UNIVERSITY OF COLORADO AT BOULDER **PHYS 2210**

CLASSICAL MECHANICS AND MATH METHODS, SPRING, 2011

Homework 7

(Due Date: Start of class on Thurs. Feb 24)

1. *The joy of Taylor Expansions. Taylor series are the single most important and common approximation technique used throughout physics. We need to keep practicing with them!*

(a) Suppose $f(x) = 1/(1-x)$. Taylor expand to find an approximate expression for $f(x)$, going out to second order. Use it to estimate f(0.1) and f(5). In both cases, compare the exact result for $f(x)$ with your series approximation, going to first order (terms proportional to x) and also second order (x^2). Does your estimate improve at second order in both cases? Comment on the general criterion about x you would guess tells you when the series approach seems to be a fruitful one here.

(b) Now let $f(x) = (1+x)/(1-x)$. Taylor expand to approximate $f(x)$ out to second order. There are many ways to do this. First , use the usual formal formula for Taylor expansion. Another way is to take your series answer in part a, and multiply it by (1+x). Try it this way too, and use it to check yourself, both methods should agree to all orders! Which method do you prefer?

(c) Now let $f(x) = (a+x)/(a-x)$, where a is a constant with units, like $a = 5m$. Taylor expand to second order. In this case, do NOT start "from scratch" blindly using the Taylor formula. Instead, factor out "a", so that it looks *exactly* like what you had in part b, except the "thing you're expanding in" isn't x. What is it? For this part, what is now the general criterion on x that tells you when this series approach seems to be a fruitful one? Why? (And please do not say "x must be small" or "$x << 1$". Small compared to what? You cannot compare meters to numbers, or apples to oranges!)

(Once you understand this trick you will be spared having to ever painfully Taylor expand much of anything - the front flyleaf of your text will tell you how to expand almost anything you'll ever encounter)

(d) One astronaut is in a circular orbit around the Earth a distance R from the center of the Earth. Another astronaut is in orbit a bit closer to the Earth, at a distance $R-d$. The difference in the magnitude of the gravitational field between their locations is $\Delta g = \frac{GM_E}{(R-d)^2} - \frac{GM_E}{R^2}$ Use an appropriate series expansion to obtain an approximate expression for Δg. What relationship between d and R are you assuming? Your answer must contain one non-zero term. Does the sign of your answer seem sensible? Lastly - use your result to find the difference in g between the bottom and top of Gamow tower. Is your answer consistent with the common claim that $g = 9.8m/s^2$ anywhere near earth? Comment.

2. A driver traveling downhill in a 1200 kg SUV on a road with a 4.5 degree slope slams on his brakes and skids 30 m before hitting a parked car. An insurance investigator hires an expert who measures the coefficient of kinetic friction between the tires and road to be $\mu_k = 0.45$. Is the investigator correct to accuse the driver of traveling faster than the 25 MPH speed limit? Explain. (While there are many ways to solve this problem, please solve it using work and energy.)

CONTINUED

3. The diagram below shows a region of space. The dashed curves indicate positions of equal potential and are labeled with the value of the potential at that curve. Three vectors originating from point Y are also shown in blue. The vector B points directly from point Y to Z. In order to a particle to be launch from Y and reach Z, which vector represents best the initial velocity? Explain your reasoning. Then, make a (very crude, no calculations required!) sketch/guess of the *path* you expect the particle to follow if launched from point Y with initial **v** given by vector A. (Explain your reasoning, briefly, so we know what you were thinking about)

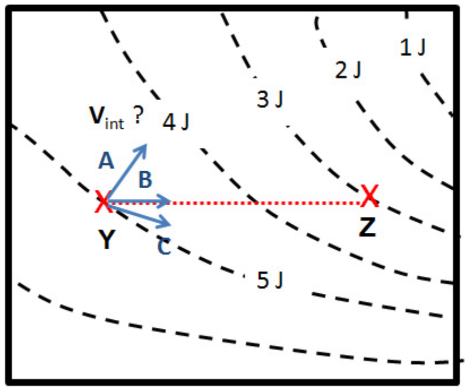

Figure 1:

4. Each diagram in Fig 2 depicts a force field in a region of space.

(a) For which force fields can you identify a closed path over which $\oint \vec{F} \cdot d\vec{l} \neq 0$? For each such case, clearly indicate an appropriate path on the diagram.

(b) For each case indicate if $\nabla \times \vec{F} = 0$ everywhere in the box?

(c) For each case, could the force depicted in the diagram be conservative? Briefly, explain.

(d) For each case, is it possible to draw a self-consistent set of equipotential contours for that situation? If so: Draw a representative set of equipotential contours for that situation. Each drawing should clearly show the correct shape of the contour lines, the correct relative spacing of the contours, and label the regions that correspond to highest and lowest potential energy. If not: Explain why drawing such contours is impossible.

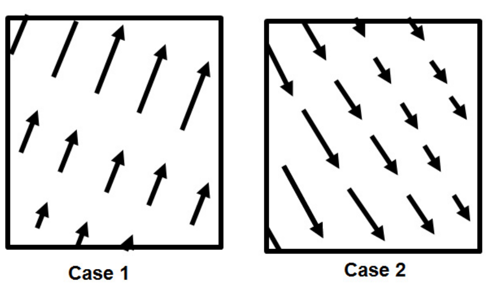

Figure 2:

5. Consider the following expression for the potential energy of a satellite in a far away solar system.

$$U(x, y, z) = 2x^2 - y^2 - xy + 3z \tag{1}$$

(a) Compare the magnitude of the force on the satellite at point 1 (0,0,1) and point 2 (1,2,0).

(b) Is this a conservative force? Clearly justify your answer.

(c) For a arbitrary scalar function f(x, y, z), evaluate the components of $\vec{\nabla} \times \vec{\nabla} f$ in Cartesian coordinates and show that the result is 0. Does this formal result relate to part b? (If not, why not? If so, briefly comment)

Extra credit! Explicitly compute the work done by this force as you follow a path from the origin (0,0,0) to point 2, along a straight line. Then, check your answer very simply using the potential function given. Explain the idea of your check.

6. Imagine a simple pendulum consisting on a point mass m fixed to the end of a massless rod (length l), whose other end is pivoted from the ceiling to let it swing freely in the vertical plane. The pendulum's position can be identified by its angle ϕ from the equilibrium position.

(a) Write the equation of motion for ϕ using Newton's second law

(b) Show that the pendulum's potential energy (measured from the equilibrium level) is $U(\phi) = A(1 - \cos\phi)$. Find A in terms of m, g and l.

(c) Write down the total energy as a function of ϕ and $\dot{\phi}$.

(d) Show that by differentiating the energy respect to t you can recover the equation of motion you found in (a). (What basic principle of physics are you using, here?)

(e) Assuming that the angle ϕ remains small throughout the motion, solve for $\phi(t)$ and show that the motion is periodic. What is the period of oscillation?