PHYS 2210 Fall 2011 Homework Set 5 Due at 4 PM on **Sept 22, 2011**

Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit. For Mathematica/Computer problems you must include a printout of your code.

1. (1.5 points) A thin metal plate of uniform density, is shaped like an equilateral triangle, as shown below. Find the location (x, y) of the center of mass of the plate. You must find the answer using calculus.

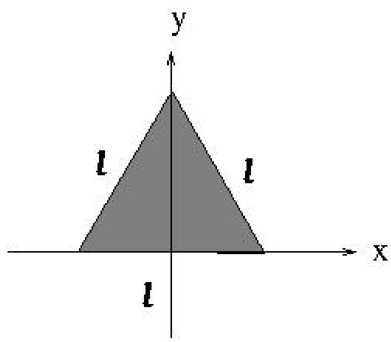

Plate for Problem 1

2. (1 point) You (mass=75 kg) are standing still at one end of a log that is floating at rest in a lake. Now you run to the other end of the log at a speed of 1 m/s and stop. If the log has a mass of 500 kg and a length of 5 m, how far does the center of the log move with respect to the shore? You may assume that there is no friction between the log and the water.

3. An astronaut has drifted too far away from the space shuttle while attempting to repair the Hubble Space telescope. She realizes that the orbiter is moving away from her at 3 m/s. She and her space suit have a mass of 90 kg. On her back is a 10 kg jetpack which consists of an 8 kg holding tank filled with 2 kg of pressurized gas. She is able to use the gas to propel herself directly towards to the orbiter. The gas exits the tank at a uniform rate with a constant velocity of 100 m/s, relative to the tank (and her).

   (a) (1 point) After the tank has been emptied, what is her velocity? Will she be able to catch up with the orbiter with that velocity?

   (b) (1 point) With what velocity (in her frame of reference!) will she have to throw the empty tank away to reach the orbiter?

4. (1.5 points) Taylor Problem 3.12

   Please turn the page over!

5. Consider a rocket of mass (with an initial mass m_0) that travels vertically. The rocket ejects fuel at a constant velocity (v_{ex}) relative to the rocket's motion.

   (a) (1 point) If this rocket is designed such that the rate of fuel ejection is constant (i.e., $\dot{m} = -k$), show that the equation of motion for this rocket is,

   $$m\frac{dv}{dt} = kv_{ex} - mg$$

   Solve this differential equation for v(t) using separation of variables. Recall that m is not a constant, but that $m = m_0 - kt$. You will assume that g doesn't change appreciably.

   (b) (0.5 points) Describe what happens to the rocket if the value of kv_{ex} was smaller than the initial value of mg.

   (c) (0.5 points) Show for a rocket that starts from rest at $y = 0$, the resulting expression for y(t) is

   $$y(t) = v_{ex}t - \frac{1}{2}gt^2 - \frac{mv_{ex}}{k}\ln\left(\frac{m_0}{m}\right).$$

   (Hint: The following integral might be useful: $\int ln(x) = xln(x) - x + const.$ If you are interested, you can integrate $ln(x)$ by parts to obtain this result.)

   (d) (0.5 point) Assume that the rocket burns fuel for 200 seconds. Use Mathematica to plot the velocity and position of the rocket as a function of time (up to 200 sec). Let the initial mass of the rocket be 2 $\times 10^6$ kg, the rate of mass ejection be 8333.33 kg/s and the exhaust speed be 3000 m/s. Obviously, the rocket starts from rest. How high is the rocket after 200 seconds? What does this tell you about your assumption about g in part (a)?

   (e) (0.5 points) Real rockets experience air drag. Due to their size and typical speeds, the air resistance is best modeled as quadratic. Consider your plots in part (d). Describe how your plots might change by adding bv^2 drag to the model.

   (f) (1 point) Using Mathematica numerically solve the equation of motion which includes air drag. Use the same parameter values as in part (d). Assume the drag coefficient b for the rocket is 0.80. Plot both the velocity and vertical position of the rocket as functions of time. Compare these plots to your predictions in part (e) and your analytic results in part (d).