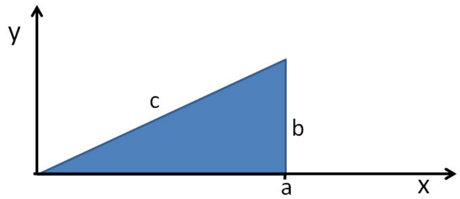
Problem List

3.1 Center of mass – Triangular plate

Given: Pollock – Spring 2011

A flat plate of mass M has a triangular shape with the dimensions shown in Fig.2. The plate has a uniform density (mass per unit area in this case).

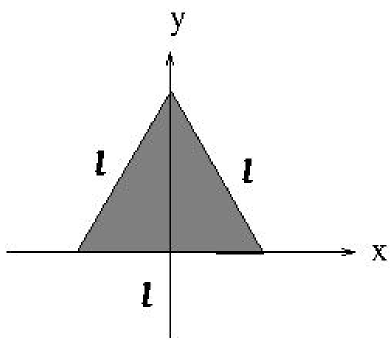


(a) Based on your physical and mathematical intuitions, without calculating, where do you predict the center of mass is, and why?

(b) Now mathematically determine its center of mass coordinates. Does your answer match your intuitions from part a? Briefly, explain (or reconcile!)

Given: Marino – Fall 2011

A thin metal plate of uniform density, is shaped like an equilateral triangle, as shown below. Find the location (x, y) of the center of mass of the plate. You must find the answer using calculus.



Given: Pollock – Spring 2012

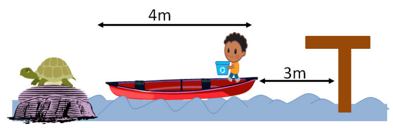
You have a flat plate (of uniform mass density, which is mass per unit *area* in this case) of total mass M. It is in the shape of a right triangle, with legs a and b (c is the hypotenuse).

(a) Draw your triangle, choosing a coordinate system origin and orientation to make it as easy as possible for you to find the center of mass. Based on your physical and mathematical intuitions, without calculating, where do you predict the center of mass is, and why?

(b) Now mathematically determine its center of mass coordinates. Does your answer match your intuitions from part a? Briefly, explain (or reconcile!)

3.2 "Walking" on water

Given: Pollock – Spring 2011, Spring 2012



Steve Jr. (mass m_s) is standing at one edge of a 4 m long canoe (mass M_c) (See Fig. 1). He observes a turtle on top of a rock standing close to the other edge of the canoe. Steve wants to catch the turtle and starts walking towards it. Ignoring water friction:

(a) Qualitatively describe the motion of the system (canoe+ Steve) as Steve walks forward. If initially the canoe is 3m away from the dock, where is Steve with respect to the dock when he reaches the other end of the canoe? (We want a formula in terms of m_s, M_c, and the given starting dimensions)

(b) If $m_s = 40$ Kg and $M_c = 30$ Kg and Steve can stretch his arm 1m away the canoe edge, can he catch the turtle? (Also, what happens in the limit of a very light canoe? A very massive canoe?)

Given: Marino – Fall 2011

You (mass=75 kg) are standing still at one end of a log that is floating at rest in a lake. Now you run to the other end of the log at a speed of 1 m/s and stop. If the log has a mass of 500 kg and a length of 5 m, how far does the center of the log move with respect to the shore? You may assume that there is no friction between the log and the water.

3.3 Lost in Space

Given: Marino – Fall 2011

An astronaut has drifted too far away from the space shuttle while attempting to repair the Hubble Space telescope. She realizes that the orbiter is moving away from her at 3 m/s. She and her space suit have a mass of 90 kg. On her back is a 10 kg jetpack which consists of an 8 kg holding tank filled with 2 kg of pressurized gas. She is able to use the gas to propel herself directly towards to the orbiter. The gas exits the tank at a uniform rate with a constant velocity of 100 m/s, relative to the tank (and her).

(a) (1 point) After the tank has been emptied, what is her velocity? Will she be able to catch up with the orbiter with that velocity?

(b) (1 point) With what velocity (in her frame of reference!) will she have to throw the empty tank away to reach the orbiter?

3.4 Rocket dynamics (with drag!)

Given: Marino – Fall 2011

Consider a rocket of mass (with an initial mass m_0) that travels vertically. The rocket ejects fuel at a constant velocity (v_{ex}) relative to the rocket's motion.

(a) If this rocket is designed such that the rate of fuel ejection is constant (i.e., $\dot{m} = -k$), show that the equation of motion for this rocket is,

$$m\frac{dv}{dt} = kv_{ex} - mg$$

Solve this differential equation for v(t) using separation of variables. Recall that m is not a constant, but that $m = m_0 - kt$. You will assume that g doesn't change appreciably.

(b) Describe what happens to the rocket if the value of kv_{ex} was smaller than the initial value of mg.

