Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit.

1. (1 point) Boas, Chapter 1, Section 13, Problem 9, parts (a) and (b)

2. An object is released from rest far above the ground. The object experiences a force due to the air (drag) that is proportional to the object's speed $\vec{F} = -b\vec{v}$. In this problem let's define the y-axis to point *down*.

   (a) (0.5 points) Your friend, who is also taking Classical Mechanics, has calculated $v_y(t)$. He/she knows that the terminal velocity of the object is 40 m/s, and has found a solution, $v_y(t) = 40(1 - e^{-t/2})$, but isn't confident that it is right. In physics we often want to check our answers by picking situations where you independently know what the answer should be without using your formula, and then checking that your proposed formula gives these answers. In this case, you know $v_y(t)$ for $t = 0$ and $t \to \infty$ from your basic understanding of physics. Check your friend's solution at these two points. How is his/her equation looking so far?

   (b) (0.5 points) Given that the drag force on this object is $\vec{F} = -b\vec{v}$, the drag force is not very significant for small times. What is the formula for $v_y(t)$ in the case of no drag?

   (c) (1 point) Given the above, check your friends solution by finding an approximate for that is valid for small t. If you find that it is incorrect, can you suggest to your friend what term(s) in the equation he/she might want to double check?

3. (1 point) Taylor, Problem 2.19

4. (2 points) In Taylor's book, he simplifies Eqn 2.42 for the range of a projectile by assuming that $R^2 \simeq R_{vac}^2$. Show that if you use the quadratic equation to solve for R in Eqn 2.42, and make the appropriate Taylor series expansion that you obtain the expression given in Equation 2.44.

Please turn over.

5. Recall from special relativity that for a particle moving at a relativistic speed, v, the energy E=γmc^2, where $\gamma = \dfrac{1}{\sqrt{(1-\frac{v^2}{c^2})}}$.

   (a) (0.75 point) Assuming that $v/c << 1$, find the first two terms of the series expansion of the energy by doing a Taylor Series expansion of $\dfrac{1}{\sqrt{1-x}}$ around $x = 0$.

   (b) (0.5 points) What is the second term? Does this make sense for a particle with v<<c?

   (c) (0.75 point) Now instead do a Taylor Series expansion of $\dfrac{1}{\sqrt{x}}$ around $x = 1$. How does this compare to your answer from part a?

   (d) (0.5 points) Could you also have taken the first 2 terms of the Taylor Series expansion of $\dfrac{1}{\sqrt{x}}$ around $x = 0$ to obtain this expression for the case where $v/c << 1$? Why or why not?

6. Consider a baseball with diameter 8 cm and mass 150 g. Use the fact quadratic drag dominates for a baseball, has a magnitude of $F(v) = 0.157\rho D^2 v^2$, where D is the diameter of the ball and ρ is the air density (all numerical values are in SI units). For this problem, we will ignore any effects due to spin on the ball. As Taylor points out on page 62, the range of a projectile subjected to quadratic drag cannot be solved analytically. But it can be evaluated numerically, using a program like *Mathematica*.

   (a) (1 point) Todd Helton recently hit a home run during a Rockies baseball game in Denver at Coors Field, where the density of air is 1.07 kg/m^3. Assuming that the ball had an initial speed of 112 mph (50 m/s) and left the bat at an angle of 35° with respect to the horizontal, how far did the ball go? (You may assume that it starts and lands at the same height.) Hint: Use NDSolve in Mathematica to solve a system of equations. You will have 2 equations to solve simultaneously, the x and y force equations, and will need to specify the initial x and y positions and the initial x and y velocities. You can then plot the solutions for y(t) and x(t) and see what the x distance is when y=0.

   (b) (0.5 points) Had Mr. Helton hit the same ball at Dodger Stadium in Los Angeles, where the density of air is 1.3 kg/m^3, how far would it have gone? In its first decade of operation, players at Coors Field hit an above-average number of home runs. Explain why this makes sense.