

Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit.

1. (3 pt) Boas 13.2.2 For full credit, you must explicitly demonstrate the separation of variables and discuss the application of all of the boundary values. Also, please include a computer plot the sum of at least the first 4 non-zero terms in the Fourier series to verify that your solution properly reproduces the boundary condition along the bottom edge of the plate ($y=0$). (Note that in Mathematica you can very easily plot the sum of a very large number of terms using the `Sum[]` function. You can easily show the sum of the first 50 terms, for example.)
2. (2 pt) Boas 13.2.9
3. We mentioned in class that the first few Legendre polynomials are

$$\begin{aligned}P_0(x) &= 1 \\P_1(x) &= x \\P_2(x) &= \frac{1}{2}(3x^2 - 1)\end{aligned}$$

- (a) (0.5pt) Use the recursion relation to find $P_3(x)$. You must show all work to receive any credit.
 - (b) (1 pt) Use the built-in Mathematica function `LegendreP` to find $P_3(x)$ and $P_2(x)$. Plot $P_3(x)$ and $P_2(x)$ from $x = -1$ to $x = 1$.
 - (c) (0.5 pt) What kind of symmetry do $P_3(x)$ and $P_2(x)$ have? Are $P_2(x)$ and $P_3(x)$ orthogonal over the interval $(-1,1)$? Explain how you can tell that without having to evaluate any integrals.
4. (3 pt) Boas 13.7.3 (Hint: Find a way to express the boundary condition in terms of spherical harmonics.)