**UNIVERSITY OF COLORADO AT BOULDER** **PHYS 2210**

CLASSICAL MECHANICS AND MATH METHODS, SPRING, 2011

Homework 4

(Due Date: Start of class on Thurs. Feb 3 )

**NOTE:** Be sure to show your work and explain what you are doing. (Correct answers, for which we cannot follow the work, are worth no credit).

**1.** Consider a sports car which is braking hard. There are two significant resistive forces acting on it, a quadratic ($cv^2$) air drag, and a constant ($\mu mg$) frictional force. When you write Newton's law, if you are interested in finding v(x) (rather than v(t)), there is a nice trick, known as the "v dv/dx rule", which uses the chain rule to rewrite $\dot{v} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$.

(a) Write down the equation of motion for $\dot{v} = f(v)$ and use the "v dv/dx" rule to solve the equation of motion directly for v(x), and show that the distance the car needs for a full stop is:

$$x_{max} = \frac{A^2}{2\mu g}\ln\left(\frac{A^2 + v_0^2}{A^2}\right) \tag{1}$$

here $\mu$ is the friction coefficient. What is the constant $A$ in this case ( in terms of given parameters in the differential equation?)

(b) The SSC Ultimate Aero TT, one of the world's fastest production cars (at a cool \$600,000+), has a maximum speed of 412 Km per hour. The engine provides maximum forward force of 11260 N on the 1200 kg car. For this car, $c$ =0.86 kg/m. On a race track, the car exits a turn with a speed of $v_0 = 300$ km/hr. As soon as the driver enters the straight track after the turn, she realizes there is another car blocking the track 2 km away. The driver slams on the brakes. Assuming a friction coefficient $\mu = 0.7$, use your above result to compute the distance required for the Aero TT to stop. Compare this distance to the corresponding distance in the absence of air drag (i.e. just kinetic road friction). (Briefly, discuss)

(c) If the blocking car was removed when the car had slowed to a velocity of 200 Km/h and then the driver starts accelerating again with constant maximum force, what is the velocity that the Aero TT reaches after it travels a distance of 2 Km from the point it started re-accelerating? (Briefly, discuss)

(d) Make a rough sketch (by hand, not with Mathematica!) of the car's position x(t) described by the "story" of parts b and c (i.e., starting from when the driver entered the straight track, and ending at the final position after part c.) Comment briefly on interesting features of your graph (e.g., signs of slope, signs of concavity, interesting points...)

**2.** (a) In Taylor's book, he simplifies Eqn 2.42 for the range of a projectile by assuming that $R^2 \sim R_{vac}^2$. Show that if instead you use the quadratic equation to solve for $R$ directly in Eqn 2.42, and make the appropriate Taylor series expansion, that you obtain the expression given in Equation 2.44.

(b) If you use your result from the previous part to compute $R$ when $v_{y0} = v_{ter}$, what do you get? Does this make sense? (Briefly, discuss) (2 pts)

**3.** Recall from special relativity that for a particle moving at a relativistic speed, $v$, the energy $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{(1-v^2/c^2)}}$. Find the first two terms of the Taylor series expansion of the energy, in the non-relativistic limit $v << c$. What is the second term? Does this make sense?

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**4.** Find the regulation size (and mass, while you're at it) for a golfball.

(a) The drag coefficient $c_D$ for a golfball is (very roughly) 0.3. (Note: the c in $cv^2$ is given by $\frac{1}{2}c_D\rho D^2$) Calculate the numerical value of the quadratic drag constant "c" for a golf ball traveling in air, and write down the equations of motion required to solve for x(t) and y(t) of a golfball with (just) quadratic drag.

(b) In class we did a MMA Tutorial using NDSolve. (The code/worksheet is still available on our course calendar for Tues Jan 25) Modify your code so that you have *quadratic* drag. For simplicity and concreteness, set the angle to 45 degrees, and pick an initial speed of 80 mi/hr (converted to SI metric, of course) Plot the trajectory for us. What is the ideal (drag-free) range in this case, and what does your code tell you the range is including quadratic drag? (Don't hunt for the optimum range like we did last week, just stick with a 45 degree angle) Also, calculate what fraction of the initial *speed* is lost from when you first hit the ball (at 45 degrees) to when it lands. (Where does the lost energy go?)

(c) When I first started thinking about this problem, I wondered whether the fractional reduction in speed was a constant, or whether it depended on the initial velocity. What do you think? (No formal calculation required, just make a qualitative physics argument.) *Check* yourself with your code, and comment.

(d) Astronaut Alan Shepard on Apollo 14 brought a golf club with him (!) Assuming he hit it at 45 degrees (and assuming he hit it at that same 80 mi/hr, a modest swing on earth, maybe optimistic on the moon, given his clumsy space suit), calculate how far the ball landed from him, and compare to what it would have been on earth. (Which effect is more significant in the end, the loss of air drag on the moon, or the difference in gravity? )

**5.** Two FBI agents (let's call them Mulder and Scully) are investigating the wreckage of the spaceship is in three large pieces around a northern Colorado town. One piece (mass = 300 kg) of the spaceship landed 6.0 km due north of the center of town. Another piece (mass = 1000 kg) landed 1.6 km to the southeast (36 degrees south of east) of the center of town. The last piece (mass = 400 kg) landed 4.0 km to the southwest (65 degrees south of west) of the center of town. There are no more pieces of the spaceship. The Air Force, which was watching the spaceship on its radar, claims it was moving with a constant speed of 5 m/s to the east at a height of 1.96 km. It was 100 m west of the center of town when the spaceship spontaneously exploded and the pieces fell to the ground. There is also evidence that none of the pieces aquired appreciable vertical velocities immediately after the explosion. Agents Mulder and Scully think a missile hit it. Are the fragments consistent with the spaceship exploding spontaneously? If not, can you tell what direction the missile came from?

**Extra credit: More fun with Taylor series** An (ideal, unstretchable, massless) string is wrapped tightly around the earth. Then, 1 extra meter is added to its length. You pull up at *one point* of the string. Approximately how high above the ground can you pull this point? (You may be quite surprised by the answer, I was!)