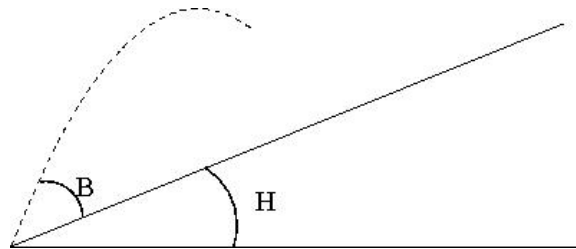


Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit.

- Point P sits on the rim of wheel of radius ρ that is rolling on the ground in the positive x direction. Its position vector can be described by the sum of a vector v_1 for circular motion about the center of the wheel plus a vector v_2 that describes the horizontal motion of the center of the wheel along the line $y = \rho$ at constant velocity. In other words, its position at time t is given by

$$\vec{r} = \vec{v}_1 + \vec{v}_2 = (\rho \sin(\omega t)\hat{x} + \rho \cos(\omega t)\hat{y}) + (\rho\omega t\hat{x} + \rho\hat{y})$$

- (1 point) Find an expression for the velocity of Point P.
 - (1 point) Are there any times when the velocity is 0? Where is Point P on the wheel at these times (on the top, on the front edge, etc)? Does this make sense?
- A slingshot launches an angry bird¹ up in the air with a speed v_o at an angle of B (with respect to the ramp) up a hill of with an incline of H above the horizontal.



- (1 point) Show that distance *along the hill* where the bird lands can be expressed as $\frac{2v_o^2 \sin(B) \cos(B + H)}{g \cos^2(H)}$. (Hint: use tilted coordinates where one axis is along the slope of the hill, and some angle addition identities might be helpful here.)
 - (1 point) For a given H , at what angle should the bird be launched to achieve the maximum distance?
 - (1 point) Show that the expression for the maximum distance is $d_{max} = \frac{v_o^2}{g(1 + \sin(H))}$
 - (1 point) For $H = 20^\circ$, use *Mathematica* to make a plot of the expression you obtained in part a from $B=0$ to 90° . Attach the plot to this assignment, and verify that the maximum distance value occurs at the angle given by your answer to part b.
- (2 points) Taylor Problem 1.48. Note that the problem is asking you to express $\hat{\rho}$, $\hat{\phi}$, and \hat{z} in terms of their \hat{x} , \hat{y} , and \hat{z} components, but these expressions may include the coordinates ρ , ϕ and z .

¹Apparently the bird is so upset over some stolen eggs that it can't fly.

4. A honeybee exits its hive and flies in spiral path. Its position \mathbf{r} at time t is given in polar coordinates by

$$r = be^{kt} \qquad \phi = ct$$

where b , c , and k are constants.

- a) (1 point) Use the expressions that we found for $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ in polar coordinates to find the velocity and acceleration vectors for the bee. Are the velocity and acceleration vectors orthogonal to each other?
- b) (1 point) Show that the angle between the velocity and acceleration vector remains constant as the bee moves away.