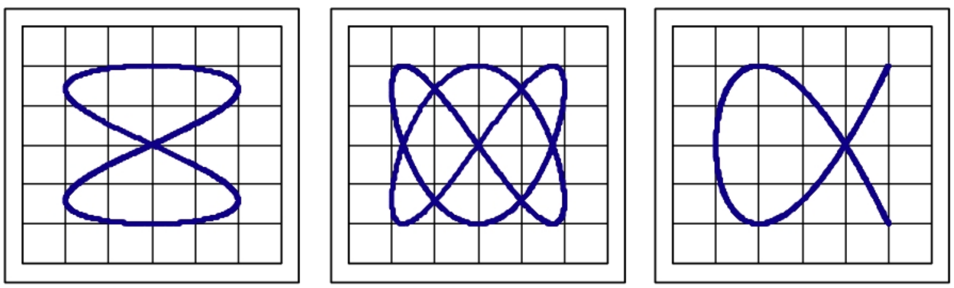
PHYS 2210 Fall 2011 Homework Set 10 Due at 4 PM on **Nov 10, 2011**

Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit.

1. (1.5 pt) Consider the motion of a two-dimensional non-isotropic oscillator, in which the spring constant in x direction (k_1) is not equal to the spring constant in the y direction (k_2). Each trajectory below depicts the possible motion of a unique non-isotropic oscillator. All three oscillators, share the property that the angular frequencies ω_1 and ω_2 for the motions along the x- and y-axes are commensurate, i.e., that the angular frequencies satisfy the following relationship: $\frac{\omega_1}{n_1} = \frac{\omega_2}{n_2}$, where n_1 and n_2 are integers. For each case below, (i) determine whether ω_1 is greater than, less than, or equal to ω_2, and (ii) determine the values n_1 and n_2 that satisfy the condition. Explain.



Trajectory #1 Trajectory #2 Trajectory #3

2. Consider an underdamped 1D oscillator of mass m and spring constant k that is subjected to a damping force of $\vec{F}_{drag} = -b\dot{\vec{x}}$. You may assume that the mass is on a horizontal frictionless table so you only need to consider the spring force and the drag force. At time $t = 0$ the system is released from rest at $x = A$.

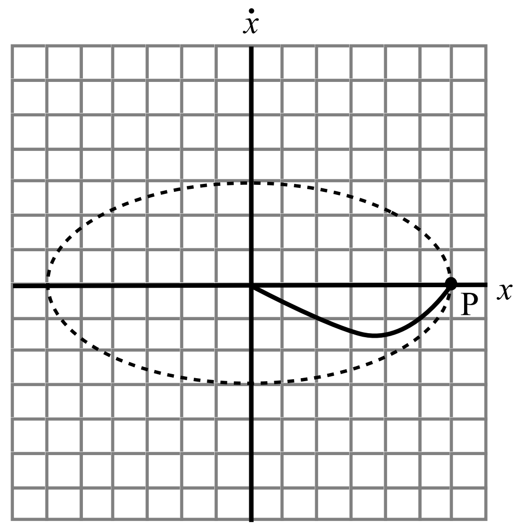
   (a) (0.5 pts) What is the solution for $x(t)$? Take the derivative to find $\dot{x}(t)$.

   (b) (0.5 pt) Use part (a) to find and expression for $E(t)$. (Hint: Some trig double angle identities might help simplify the expression a bit.)

   (c) (0.75 pts) Find the rate of energy loss, dE/dt, and show that it is proportional to $\dot{x}^2$.

   (d) (0.25 pts) Is the rate of energy loss a constant? Are the any times when the energy loss rate is 0? Explain why this makes sense.

3. (1.5 pt) Shown one the next page are phase space plots for (i) a simple harmonic oscillator (dashed) and (ii) the same oscillator with a retarding force applied (solid). Point P represents the initial conditions of the oscillator in both instances. In the diagram, you may assume each division along the position axis corresponds to 0.1 m; along the velocity axis, 0.10 m/s.



(a) Explain how you can tell that the damped oscillator is not underdamped.

