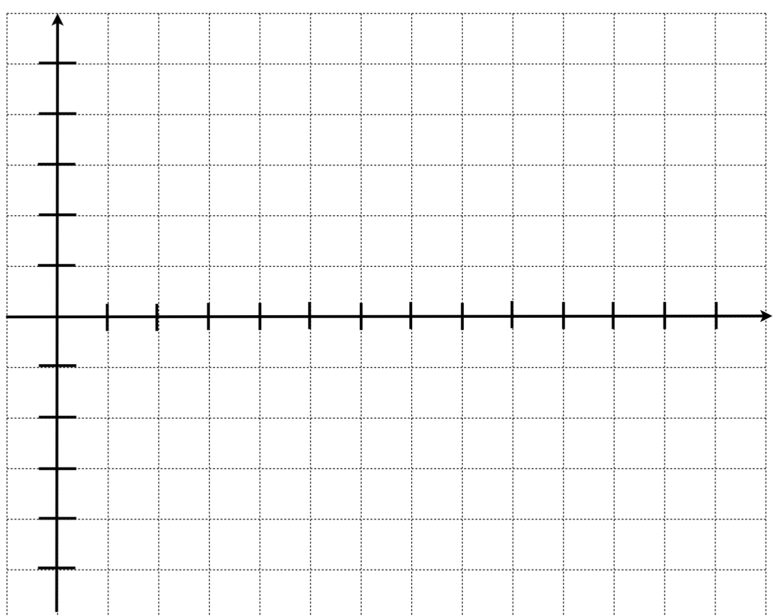
Bessel Functions in Mathematica

PHYS 2210 Fall 2010

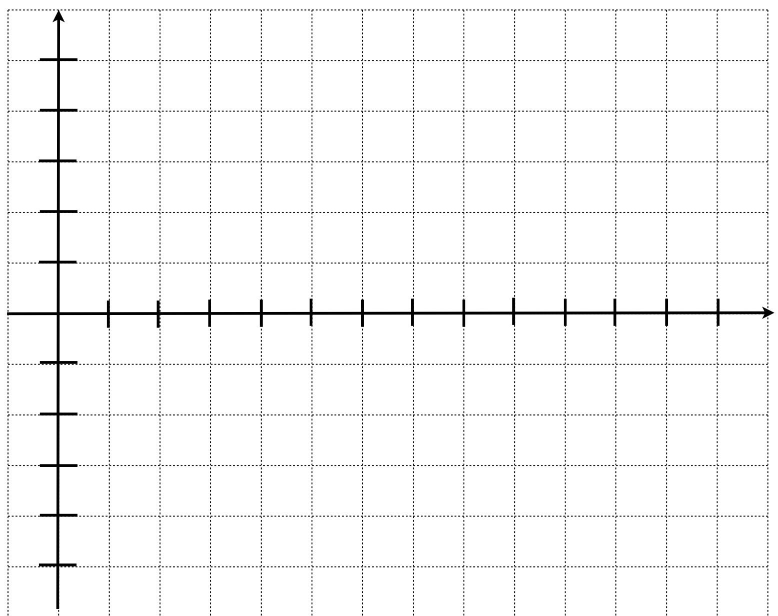
YOUR NAME IS:

In this tutorial, we will investigate some of the properties of Bessel functions using Mathematica.

1. Make a plot of $J_0(x)$, and $J_1(x)$ from $x = 0$ to $x = 12$. Make an accurate sketch of the functions on the axes below.



2. Make a plot of $Y_0(x)$, and $Y_1(x)$ from $x = 0$ to $x = 12$. Make an accurate sketch of the functions on the axes below.



3. Experiment with making similar plots for some of the higher order Bessel functions (though you do not need to sketch them here). What in general can you say about the value of the J and Y Bessel functions at x=0? What can you say about the locations of the zeros as n increases?

4. Use Mathematica to evaluate the value of $J_2(19.5)$.

In class we discussed how for large values of x, $J_p(x) \simeq \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{4} - \frac{\pi p}{2})$.

5. Evaluate this approximate form (with Mathematica, of course!) for $p = 2$ at $x = 19.5$. How well does this approximation compare to the value that you found in Question 4?

6. The approximate expression suggests that the locations of the zeros of the Bessel functions are at approximately $x \simeq \pi(n - \frac{1}{4} + \frac{p}{2})$ where n is an integer. For J_0 and J_1 find the locations of the first 3 zeros (not including the one at the origin) using the exact Bessel function and the approximate form. (Hint: take advantage of function BesselJZero in Mathematica.) Tabulate your results below. How well do they agree?

If you have time, give these a try....

7. Plot both the exact J_2 Bessel function and the approximation given above on a single plot. Above roughly what value of x does the large value approximation work well?

8. As was mentioned in class, the orthogonality relation for the Bessel functions is

$$\int_0^1 x J_p(ax) J_p(bx) dx = 0 \text{ for } a \neq b.$$

Note that here a and b are both zeros of J_p. Verify that this relation holds for J_1 when a and b are equal to the second and third zeros. (Hint: Use the NIntegrate function.)

9. The other part of the orthogonality relation for the Bessel functions is

$$\int_0^1 xJ_p(ax)J_p(bx)dx = \frac{1}{2}J_{p-1}^2(a) \text{ for } a = b.$$

Verify that this relation holds for J_1 when a is equal to the second zero.

10. Make a 3D plot of $J_0(r)$, where $r = \sqrt{x^2 + y^2}$. This is a snapshot of the surface of vibrating cylindrical membrane, like the surface of a drum.