Plot the trajectory for us. What is the ideal (drag-free) range in this case, and what does your code tell you the range is including quadratic drag? (Don't hunt for the optimum range like we did last week, just stick with a 45 degree angle) Also, calculate what fraction of the initial *speed* is lost from when you first hit the ball (at 45 degrees) to when it lands. (Where does the lost energy go?)

(c) When I first started thinking about this problem, I wondered whether the fractional reduction in speed was a constant, or whether it depended on the initial velocity. What do you think? (No formal calculation required, just make a qualitative physics argument.) *Check* yourself with your code, and comment.

(d) Astronaut Alan Shepard on Apollo 14 brought a golf club with him (!) Assuming he hit it at 45 degrees (and assuming he hit it at that same 80 mi/hr, a modest swing on earth, maybe optimistic on the moon, given his clumsy space suit), calculate how far the ball landed from him, and compare to what it would have been on earth. (Which effect is more significant in the end, the loss of air drag on the moon, or the difference in gravity?)

2.17 Playing soccer in La Paz

Given: Pollock – Spring 2012

Find the regulation size (and mass, while you're at it) for a soccer ball.

(a) The drag coefficient c_D for a soccer ball is about 0.25. (Note: the c in cv^2 is given by $\frac{1}{2}c_D\rho D^2$) Calculate the numerical value of the quadratic drag constant "c" for a soccer ball traveling in air at STP, and write down the equations of motion required to solve for x(t) and y(t) with (just) quadratic drag.

(b) On our course calendar, right next to this homework, you can find a sample Mathematica notebook that solves for trajectory with *linear air drag.* Modify that code so that you have *quadratic* drag. For simplicity and concreteness, set the kick angle to 45 degrees, and pick an initial speed of 90 mi/hr (converted to SI metric, of course) Plot the trajectory. What is the ideal (drag-free) range in this case, and what does your code tell you the range is with quadratic drag? (Don't hunt for the optimum range like last week, stick with a 45 degree angle) Also, calculate what fraction of the initial *speed* is lost from when you first hit the ball (at 45 degrees) to when it lands. (Where does the lost energy go?)

(c) A few years ago, there was big controversy when the Soccer Federation banned international soccer competitions in cities above 2500 m elevation (This was out of concern about players' health, and has since been eliminated.) But, what about the impact on the physics of the game? Use your code to estimate how much farther that same kick would take a soccer ball in La Paz, Bolivia (elevation, 3000 m) compared to a game at sea level (which your code was assuming.) Do you think this difference might be measurable/noticeable? *Note: The air density at 3000 m is about 70% what it is at sea level.*

2.18 Can a dropped penny kill a person?

Given: Pollock – Spring 2012

In Season 1 Episode 4 of Mythbusters (originally aired in 2003), Jamie and Adam considered the myth that a penny dropped from the Empire State building can kill a person walking on the sidewalk below. You can watch a clip from this episode here: `http://youtu.be/PHxvMLoKRWg`. Let's do our own investigation!

(a) Look up the size and mass of a penny. Using the drag coefficients given by Taylor pp 44-45, which means we are assuming a spherical penny ⌣̈, find the terminal speed of a dropped penny, taking into account both linear and quadratic terms together. Then, write down the differential equation for the motion of a falling penny (Keep both linear and quadratic terms. Clearly articulate your sign conventions)

(b) For what range of speeds will (i) the linear term of air resistance dominate over the quadratic term? (ii) the quadratic term dominate over the linear? Discuss the relative importance of linear and quadratic drag here - if you had to pick just ONE term (linear or quadratic), which would you use, and why?

(c) If we keep both linear and quadratic terms, we have a non-linear differential equation which we can only solve numerically. Use your favorite numerical differential equation solver to determine and plot velocity and position of the penny as a function of time, as it falls from the top of the Empire State building (381 m). Also find the time it hits the ground, and the speed with which it hits. Include your code and plots with your homework.
Mathematica users might find this screencast on using `NDSolve` (Mathematica's built-in numerical differential equations solver) helpful: `http://youtu.be/zKO6vOwOKdI`
Be Elegant: *You could use Mathematica's* `EventLocator` *method to stop the integration when the penny hits the ground. Helpful link for that:* `http://goo.gl/Mkz3i`

(d) Compare the result for "final velocity" from your numerical results (part c) with what you got in part a, and also what you get by assuming JUST the one dominant drag term you chose in part b. Comment.

(e) Now for the interesting part - could this penny kill someone? Use basic freshman physics and make crude (but quantitative) estimates to discuss whether you think the myth is busted or not! *(I can think of many physics approaches to evaluate what might injure you - think about it a bit! Just e.g., you might assume a head applies a constant force to stop the penny during impact. If so, we've got about 1mm of tissue to stop a penny before it drives into the skull. Or, how about this - "Wikipedia: hydrostatic shock" says a skull pressure of* $10^7 N/m^2$ *can seriously injure you. Could that be useful?)*

2.19 Solving ODEs with Euler-Cromer Numerical Integration

Given: Pollock – Spring 2012

Consider a familiar horizontal spring-mass system. Recall that the the solutions for both the velocity and position of the mass are oscillatory.

(a) Write down the second-order differential equation which describes the position of the mass. Then, as we did in the numerical integration tutorial in class, write this differential equation as two first order differential equations (one for dx/dt, one for dv/dt) *(That in-class Tutorial is also available for review/download on this week's course calendar)*

(b) Assume the oscillator starts from rest at $x = +1$ m, with $m = 0.1$ kg and $k = 10$ N/m. Recalling that the period of oscillations is given by $T = 2\pi\sqrt{m/k}$, let's pick the time step for our numerical integration to be 10% of T (so 1 period has 10 time steps). Using the computational tool of your choice, you will be writing a `for` loop to numerically integrate these equations for several periods. (For help writing `for` loops in Mathematica, review the numerical integration tutorial, use the documentation or check out this screencast: `http://youtu.be/UpkcScYeQTc`.)
IMPORTANT:
We'd like you to write TWO codes: (code 1, "Euler-Cromer" method) Compute the force at an instant, update the velocity (using that force), then update the position (using that *updated* velocity!) and then repeat. Alternatively, (code 2, "Simple Euler method") Compute the force at an instant, update the position first, then update the velocity (and repeat.) See the difference? (Compare these two methods by using each of them to integrate the spring mass system and plot the position of the mass as a function of time.) What do you notice?
One anecdote claims the Euler-Cromer method was discovered by a high school physics student. The story is given in this paper.

(c) Try decreasing your integration time step by an order of magnitude. What changes do you have to make to your code? What happens to your solutions? When numerically integrating, what would help you pick the time step? (Why not, say, choose a billionth of a period in this case, to get super accuracy?)

(d) Several good integration algorithms are built in to a lot computational tools (e.g., Runge-Kutta, Dorman-Prince, etc.). Use one of these built-in integration methods to integrate the equations of motion and compare the results to those in part (b) or (c). Mathematica users can use NDSolve, a function which attempts to select the best method given the equation of motion. For help using NDSolve, check out this screencast: `http://youtu.be/zKO6v0w0KdI`. Notice, you don't have to pick a time step!