**UNIVERSITY OF COLORADO AT BOULDER** **PHYS 2210**

CLASSICAL MECHANICS AND MATH METHODS, SPRING, 2011

Homework 6

(Due Date: Start of class on Thurs. Feb 17 )

**NOTE:** In general, lettered parts are worth 4 pts each, unless we state otherwise.

**1.** A collision between two bodies is defined to be elastic if the total kinetic energy before and after the collision is the same. Consider an elastic collision between two identical atoms one which is initially at rest $\vec{v}_2 = 0$ and the other is moving with velocity $\vec{v}_1 \neq 0$. Denoting $\vec{v}'_{1,2}$ the corresponding velocities after the collision

(a) Write down the vector equations representing the conservation of momentum and the scalar equation representing the conservation of kinetic energy in an elastic collision.

(b) Use this to prove that the angle between $\vec{v}'_1$ and $\vec{v}'_2$ is $\pi/2$. Think of a situation where this fact might prove to be useful or relevant.

**2.** A rocket having two or more engines, stacked one on top of another and firing in succession is called a multi-stage rocket. Normally each stage is jettisoned after completing its firing. The reason rocketeers stage models is to increase the final sped (and thus, altitude) of the uppermost stage. This is accomplished by dropping unneeded mass throughout the burn so the top stage can be very light and coast a long way upward. Let us understand better the advantages of a multi-stage rocket. Imagine that the rocket carries 70% of its initial mass as fuel (i.e. the mass of all the fuel is $0.7m_0$)

(a) What is the rocket final speed accelerating from rest in free space, if it burns its fuel in a single stage? Express your answer in terms of $v_{ex}$

(b) Now suppose instead that it burns the fuel in two stages like this: In the first stage it burns a mass $0.35m_0$ of fuel. It then jettisons the (empty) first stage fuel tank. Let's assume this empty tank has a mass of $0.1m_0$. It then burns the remaining $0.35m_0$ of fuel. (So, we've burned the same total amount of fuel as part a, right? We simply jettisoned an empty fuel-stage in the middle) Find the final speed in this case, assuming the same value of $v_{ex}$ as in part a. Compare and discuss briefly.

**3.** Taylor works out the rocket equation in deep space. But at launch, obviously you cannot neglect gravity - the net external force, dP/dt, is no longer zero.

(a) Follow Taylor's derivation on page 86, and fix it up, getting to Eq 3.6 and find the "correction" term you need to add to include gravity. Assuming that $v_{exh}$ is a fixed (constant) number, and assuming that you want the rocket to simply "hover" above the ground (rather than really launching), solve the ODE you get to find rocket mass as a function of time. (Does $dm/dt$ turn out to be a constant? Explain physically why your answer to that question makes sense)

(b) If your payload (the mass that is left over after all the fuel is gone) is roughly $e^{-2} = .135$ of the initial mass, how long can you hover? Given the (very optimistic!) value of $v_{exh} = 2000 m/s$, comment on why we don't all commute around with jetpacks.

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**4.** So far we have considered the ideal case of a rocket without drag. In real life, however, drag can be an important limitation and must be considered. Imagine the situation of a linear drag $\vec{f} = -b\vec{v}$ acting on the rocket body only (with no other external forces, so we're back to the "gravity free" case of deep space)

(a) Once again, the net external force, dP/dt, is not zero. Follow Taylor's derivation on page 86, and fix it up, getting to Eq 3.6 and find the "correction" term you need to add, caused by drag.

(b) Now solve your ODE, to show that if the rocket starts from rest and ejects a mass at constant rate $\dot{m} = -k$ (with k a given constant), the its speed is given by $v = \frac{k}{b} v_{ex} \left[ 1 - \left( \frac{m}{m_0} \right)^A \right]$

What is $A$ in terms of $k, b, v_{ex}$ and $m_0$.

HINT: since dm/dt=-k, you can eliminate any stray "dt" terms that appear in your ODE.

(c) What is the corresponding speed if we ignore drag? Show that the eq'n above reproduces the speed for the drag free case if $b \to 0$ (see hint below). Calculate the first non-vanishing correction introduced by a finite drag to the speed. Does the sign of your correction make physical sense? Briefly, discuss.

HINT: A trick that may be helpful here: you can always rewrite the function $f(x) = c^x$ as $f(x) = e^{ln(c^x)} = e^{x\, ln(c)}$.

**5.** The magnetic field inside a particular very long current carrying wire is given by $\vec{B} = -(\frac{B_0}{R}) r\hat{\phi}$, where R is the radius of the wire, and r is the distance from the center of the wire. Ampere's law, discovered by Ampere in 1826, relates the integrated magnetic field around any closed loop to the total electric current passing through the loop, $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{through}$. If we want to determine the current passing through the loops shown in Fig.1, we need to evaluate the line integral of $\vec{B} \cdot d\vec{r}$.

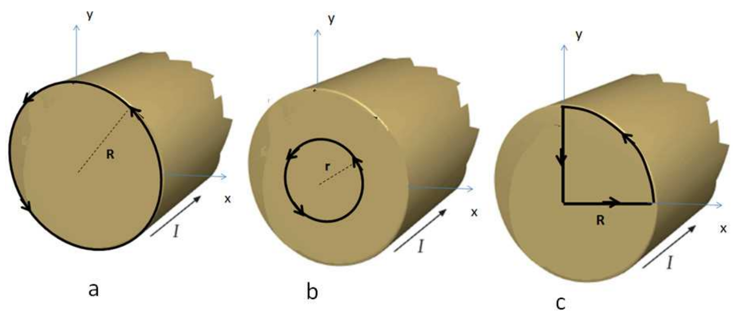


Figure 1:

(a) Explicitly compute $\oint \vec{B} \cdot d\vec{r}$ along the full circle path of radius R, shown in Fig.1a. Use this to find $I_{through}$. (Briefly, discuss the physical meaning of the sign of your answer). (2 pts)

(b) Then, compute $\oint \vec{B} \cdot d\vec{r}$ along the path of radius $r_0$ in Fig 1b. How does $I_{thru}$ compare with part a? (2 pts)

(c) Compute $\oint \vec{B} \cdot d\vec{r}$ along the quarter circle path in Fig 1c. Compare your answers to the above three parts, and discuss. (What do you conclude about how the current is distributed through the wire?) (2 pts)

(d) Sketch a vector plot of $\vec{B} = -(\frac{B_0}{R}) r\hat{\phi}$. Rewrite $\vec{B}$ entirely in Cartesian coordinates, and then, use the command VectorPlot in Mathematica to generate a plot of $\vec{B}$ to check your hand-drawn sketch. (4 pts)

**6.** *This one is pure extra credit!* A puck (mass $m$) on a frictionless air hockey table is attached to a cord passing through a hole in the surface as in the figure. The puck is moving in a circle of radius $r_i$ with angular velocity $\omega_i$. The cord is then slowly pulled from below, shortening the radius to $r_f$ (r-final)

(a) What is angular velocity of the puck when the radius is $r_f$? (2 pts)

(b) Assuming that the string is pulled so slowly that we can approximate the puck's path by a circle of slowly shrinking radius, calculate the work done in moving the puck from $r_0$ to $r$. (Look back at Taylor Eq 1.48. "Slowly" means that $\dot{r}$ is tiny, as is the angular acceleration, so only the centripetal force will be important). Compare your answer with the puck's gain in kinetic energy, and comment briefly.