

Show and explain all of your work! Correct answers for which we cannot follow your work are worth no credit.

- (1 point) Boas, Chapter 8, Section 3, Problem 7. Find the general solution. You do not need to compare it to a computer solution.
- Consider a softball with diameter 10 cm and mass 200 g. Use the fact that the force of air resistance on a spherical object of diameter  $D$  can be approximated using coefficients  $c_1 = (1.55 \times 10^{-4})D$  and  $c_2 = 0.22D^2$  (all numerical values are in SI units).

- (1 point) Calculate the terminal speed of the softball taking into account both the linear and quadratic terms. Show all work.
- (0.5 points) The ratio of the quadratic and the linear terms of the force of air resistance can therefore be expressed as:

$$\frac{|c_2 v^2|}{|c_1 v|} = \frac{0.22D^2 v^2}{(1.55 \times 10^{-4})Dv} = (1.4 \times 10^3)|v|D$$

Using the above ratio, for what range of speeds will (i) the linear term of air resistance dominate over the quadratic term? (ii) the quadratic term dominate over the linear term? If it were desired to approximate the effect of air resistance on a falling softball with either the linear term or the quadratic term (not both), which term would you keep? Explain your reasoning.

- (1 point) We have already seen in class an exact solution for velocity in the case where there is only linear or quadratic drag. So, based on your answer to part b, what is the approximate speed of a softball 2 seconds after it was released from rest?
  - (1.5 points) Let's investigate how good this approximation is. If we keep both the linear and quadratic terms, we have a non-linear differential equation. In general we do not know how to solve this kind of differential equation, but we can solve it numerically. Use the `NDSolve` function in *Mathematica* to find a the numerical solution for the velocity at  $t=2$  sec. Compare this to the approximate one that you found in part c. Was it a good approximation?
- (2 points) The Bugatti Veyron Super Sport, the world's fastest and most expensive production car, has a maximum speed of 268 miles per hour<sup>1</sup>. The engine provides maximum forward force of 12,350 N on the 1900 kg car. Quadratic air resistance dominates, with a drag coefficient of .86 kg/m. The car starts from rest at one end of a 5 km long test track. Can the Bugatti reach a speed of 265 mph (very close to the maximum speed) before it runs out of road? (Watch your units on this one!)

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<sup>1</sup>James May noted on the BBC show "Top Gear" that at this top speed "the tires will only last for about 15 minutes, but it's okay because the fuel runs out in 12 minutes."

Problem 4 is of interest for an ongoing study on the use and effectiveness of Phet computer simulations. If you consent to allow the researchers to have access to your homework solution to help improve the simulations, please sign the attached consent form and turn it in with your homework. Your solutions will be anonymous to the researchers and your decision on participation in this study will not impact your grade in this class.

4. In your textbook (section 2.4), Taylor solves for the case of a baseball being dropped from a high tower subject to quadratic air resistance,  $F_D = -cv^2\hat{v}$ . Let's now instead look at the case of a ball being shot *up* at an initial speed  $v_0$ .
- (a) (0.5 points) Draw a free body diagram for a ball moving vertically upwards, subject to quadratic air drag. Write down a differential equation for this situation and solve this differential equation for  $v(t)$ . Make a rough sketch of  $v(t)$  vs.  $t$ , and briefly discuss any key features.
  - (b) (0.5 points) Using your result from part a, find an expression for the time it takes to reach the top of the trajectory. (It will look simpler if you write it in terms of terminal velocity, which satisfies  $v_t^2 = mg/c$ .)
  - (c) (0.5 points) Now download the PhET simulation at: <http://phet.colorado.edu/en/simulation/projectile-motion>. On the top right, switch the object to baseball. This sim uses quadratic drag:  $F_D = -\frac{1}{2}c_0A\rho_{air}v^2\hat{v}$ , where  $c_0$  is the drag coefficient,  $A$  is the cross-sectional area of the object being shot, and  $\rho_{air}$  is the density of air =  $1.3 \text{ kg/m}^3$ . (The sim shows you the value of  $c_0$ .) By experimenting with the sim, what initial velocity makes the ball reach the top at approximately 3 sec?
  - (d) (0.5 points) Now use your formula in part b for “time to top” to deduce what numerical initial velocity,  $v_0$ , you need to get the ball to reach the top of its trajectory at precisely  $t = 3$  sec. How does your calculated value compare to your “experimental” value?
  - (e) (0.5 points) When playing with the PhET sim, does it seem to take longer for a ball to go from the ground to the top of a trajectory or from the top of the trajectory to the ground? Explain why this is the case.
  - (f) (0.5 points) Again, playing with the sim, write down the initial velocity that makes the ball reach the top of its trajectory at 4 sec, then 5 sec, then 6 sec, and so on. What do you notice happening? Make a plot of  $t_{top}$  vs.  $v_0$  and explain in words how this relates to what you see on the sim.

