PHYS 2210 Fall 2011 Homework Set 12 Due at 4 PM on **Dec 1st, 2011**

Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit.

1. For this problem, consider the periodic function $f(x)$: (one period of the function is specified here)

$$\begin{aligned} f(x) &= -x, \text{ for } -\frac{\pi}{2} < x < 0 \\ &= x, \text{ for } 0 < x < \frac{\pi}{2} \end{aligned}$$

   (a) (2 pt) Find the Fourier Series expansion for $f(x)$. Be sure to include a sketch of the function.

   (b) (1 pt) To check that your solution is correct, use a computer, plot the sum of the **first 3 non-zero terms** in this series from $-\pi$ to π. It should resemble the sketch that you drew in part a.

2. For this problem, consider the periodic function $g(x)$: (one period of the function is specified here)

$$\begin{aligned} g(x) &= 1, \text{ for } -1 < x < 1 \\ &= 0, \text{ for } 1 < x < 5 \end{aligned}$$

   (a) (2 pt) Find the Fourier Series expansion for $g(x)$. Be sure to include a sketch of the function.

   (b) (1 pt) To check that your solution is correct, use a computer, plot the sum of the first **first 4 non-zero terms** in this series from -10 to 10. It should resemble the sketch that you drew in part a.

3. (0.5 pt) Show that the functions x^2 and $\sin x$ are orthogonal on the interval (-1,1). (Hint: You should not need to work out the integral. What do you know about even and odd functions?)

4. Consider the following function:

$$\begin{aligned} f(x) &= \sin x, \text{ for } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ &= 0, \text{ for } |x| > \frac{\pi}{2} \end{aligned}$$

   (a) (1 pt) Demonstrate that the Fourier transform of $f(x)$ is $g(\alpha) = \dfrac{-i\alpha \cos\left(\frac{\alpha\pi}{2}\right)}{\pi(1-\alpha^2)}$. Hint: It might help to express $e^{-i\alpha x}$ in terms of sines and cosines. Recalling the properties of integrals of odd and even functions can also save you some work.

   (b) (0.5 pt) Using your answer for $g(\alpha)$, express $f(x)$ as a Fourier integral (i.e. substitute your result for $g(\alpha)$ into the Fourier integral for $f(x)$). Leave the answer as an integral. Do not evaluate the integral.

5. (2 pt) Evaluate the following integrals:

   (a) $\int_0^{\pi} \sin(x)\delta(x - \frac{\pi}{2})dx$

   (b) $\int_{-\infty}^{\infty} e^x \delta(3x)dx$

   (c) $\int_0^{2\pi} \cos(x)\delta(x + \pi)dx$

   (d) $\int_{-10}^{10} x^2\delta(1 - x)dx$