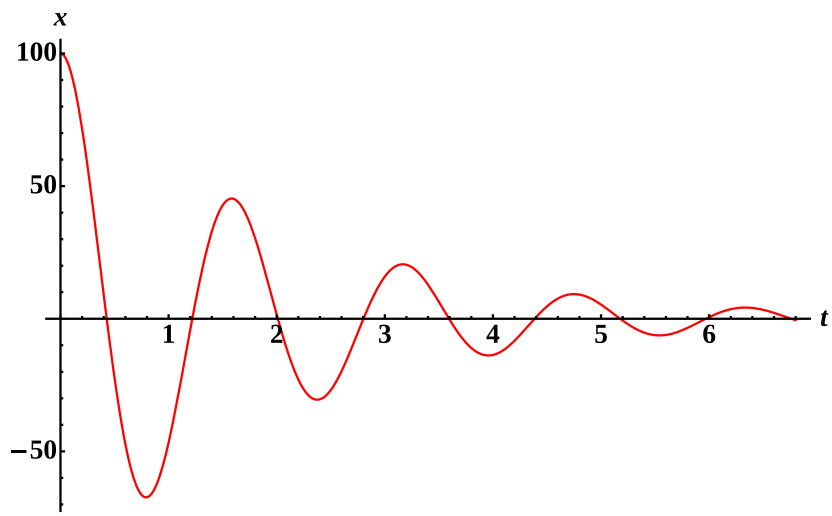
(b) Is the damped oscillator critically damped or overdamped? Explain how you can tell.

(c) If you said in part b that the oscillator is {critically damped, overdamped}, then draw how the phase space plot would be different if the oscillator (starting at point P) were instead {overdamped, critically damped}. Explain your reasoning

4. In this pair of problems, you will compare our model for a damped harmonic oscillator to real data collected from a video of a spring-mass system immersed in viscous oil. The video from which this position vs time data is collected is available here:
`http://www.youtube.com/watch?v=ZYFVKZPut9w`
Data was obtained using free video tracking software (Tracker) and saved in a comma-separated variables (CSV) file. First, you will describe a few properties of a damped oscillator to assist you in your analysis of the data.

(a) (0.5 pts) Write down the differential equation for a damped harmonic oscillator with undamped natural frequency, ω_0, and damping parameter, β. Write down the general solution for this differential equation for the underdamped case ($\beta < \omega_0$).

Consider the following sketch of a solution to this differential equation for a particular choice of β and ω_0 when the oscillator was displaced and let go.



(b) (0.5 pts) You can obtain an estimate for $\omega_1 = \sqrt{\omega_0^2 - \beta^2}$ using the time between successive zero crossing. Estimate ω_1 and explain how you obtained your result.

(c) (0.5 pts) You can also obtain an estimate for β using successive maxima or minima from the plot. Estimate β and ω_0. Explain how you obtained your result.

(d) (0.5 pts) If ω_0 were kept constant but β were increased (but still below ω_0), sketch how the plot above would change. What physical change to a spring-mass immersed in oil can be done to increase β? Answer both questions for the case where ω_0 is kept constant but β is decreased. What property of the sketch does β appear to control?

(e) (0.5 pts) If β were kept constant but ω_0 were decreased (but still above β), sketch how the plot above would change. What physical change to a spring-mass immersed in oil can be done to decrease ω_0? Answer both questions for the case where β is kept constant but ω_0 is increased. What property of the sketch does ω_0 appear to control?

5. You will now analyze a real spring-mass system immersed in viscous oil for which position versus time data was collected for the oscillator.

(a) (0.5 pts) The position versus time data appears in a comma-separated variables (CSV) file on CULearn. Importing this data is straight-forward in Mathematica using the `Import` command. You'll need to store the data in a variable, so you can plot it.

For example, `data = Import["/file/to/datafile.csv"]` will import the values in the CSV file into the variable `data`. You'll need to specify the full path to the datafile.

If you need a little more help, a video describing how to import data into Mathematica is located here: `http://www.youtube.com/compphysatcu#p/u/3/MmS3JNk7JE4`

Plot this data using the `ListPlot` command,
`experiment = ListPlot[data[[All, {1, 2}]], PlotRange -> All]`.
You are storing this plot (as `experiment`) for later, so you can display the experimental data and the results from your model on the same plot.

(b) (0.5 pts) Using the plot of the data, estimate values for ω_1, β, and ω_0. Determine the initial position and initial velocity for the oscillator. This oscillator was displaced and released from rest.

(c) (0.5 pts) Plot the particular solution to damped harmonic oscillator for your model parameters (ω_1, β, and ω_0) and initial conditions. Hint: Used what you learned from Problem 1, parts b and c. You should store this plot (e.g., `model = Plot[...]`).

(d) (0.5 pts) Plot the result of your model and the experimental data on the same axes using the `Show` command. This is why you stored the plots earlier (e.g., `Show[experiment, model]`). How does your model match the experimental data? Can you tweak your model parameters to make the fit better? Hint: Used what you learned by solving Problem 1, parts d and e.

(e) (0.5 pts) Why did we do this? Experimental physicists collect data and often attempt to fit a model of that system to their data. It's likely that you got pretty good but not great agreement with the data that was collected. Can you identify at least one aspect of the physical system (spring mass immersed in oil) that might have improved the model? Can you identify at least one aspect of the data collection procedure (video tracking software) that might have helped you better estimate your model parameters (ω_1, β, and ω_0)?