PHYS 2210 Fall 2011 Homework Set 7 Due at 4 PM on **Oct 13th, 2011**

1. (1 point) Consider a spherical shell that extends from $r = R$ to $r = 2R$ with a **non-uniform density** $\rho(r) = \rho_0 r$. What is the total mass of the shell?

2. (2.5 points) Imagine that NASA digs a straight tunnel through the center of the moon (see figure) to access the Moon's ^{3}He deposits. An astronaut places a rock in the tunnel at the surface of the moon, and releases it (from rest). Show that the rock obeys the force law for a mass connected to a spring. What is the spring constant? Find the oscillation period for this motion if you assume that Moon has a mass of 7.35×10^{22} kg and a radius of 1.74×10^{6} m. Assume the moon's density is uniform throughout its volume, and ignore the moon's rotation.

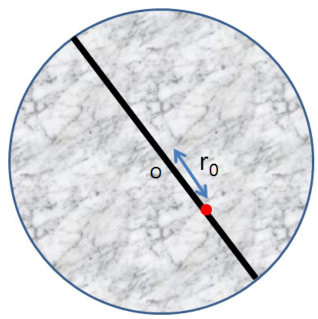

Figure 1:

3. (3 points) Taylor 4.24

4. Consider a very (infinitesimally!) thin but massive circular loop, radius R (total mass M), centered around the origin, sitting in the x-y plane. Assume it has a uniform linear mass density λ (which has units of kg/m) all around it. (So, it's like a skinny donut that is mostly hole, centered around the z-axis)

   (a) (0.5 points) What is λ in terms of M and R? What is the direction of the gravitational field generated by this mass distribution at a point in space a distance z above the center of the donut, i.e. at $(0, 0, z)$ Explain your reasoning for the direction carefully.

   (b) (1 points) Compute the gravitational potential at the point $(0, 0, z)$ by directly integrating $-Gdm/r$, summing over all infinitesimal "chunks" dm along the loop. Then, take the z-component of the gradient of this potential to find the graviational field.

   (c) (1 points) Compute the gravitational field, $\vec{g}$, at the point $(0, 0, z)$ by directly integrating Newton's law of gravity, summing over all infinitesimal "chunks" of mass along the loop. Check that you agree with your result from the previous part.
   (please turn over!)

(d) (0.5 points) In the two separate limits $z << R$ and $z >> R$, Taylor expand your g-field (in the z-direction) out only to the first non-zero term, and convince us that both limits make good physical sense.

(e) (0.5 points) Can you use Gauss' law to figure out the gravitational potential at the point $(0, 0, z)$? (If so, do it and check your previous answers. If not, why not?)