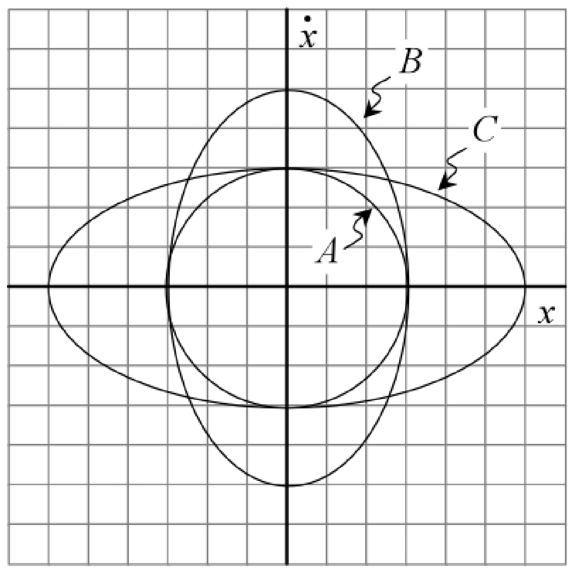
Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit. For Problems 1-3, do not use a computer, and you do not need to compare the solution to a computer solution.

1. (1 point) Boas 8.5.2
2. (1 point) Boas 8.5.8
3. (1 point) Boas 8.5.15
4. (0.5 point) Boas 8.5.36 (Note that figure 1.1 is on page 390)
5. (1 point) Taylor 5.2
6. Consider the phase space plots (A, B, and C) shown below.



   (a) (0.5 points) Could all three plots correspond to the same simple harmonic oscillator (i.e., same mass and same spring constant)? Explain why or why not.

   (b) (0.5 points) Which pair of plots could be used to show the effect of keeping the total energy constant but increasing the mass (while keeping the spring constant fixed)? Clearly indicate which plot would correspond to the smaller mass. Explain without performing any calculations.

   (c) (0.5 points) Suppose that plots A and C correspond to systems with springs with the same spring constant, but different masses. Do these two systems have the same total energy? If not, which one has more total energy? Explain how you can tell.

(d) (0.5 points) Suppose that in the diagram each division along the position axis corresponds to 0.1 m; along the velocity axis, 0.10 m/s. What is the period of the oscillator shown by path B?

7. We often consider the simple pendulum to be an example of simple harmonic motion. In this problem will will see how accurate this is.

   (a) (0.5 points) Imagine a vertical pendulum of length l and mass m. By considering the forces on the pendulum and applying Newton's second law, obtain differential equation for the forces on the pendulum in terms of θ, the angle with respect to the vertical axis. (Hint: It might be easiest to work in polar coordinates.)

   (b) (0.5 points) Show that in the limit where θ is small that you obtain a second-order linear differential equation with constant coefficients. What is the general solution to this equation?

   (c) (0.5 points) Consider m=1 kg, and l=1 m. If the pendulum is released at rest at $t = 0$ from θ=0.05 radians, what is the particular solution for $\theta(t)$ in your expression from part b? What is the period of this solution?

   (d) (0.25 points) If instead the initial condition is that θ=1.25 radians at $t = 0$, what happens to the period of the motion?

   (e) (0.5 points) Now use Mathematica to solve the exact equation that you found in part a (keeping the $\sin(\theta)$ in the expression) for the case where the pendulum is released from rest at $t = 0$ from θ=0.05 radians. Make sure that you figure out how to apply initial conditions. Is there an exact solution if you use `DSolve` or do you have to use `NDSolve`?

   (f) (0.5 points) Make a plot of the solution. Now use the `FindRoot` command to figure our where the solution is zero.[1] (You should have some idea of where to look for the roots from the plot that you made.) Use the locations of two roots to determine the period. How does this compare the to the period of the approximate small angle solution found above in part c?

   (g) (0.5 points) Now use Mathematica to determine a numerical solution for the case where the pendulum is released from rest at $t = 0$ from θ=1.25 radians. Plot the solution. Once again, use the `FindRoot` command to determine the locations of two roots to determine the period. How does this compare to the case where it was released from $\theta = 0.05$ in part f?

   (h) (0.25 points) Based on these results, what can you say about how well simple harmonic motion approximates the behavior of a pendulum?

[1] For a sample Mathematica notebook that solves a second-order ODE and uses the FindRoot function, please look at "Another ND Solve Example" on CULearn, in the Mathematica Examples folder. You might find this to be a useful starting point.