(c) Show for a rocket that starts from rest at $y = 0$, the resulting expression for y(t) is

$$y(t) = v_{ex}t - \frac{1}{2}gt^2 - \frac{mv_{ex}}{k}\ln\left(\frac{m_0}{m}\right).$$

(Hint: The following integral might be useful: $\int ln(x) = xln(x) - x + const$. If you are interested, you can integrate $ln(x)$ by parts to obtain this result.)

(d) Assume that the rocket burns fuel for 200 seconds. Use Mathematica to plot the velocity and position of the rocket as a function of time (up to 200 sec). Let the initial mass of the rocket be 2 $\times 10^6$ kg, the rate of mass ejection be 8333.33 kg/s and the exhaust speed be 3000 m/s. Obviously, the rocket starts from rest. How high is the rocket after 200 seconds? What does this tell you about your assumption about g in part (a)?

(e) Real rockets experience air drag. Due to their size and typical speeds, the air resistance is best modeled as quadratic. Consider your plots in part (d). Describe how your plots might change by adding bv^2 drag to the model.

(f) Using Mathematica numerically solve the equation of motion which includes air drag. Use the same parameter values as in part (d). Assume the drag coefficient b for the rocket is 0.80. Plot both the velocity and vertical position of the rocket as functions of time. Compare these plots to your predictions in part (e) and your analytic results in part (d).

3.5 Colliding atoms

Given: Pollock – Spring 2011

A collision between two bodies is defined to be elastic if the total kinetic energy before and after the collision is the same. Consider an elastic collision between two identical atoms one which is initially at rest $\vec{v}_2 = 0$ and the other is moving with velocity $\vec{v}_1 \neq 0$. Denoting $\vec{v}'_{1,2}$ the corresponding velocities after the collision

(a) Write down the vector equations representing the conservation of momentum and the scalar equation representing the conservation of kinetic energy in an elastic collision.

(b) Use this to prove that the angle between $\vec{v}'_1$ and $\vec{v}'_2$ is $\pi/2$. Think of a situation where this fact might prove to be useful or relevant.

3.6 Multi-stage rockets

Given: Pollock – Spring 2011

A rocket having two or more engines, stacked one on top of another and firing in succession is called a multi-stage rocket. Normally each stage is jettisoned after completing its firing. The reason rocketeers stage models is to increase the final sped (and thus, altitude) of the uppermost stage. This is accomplished by dropping unneeded mass throughout the burn so the top stage can be very light and coast a long way upward. Let us understand better the advantages of a multi-stage rocket. Imagine that the rocket carries 70% of its initial mass as fuel (i.e. the mass of all the fuel is $0.7m_0$)

(a) What is the rocket final speed accelerating from rest in free space, if it burns its fuel in a single stage? Express your answer in terms of v_{ex}

(b) Now suppose instead that it burns the fuel in two stages like this: In the first stage it burns a mass $0.35m_0$ of fuel. It then jettisons the (empty) first stage fuel tank. Let's assume this empty tank has a mass of $0.1m_0$. It then burns the remaining $0.35m_0$ of fuel. (So, we've burned the same total amount of fuel as part a, right? We simply jettisoned an empty fuel-stage in the middle) Find the final speed in this case, assuming the same value of v_{ex} as in part a. Compare and discuss briefly.

3.7 Hovering rockets

Given: Pollock – Spring 2011

Taylor works out the rocket equation in deep space. But at launch, obviously you cannot neglect gravity - the net external force, dP/dt, is no longer zero.

(a) Follow Taylor's derivation on page 86, and fix it up, getting to Eq 3.6 and find the "correction" term you need to add to include gravity. Assuming that v_{exh} is a fixed (constant) number, and assuming that you want the rocket to simply "hover" above the ground (rather than really launching), solve the ODE you get to find rocket mass as a function of time. (Does dm/dt turn out to be a constant? Explain physically why your answer to that question makes sense)

(b) If your payload (the mass that is left over after all the fuel is gone) is roughly $e^{-2} = .135$ of the initial mass, how long can you hover? Given the (very optimistic!) value of $v_{exh} = 2000 m/s$, comment on why we don't all commute around with jetpacks.

3.8 Rocket with linear drag

Given: Pollock – Spring 2011

So far we have considered the ideal case of a rocket without drag. In real life, however, drag can be an important limitation and must be considered. Imagine the situation of a linear drag $\vec{f} = -b\vec{v}$ acting on the rocket body only (with no other external forces, so we're back to the "gravity free" case of deep space)

(a) Once again, the net external force, dP/dt, is not zero. Follow Taylor's derivation on page 86, and fix it up, getting to Eq 3.6 and find the "correction" term you need to add, caused by drag.

(b) Now solve your ODE, to show that if the rocket starts from rest and ejects a mass at constant rate $\dot{m} = -k$ (with k a given constant), the its speed is given by $v = \frac{k}{b} v_{ex} \left[1 - \left(\frac{m}{m_0} \right)^A \right]$

What is A in terms of k, b, v_{ex} and m_0.

HINT: since dm/dt=-k, you can eliminate any stray "dt" terms that appear in your ODE.

