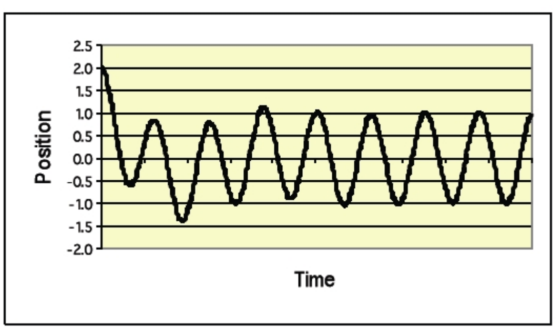
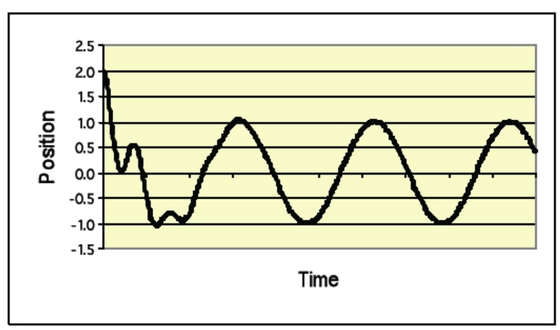
Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit.

1. (1 point) Boas 8.6.6 (You do not need to find a computer solution.)
2. (1 point) Boas 8.6.8 (You do not need to find a computer solution.)
3. (1 point) Boas 8.6.10 (You do not need to find a computer solution.)
4. (1 point) Boas 8.6.14 (You do not need to find a computer solution.)
5. (2 pts) A harmonic oscillator with a restoring force $25m\alpha^2 x$ is subject to a damping force $4m\alpha\dot{x}$ and a sinusoidal driving force $\mathrm{F}_0\cos(2\alpha t)$.
   (a) Write down the differential equation that governs the motion of this oscillator.
   (b) Show that if the driving force were removed the oscillator would become underdamped, and express the frequency of the oscillator in terms of the given quantities. Explain your reasoning.
   (c) For any damped oscillator that is driven by a sinusoidal external force, we know that the eventual (steady-state) motion is sinusoidal in nature. However, before the oscillator reaches steady state, its motion can be thought of as the algebraic sum of the steady-state motion plus a transient oscillatory motion whose amplitude dies exponentially with time. The transient component of the motion (considered by itself) could accurately describe the motion of the same oscillator with the driving force turned off (but with the damping still present).
      i. Each x vs. t graph below illustrates the actual motion (transient plus steady-state) of a damped, driven oscillator starting at t = 0. For each case, is the frequency of the steady-state motion greater than, less than, or equal to that of the transient motion? Explain.



Graph 1



Graph 2

      ii. Identify which graph (1 or 2) would better correspond to the damped, driven oscillator described in parts a-b of this problem. Explain your reasoning.

6. (1.5 points) Taylor 5.41 You may assume that $\omega \simeq \omega_0$ at the peak value of A^2, but definitely no at the half-max. You are also told that $\beta << \omega_0$, so for example if you have terms like $\beta^2\omega_0^2$ and β^4, you can drop the β^4 term since it will be very small compared to the other term.

7. This problem is based on the system described in Taylor Problem 5.38.

   (a) (1 points) Do Taylor Problem 5.38. (Note that you must plot the result using Mathematica, Matlab or some other computer program)

   (b) (1 points) Redo problem 5.38 with $\omega \simeq \omega_0 + \beta = 1.1$. Once again find A, δ, B_1, and B_2. Make a plot of $x(t)$ the first ten or so periods. In Problem 6, you proved that A^2 is equal to half its maximum value at $\omega \simeq \omega_0 \pm \beta$. Show that indeed for this value of ω, A^2 is roughly half of the value that you found in part a.

   (c) (0.5 points) Now plot your solutions for part a and b on the same axes. How does the solution to part b compare to the solution for part a? What differences do you notice? (Hint: In Mathematica you can plot two functions together like this `Plot[{f1[t],f2[t]},{t,0,10}, PlotRange->All]`. Just make sure that you do not reuse the same variables names for both functions.)