UNIVERSITY OF COLORADO AT BOULDER **PHYS 2210**

CLASSICAL MECHANICS AND MATH METHODS, SPRING, 2011

Homework 12

(Due Date: Start of class on Thurs. Apr 7)

1. On the last exam, we had a problem with a flat ring, uniform mass per unit area of σ, inner radius of R, outer radius of $2R$. A satellite (mass m) sat a distance z above the center of the ring. We asked for the gravitational potential energy, and the answer was

$$U(z) = -2\pi G\sigma m(\sqrt{4R^2 + z^2} - \sqrt{R^2 + z^2}) \tag{1}$$

(a) If you are far from the disk (on the z axis), what do you *expect* for the formula for U(z)? (Don't say "0" - as usual, we want the functional form of $U(z)$ as you move far away. Also, explicitly state what we mean by "far away". (Please don't compare something with units to something without units!)

(b) Show explicitly that the formula above does indeed give precisely the functional dependence you expect.

2. *On the midterm, we asked you to check an answer to a calculation, and we observed a lot of people struggled with this. Let's practice!* In each case below, I will pose an abbreviated problem and a proposed answer. Your task is NOT to solve the problem (!!) but rather, to simply CHECK the given answer. Briefly, comment!

(a) In a tragic accident, a car (mass m) plunges into an icy lake. The water produces a linear drag force on the car $\vec{f_d} = -b\vec{v}$. Find the velocity of the car as a function of time. The student's response after several lines of careful calculations was:

$$v(t) = v_0 e^{bt/m} \tag{2}$$

(Remember, you are NOT being asked to solve this problem!! You only need to make two **checks** of this answer to see if it is reasonable. If your checks indicate it is not reasonable, do they suggest any possible fix, or something you might look into?)

(b) A buoy bobs in the Boulder reservoir. It is cylindrical in shape, with a circular base of radius r, and height h. Small periodic wavelets (of amplitude A meters, frequency ω) drive the buoy. (Water has density ρ) What is the resulting period of oscillation, averaged over a very long time? The student's response after several lines of careful calculations was:

$$T = 2\pi/\omega - 2\pi\sqrt{m/(\rho g(2\pi r h))} \tag{3}$$

(Again, don't solve the problem, just check the solution - and then, can you make any suggestions about what might be wrong even without having solved the problem?)

(c) A formula for potential energy U(x,y,z) was given, and the question asked for the resulting force. The student's response after several lines of careful calculations was:

$$\vec{F}(x, y, z) = cx\ \hat{j} \tag{4}$$

with the constant $c = 4.9/s^2$ (Again, don't solve the problem, just check the solution - and then, can you make any suggestions about what might have gone wrong even without having solved, or in this case even seen, the original problem?)

CONTINUED

3. (a) Solve Boas Problem 8.6.12 (That's chapter 8, section 6, problem 12, on page 423). We just want the general solution.

Please tell us explicitly if the associated (complementary) homogenous problem is (choose one) {over-damped, under-damped, undamped, critically damped} How do you tell?

(b) Going back to the full Boas problem - invent a physics problem for which that ODE is the solution. Be explicit - what are the values of all relevant physical parameters, in SI units?

Now, given that the system starts with $y(0) = 0$, $y'(0) = 0$, solve that problem completely. There should be no undetermined coefficients left!

(c) Sketch your solution in part b) above, noting any important aspects of the sketch. (Don't worry about precision, this is a sketch, not a plot. We want to know the *features.*) Then, use MMA to plot the actual solution. Print it out and discuss - in particular, discuss if it did not exactly match with your sketch. (The differences may or may not be subtle, depending on the care of your initial sketch)

4. A series LRC circuit (Taylor Fig 5.10) is connected across the terminals of an AC power supply that produces a voltage $V(t) = V_0 e^{i\omega t}$. The "equation of motion" for the charge $q(t)$ across the capacitor is as follows:

$$L\ddot{q} + R\dot{q} + q\frac{1}{C} = V_0 e^{i\omega t} \tag{5}$$

The above differential equation will have a steady-state solution of the form:

$q(t) = q_0 e^{i(\omega t - \phi)}$ [Note: The parameters q_0 and ϕ are actually functions of ω, the frequency of the AC power supply. However, in this problem you will not have to write out these functions in full.]

(a) In terms of q_0, ω, ϕ and the relevant coefficients from the differential equation, write down the following functions:

i. the potential difference $\Delta V_C(t)$ across the capacitor

ii. the potential difference $\Delta V_R(t)$ across the resistor

iii. the potential difference $\Delta V_L(t)$ across the inductor

(b) Determine the smallest positive values of α, β, and γ (in radians) that satisfy the following Euler relations: $e^{i\alpha} = i$, $e^{i\beta} = -1$, $e^{i\gamma} = -i$

(c) Using your results from part b), rewrite the functions in part a) so that each function can be written as a positive real number times (the same) complex exponential. Use your rewritten functions to answer the following questions.

i. What is the phase difference between $\Delta V_C(t)$ and $\Delta V_R(t)$?

Do the peaks of $\Delta V_C(t)$ come just *before* ("leads") or just *after* ("lags") the peak of $\Delta V_R(t)$? (Show/explain your reasoning)

ii. What is the phase difference between $\Delta V_R(t)$ and $\Delta V_L(t)$?

Does ΔV_L lead or lag ΔV_R?

5. In section 5.6, Taylor states that the amplitude of an oscillator subject to a sinusoidal driving force is

$$A^2 = \frac{f_0^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$

(a) Find an expression for ω such that the amplitude A is maximized. That is, derive our text's Equation 5.73. (*Hint:* Rather than differentiate and extremize the entire expression for $A(\omega)$, make your job easier by judiciously picking which part of that expression to differentiate and extremize.)

Then, use your expression to find the exact maximum amplitude, A_{max}.

Write a second expression for the amplitude A_0 which corresponds to $\omega = \omega_0$.

Briefly, comment on the difference between A_{max} and A_0.

(b) Open up the resonance PhET sim, found at

`http://www.colorado.edu/physics/phet/dev/resonance/`

Select the highest number version, and then click on "dev" (near the top). Play around to see what the sim can do.

If you set the damping at a moderate value (say, 1 N m/s) and then decrease the driving frequency of the oscillator to roughly 10% below the natural frequency, what do you predict will happen? Focus first on simply position as a function of time, and predict, based on Taylor 5.68 and the formula for A above what you should see (just qualitatively is fine.) Then, watch the sim carefully. Explain how what you see is or is not consistent with what you predicted. How long does it take for the motion to settle down? (you will only be able to crudely approximate this, but if you are way off, chances are that something went wrong)

After waiting for any transients to die out in the system you have set up, focus on the relationship between the phase of the driver and the mass. What does Taylor's Fig 5.19 predict? What does it look like on the sim? (It's hard to tell exactly, just be approximate. Note that there is a "slo motion" slider, you can put rulers on the screen, and you can set up multiple oscillators with different parameters if that helps you)

(c) Now increase the frequency of the driver to about 10% *higher* than the resonant frequency. What relationship between the phase of the driver and the mass do you expect? Does the PhET sim confirm this? How does the time it takes to reach this final value compare to what it took for the case of 10% *lower* driving frequency? (Briefly, comment)

(d) Check out the case of driving the mass at ω_0. What is the phase difference between driver and mass?

Building on what you have seen in the last three parts, explain Taylor Figure 5.19 in your own words.

Extra credit. Play with the sim a little more, and write down one additional (real) question that you have about something you notice!