(c) What is the corresponding speed if we ignore drag? Show that the eq'n above reproduces the speed for the drag free case if $b \to 0$ (see hint below). Calculate the first non-vanishing correction introduced by a finite drag to the speed. Does the sign of your correction make physical sense? Briefly, discuss.

HINT: A trick that may be helpful here: you can always rewrite the function $f(x) = c^x$ as $f(x) = e^{ln(c^x)} = e^{x\,ln(c)}$.

3.9 Rocket science!

Given: Pollock – Spring 2012

(a) A rocket having two or more engines, stacked one on top of another and firing in succession is called a multi-stage rocket. Normally each stage is jettisoned after completing its firing. The reason rocketeers stage models is to increase the final sped (and thus, altitude) of the uppermost stage. They do this by dropping unneeded mass throughout the burn so the top stage can be very light and coast a long way upward. Let's examine the advantages of a multi-stage rocket. Suppose the rocket carries 80% of its initial mass as fuel (i.e. the mass of all the fuel is $0.8m_0$) What is the rocket final speed accelerating from rest in free space, if it burns its fuel in a single stage? Express your answer in terms of v_{ex}.

(b) Now suppose instead that it burns the fuel in two stages like this: In the first stage it burns a mass $0.4m_0$ of fuel. It then jettisons the (empty) first stage fuel tank. Assume this empty tank has a mass of $0.1m_0$. It then burns the remaining $0.40m_0$ of fuel. (So, we've burned the same total amount of fuel as part a, right? We simply jettisoned an empty fuel-stage in the middle) Find the final speed in this case, assuming the same value of v_{ex} as in part a. Compare and discuss!

(c) Taylor worked out the rocket equation in deep space. But at launch, you can't neglect gravity - the net external force, dP/dt, is no longer zero! Follow Taylor's derivation on p. 86, and fix it up, getting to Eq 3.6 and find the "correction" term you need to add to include gravity. Assuming that v_{exh} is a fixed (constant) number, and assuming that you want the rocket to simply "hover" above the ground (rather than really launching), solve the ODE you get to find rocket mass as a function of time. (Does dm/dt turn out to be a constant? Explain physically why your answer to that question makes sense)

(d) If your payload (the mass that is left over after all the fuel is gone) is roughly $e^{-2} = .135$ of the initial mass, how long can you hover? Given the (very optimistic!) value of $v_{exh} = 2000m/s$, comment on why we don't all commute around with jetpacks.

The rest of this question is pure EXTRA CREDIT

So far we have considered the ideal case of a rocket without drag. In real life, however, drag can be an important limitation. Imagine the situation of a linear drag $\vec{f} = -b\vec{v}$ acting on the rocket body only (with no other external forces, so we're back to the "gravity free" case of deep space.) Once again, the net external force, dP/dt, is not zero. Follow Taylor's derivation on page 86, and fix it up, getting to Eq 3.6 and find the "correction" term you need to add, caused by drag. Solve your ODE, to show that if the rocket starts from rest and ejects a mass at constant rate $\dot{m} = -k$ (with k a given constant), its speed is given by $v = \frac{k}{b} v_{ex} \left[1 - \left(\frac{m}{m_0}\right)^A\right]$ What is A in terms of k, b, v_{ex} and m_0? Alternatively, if you'd rather not do the integral analytically, put your ODE into Mathematica, pick some reasonable numbers (see my lecture notes) and simply plot v as function of m!

HINT: since dm/dt=-k, you can eliminate any stray "dt" terms that appear in your ODE.

If you want yet another 2 points of extra credit and more practice with Taylor expansions: what is the corresponding speed if we ignore drag? Show that the equation you got reproduces the speed for the drag free case if $b \to 0$ (see hint below). Calculate the first non-vanishing correction introduced by a finite drag to the speed. Does the sign of your correction make physical sense? Briefly, discuss.

HINT: A helpful bit of math: you can always rewrite the function $f(x) = c^x$ as $f(x) = e^{ln(c^x)} = e^{x\,ln(c)}$.

3.10 Mulder and Scully investigate a crash

Given: Pollock – Spring 2011

Two FBI agents (let's call them Mulder and Scully) are investigating the wreckage of the spaceship is in three large pieces around a northern Colorado town. One piece (mass = 300 kg) of the spaceship landed 6.0 km due north of the center of town. Another piece (mass = 1000 kg) landed 1.6 km to the southeast (36 degrees south of east) of the center of town. The last piece (mass = 400 kg) landed 4.0 km to the southwest (65 degrees south of west) of the center of town. There are no more pieces of the spaceship. The Air Force, which was watching the spaceship on its radar, claims it was moving with a constant speed of 5 m/s to the east at a height of 1.96 km. It was 100 m west of the center of town when the spaceship spontaneously exploded and the pieces fell to the ground. There is also evidence that none of the pieces acquired appreciable vertical velocities immediately after the explosion. Agents Mulder and Scully think a missile hit it. Are the fragments consistent with the spaceship exploding spontaneously? If not, can you tell what direction the missile came from?