UNIVERSITY OF COLORADO AT BOULDER **PHYS 2210**

CLASSICAL MECHANICS AND MATH METHODS, SPRING, 2011

Homework 14

(Due Date: Start of class on Thurs. Apr 21)

1. For the periodic function $f(t) = At$ for $-1 < t \leq 1$ (with A a given constant)

(a) Compute the Fourier coefficients a_n and b_n

(b) Suppose this force drives a weakly damped oscillator with damping parameter $\beta = 0.05$ and a natural period $T = 2$. Find the long time motion $x(t)$ of the oscillator. Discuss your answer. Use Mathematica to plot $x(t)$ using the first four non-zero terms of the series, for $0 < t \leq 10$. (Set A=1 for this)

(c) Repeat part b) for the same damping parameter, but with natural period $T = 1$. Briefly discuss any major differences between this result and what you had in the previous part.

(d) For most of the possible natural frequencies the response of this particular driven oscillator is going to be weak. However, there are some particular natural frequencies at which you are going to see an enhanced response - determine which those are. (Be careful, think a bit about this!)

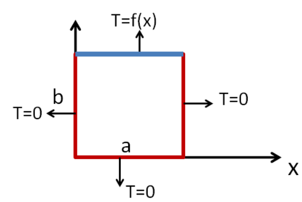

Figure 1:

2. For the rectangular plate problem shown in Fig.1 calculate the steady state temperature $T(x, y)$ everywhere inside the plate if the boundary condition on the top surface is given by

$$f(x) = \begin{cases} T_0 & 0 < x \leq a/2 \\ 0 & a/2 < x \leq a \end{cases} \tag{1}$$

3. For the previous problem:

(a) Code up your formula in Mathematica, adding up 30 terms in your sum to find T(x,y). For concreteness, please set a=1, b=2, and T_0=100.
Note that to define a function in Mathematica you need to be careful about syntax. If, for instance, $T(x, y) = \Sigma_n n\ sin(n\pi x)e^{-n\pi y}$, *the Mathematica syntax for that might be*
t[x_,y_] := Sum[n Sin[n Pi x] Exp[-n Pi y],{n,1,30}]
Watch out for the underscores for variables x and y on the left side (they do NOT appear again on the right side), and the colon before the equals sign. Don't capitalize functions that you invent, caps are reserved for MMA built-in functions.

(b) As a check, set y=2, and just plot your T[x,2] (from x=0 to 1) to make sure you did that Fourier sum right. (That's the boundary condition we gave you, so you know what it should look like!) Comment on any interesting or surprising features you observe about this plot, is it exactly what you expected?

(c) Plot your full T(x,y) using the function Plot3D. See the documentation center for the basic syntax of Plot3D. Briefly, discuss/document the key features of your plot. Please tell us explicitly what does the height of this graph represent, physically? *Plot3D is very cool. You can rotate the resulting curve with your mouse to really get a good look at your result.*

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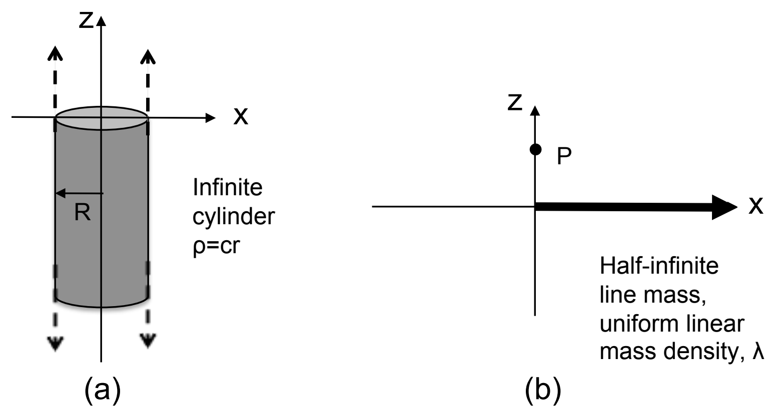

Figure 2: (a) An infinite cylinder of radius R centered on the z-axis, with *non-uniform* volume mass density $\rho = cr$, where r is the radius in cylindrical coordinates. (b) A half-infinite line of mass on the x-axis extending from $x = 0$ to $x = +\infty$, with uniform linear mass density λ.

4. There are two general methods we use to solve gravitational problems (i.e. find $\vec{g}$ given some distribution of mass).

(a) Describe these two methods. We claim one of these methods is easiest to solve for $\vec{g}$ of mass distribution (a) above, and the other method is easiest to solve for $\vec{g}$ of the mass distribution (b) above. Which method goes with which mass distribution? Please justify your answer.

(b) Find $\vec{g}$ of the mass distribution (a) above for any arbitrary point outside the cylinder.

(c) Find the x component of the gravitational acceleration, g_x, generated by the mass distribution labeled (b) above, at a point P a given distance z up the positive z-axis (as shown).

Extra credit!

(i) For the rectangular plate problem shown in Fig.1 (for problem 2), calculate the steady state temperature $T(x, y)$ everywhere inside the plate if the boundary condition on the top surface was $f(x) = \sin(3\pi x/a)$. (Note - this is much *easier* than problem 2, if you think about it! In fact, if you look back at what you did there, you shouldn't need to do any new calculations at all.)

(ii) Keep the boundary condition the same as part i on the bottom and left edges (T=0 there), and keep the boundary condition the same at the top edge too, i.e. once again T(x,y=b)= $\sin(3\pi x/a)$. But let's change the right edge condition: Let T(x=a,y) = $\sin(3\pi y/b)$. Find T(x,y) everywhere else. *Hint: Once again, you should not need to do any real calculations here - you can pretty much just write down the answer if you think about it right!*