*Extra Credit:* Play with the PhET sim a little more and explore anything you are interested in. Write down one question that you have about something you notice when playing with the sim.

Consent form for collection of course information and materials

You are invited to participate in a research project to improve the learning and appreciation of science with the use of technology.

The purpose of this project is to investigate the use of PhET computer simulations and other presentations of science materials in introductory science courses. Specifically, we are looking for any differences that may exist between students' use of different presentations of science content, with those presentations being computer-based and / or paper based, such as homework or in-class tutorials. The PhET simulations are designed to enhance your conceptual understanding of the underlying science principles. We would like to learn about your experiences with these presentations of science concepts to help us design future materials for the science classroom. Participation in this study is entirely your choice.

You are being asked to provide us with access to your materials completed as a part of this course, such as homework solutions, exam scores, and other evaluations and surveys. We are interested in responses to the questions related to the PhET simulations and other PER-based instructional activities. Your responses to these questions will be transferred in a flat text file to our server, or with your direct submission of homework to the investigator.

You will be asked to complete evaluations to the best of your ability. These evaluations will be administered in lecture or online in the throughout the semester. Your score on some of these evaluations will not affect your course grade, i.e. these will not count towards your final grade in this class. Class instructors who are members of the research group (listed above) will not have access to individual survey responses until after the course has been completed and final grades submitted. No other class instructors will ever have access to individual responses; they will only receive information on who participated and the results for the class as a whole – that is results with no identifying information about individuals. If your professor is offering course credit or other incentive in return for participating in this research, you may collect this credit without participating by simply submitting the survey uncompleted.

The potential risks of participating in this study are minimal. Your participation in this project is strictly voluntary, and a decision not to participate or to discontinue participation at any time will in no way affect your grade. There are minimal risks to you involved with providing us with access to the above mentioned information. Information from this research and evaluation will contribute to the ongoing changes being made to the PhET explorations and will contribute to improving science courses. Your individual privacy will be maintained in all published and written data resulting from this study. No one except the researchers will have access to your identity. Talks, based on this study, will be given to various professional audiences. In these talks, only the statistical analysis of your coursework will be released. The data from the analysis will be stored on password protected computers. After the completion of this project these materials will be stored for a period of 3 years and then destroyed.

This project is conducted under the direction of Dr. Kathy Perkins, Dr. Carl Wieman, Dr. Wendy Adams, Dr. Noah Finkelstsein, and Dr. Robert Parsons, 390 UCB, University of Colorado at Boulder, Boulder, Colorado 80309 (303-492-4367). Other investigators include Dr. Noah Podolefsky, Dr. Archie

Initials \_\_\_\_\_

Paulson and others of the Physics, Chemistry, Geological Science, Integrative Physiology and Biology Departments at the University of Colorado, Boulder, Campus Box 390, Boulder, Colorado, 80309.

If you have questions regarding your rights as a participant, any concerns regarding this project or any dissatisfaction with any aspect of this study, you may report them -- confidentially, if you wish -- to the Institutional Review Board, 3100 Marine Street, Rm A15, 563 UCB, (303) 735-3702. Copies of the University of Colorado Assurance of Compliance to the federal government regarding human subject research are available upon request from the graduate school at the address listed above. In addition, research and evaluation personnel will be happy to answer any questions you may have about this evaluation.

**Authorization:**

I have read this paper about the study or it was read to me. I know the possible risks and benefits. I know that being in this study is voluntary. I choose to be in this study. I know that I can withdraw at any time. I have received, on the date signed, a copy of this document containing 2 pages.

Name of Participant (printed) \_\_\_\_\_

Signature of Participant \_\_\_\_\_ Date \_\_\_\_\_.  
(Also initial all previous pages of the consent form.)