CLASSICAL MECHANICS AND MATH METHODS, SPRING, 2011

Homework 9

(Due Date: Start of class on Thurs. March 10)

1. You are stranded on the surface of the asteroid Vesta. If the mass of the asteroid is M and its radius is R, how fast would you have to jump off its surface to be able to escape from its gravitational field? (Your estimate should be based on parameters that characterize the asteroid, not parameters that describe your jumping ability.) Given your formula, look up the approximate mass and radius of the asteroid Vesta 3 and determine a numerical value of the escape velocity. Could you escape in this way? (Briefly, explain) If so, roughly how big in radius is the maximum the asteroid could be, for you to still escape this way? If not, estimate how much smaller an asteroid you would need, to escape from it in this way?

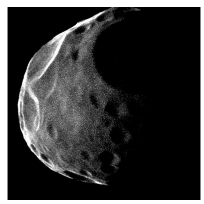

Figure 1:

2. Consider two identical uniform rods of length L and mass m lying along the same line and having their closest points separated by a distance d as shown in the figure

(a) Calculate the mutual force between these rods, both its direction and magnitude.

(b) Now do several checks. First, make sure the units worked out (!) The, find the magnitude of the force in the limit $L \to 0$. What do you expect? Briefly, discuss. Lastly, find the magnitude of the force in the limit $d \to \infty$? Again, is it what you expect? Briefly, discuss.

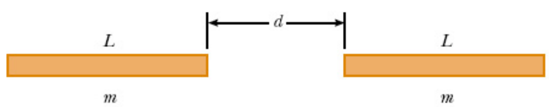

Figure 2:

3. *Practicing with complex numbers. (1 pt each)*

(a) If $z_1 = -\sqrt{3} + i$, draw z_1 in the complex plane. Compute its real and imaginary parts and magnitude. Write z_1 as $Ae^{i\theta}$ and determine A and θ.

(b) If $z_2 = 1/(1 + i)$, draw z in the complex plane. Compute its real and imaginary parts and magnitude. Write z as $Ae^{i\theta}$ and determine A and θ.

(c) What are the real and imaginary parts of $1/z_3^2$ with $z_3 = 0.5e^{-i\pi/3}$.

(d) Compute $Z = z_1 * z_3$ with z_1 and z_3 given in parts (a) and (c). Draw Z in the complex plane. Compute its real and imaginary parts and magnitude. Write Z as $Ae^{i\theta}$ and determine A and θ.

CONTINUED

4. Consider again the simple pendulum problem. In homework 7 you showed that the pendulum's potential energy (measured from the equilibrium level) is $U(\phi) = mgL(1 - \cos\phi)$ (where L is the pendulum length) and that under the small angle assumption the motion is periodic with period $\tau_0 = 2\pi\sqrt{l/g}$. In this problem we will use Mathematica to find the period for large oscillations as well:

(a) Using conservation of energy, determine $\dot{\phi}$ as a function of ϕ, given that the pendulum starts at rest at a starting angle of Φ_0. Now use this ODE to find an expression for the time the pendulum takes to travel from $\phi = 0$ to its maximum value Φ_0. (You will have a formal integral to do that you cannot solve analytically! That's ok, just write down the integral, being very explicit about the limits of integration) Because this time is a quarter of the period, write an expression for the full period, as a multiple of τ_0.

(b) Use MMA to evaluate the integral and plot τ/τ_0 for $0 \leq \Phi_0 \leq 3$ rad. For small Φ_0, does your graph look like that you expect? (Discuss - what value DO you expect?) What is τ/τ_0 for $\Phi_0 = \pi/8$ rad? How about $\pi/2$ rad? What happens to τ as the amplitude of the oscillation approaches π? Explain.

Note: The integral has some curious numerical pathologies. If your plot has little gaps in it, it's because MMA is generating tiny imaginary terms added to the result. You might just try plotting the real part of the integral, using the command Re[ ]. Similarly, if it gives you symbolic results, you might fix that by using the command N[ ], which forces MMA to evaluate the expression as a number, if it can! By the way, this integral is called the "complete elliptic integral of the first kind".

(c) A real grandfather clock has a length of about half a meter, and you pull the pendulum, oh, about 10 cm to the side. When designing the clock for practical use, would the clockmaker be safe in making the "small angle" approximation? (What we mean here is to justify your answer with a calculation that tells you whether this clock would be annoying or useful to have in your house! Can you be concrete about what you would consider "annoying" about a clock?)

5. Two large solid spheres, each with mass M, are fixed in place a distance 2l apart, as shown. A small ball of mass m is constrained to move along the x-axis, shown as a dashed line. (Let x = 0 represent the point on the axis directly between the spheres, with +x to the right.)

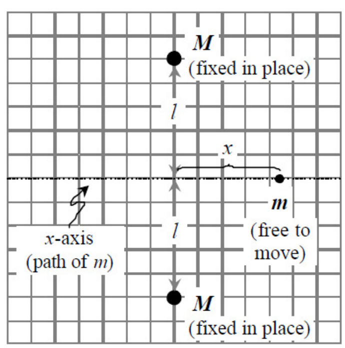

Figure 3:

(a) Compute the net force on the ball by both large spheres, and write down a differential equation that governs the motion of the small ball.

(b) Is the net force exerted on the small ball a restoring force? Explain how you can tell from your differential equation above? Under what limiting conditions for x is it possible to say that the motion of the mass m can be approximated as simple harmonic motion? (For large x? For small x? Large or small compared to what?) In this limiting case, determine an expression for the period of motion in terms of the given parameters. Explain your reasoning.

6. An ideal (massless) spring with force constant k is used to hang a crate from the ceiling, as shown (left, below). Suppose that the spring were treated as two separate springs (1 and 2) connected end-to-end. (See Figure) Treat each spring as having its own spring constant (k_1 or k_2) given by Hooke's law, $|F_i| = k_i x_i$, where $|F_i|$ is the magnitude of the force exerted on (or by) spring i when the length of that spring *changes* by an amount x_i.

(a) Relate the magnitudes of the forces F_1 and F_2 exerted by each spring individually, and the magnitude of the force F exerted on the crate. Explain. (Hint: Sketch separate free body diagrams for the springs and the crate.) Then, find the relationship between the stretches x_1 and x_2 of the individual springs and the stretch x of the original (single) spring. Explain your reasoning. Based on these results, determine an expression for the spring constant of the original spring (k) in terms of the spring constants (k_1 and k_2) of the two individual " spring pieces". Based on this problem, if you chop an ideal spring in half, what happens to its spring constant? Show all work.

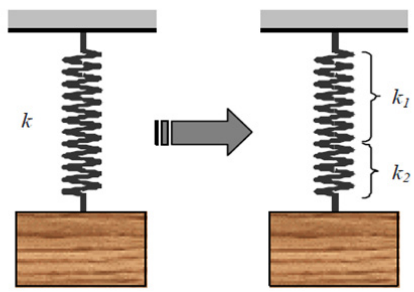

Figure 4:

(b) Now consider the case in which you replace the original spring with two new springs (3 and 4) that connect directly to the crate and to the ceiling. (See Figure.) Assume that the crate ends up at the same equilibrium height in this new situation as when there was just a single spring (k). Following the same logic as you did above, determine an expression for the spring constant k of the original spring in terms of the spring constants k_3 and k_4 of the new springs. Explain your reasoning.

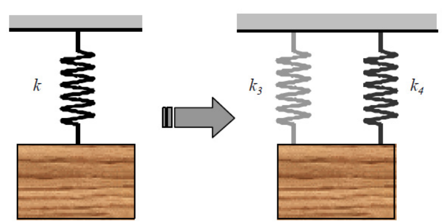

Figure 5:

NOTE: There is an extra-credit survey online - just go to our course main page, you'll see it linked prominently on our "Special notes" section, under the current fun picture. Or go directly to

http://www.colorado.edu/physics/phys2210/phys2210_sp11/preflights/survey_midterm_2210.html

We really appreciate your feedback, it helps us make this a better course!