Homework 1

(Due Date: Start of class on Thurs. Jan 19)

NOTE: Show your work neatly, explain what you are doing. Correct answers, for which we cannot follow the work, are worth no credit! All lettered sub-parts are worth 4 points each, unless otherwise specified.

1. Obtain a computational tool and start using it (4 pts):

Please install Mathematica 8.0 on your computer, or find a computer that you can regularly use that has it installed already (e.g. in the computer room). Other software, e.g. Python with Numpy, is also OK, but Mathematica is what physics faculty in future courses will probably assume you have some familiarity with.

As a registered CU student, you can obtain a (free) student license for Mathematica: `http://oit.colorado.edu/software-hardware/site-licenses/mathematica`. Students interested in using Python can download the Enthought Python Distribution for free: `http://www.enthought.com/products/edudownload.php`. *All future problem sets will require some use of a computational tool.* If you have never used Mathematica before, you might want to watch a couple of short (5-6 minute) screencasts developed for this class: `http://www.youtube.com/compphysatcu`

Getting help: Mathematica users may contact either Prof. Pollock or Dr. Caballero for assistance. Python users may Dr. Caballero for assistance.

(a) Once you download the computational tool of your choice, try using it as a calculator. Search online or use built-in help for a few of your computational tool's built-in functions. Perform some simple operations using these built-in functions. Print out your code and include this print-out as part of your homework set.

2. Mass density of a star (12 pts: 4+4+4):

(a) A very simplistic model of a gaseous star might give you the volume mass density, ρ as a function of radius, r as

$$\rho(r) = \begin{cases} \rho_0 e^{-r/R_0} & r < R \\ 0 & r \geq R \end{cases} \tag{1}$$

Just looking at the expression, try to give some simple, physical interpretation of the three parameters ρ_0, R_0, and R. What are their units? What do they mean, or tell you, physically?

(b) Make a simple sketch of the mass density as a function of radius. (Your axes, where possible, should show how ρ_0, R_0, and R are involved)

(c) An even more simplistic model might instead give

$$\rho(r) = \begin{cases} a/r & r < R \\ 0 & r \geq R \end{cases} \tag{2}$$

In this case, can you compute the total mass of the star in terms of the two parameters a and R? (If so, do it! If not, explain why not.) What are the units of a?

3. *A couple of math/calculus review questions* (12 pts: 4+4+4)

(a)

$$\int dx \frac{4x}{(x^4 + 2(bx)^2 + b^4)^{3/2}} \tag{3}$$

(where b is a known constant. Note: this is an indefinite integral)

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(b)

$$\frac{d}{dy}\int_c^y dx f(x) \qquad (4)$$

(where $f(x)$ is some given, known (well behaved) function of x and c is a constant.)

(c)

$$\frac{d}{dy}\int_0^2 dx(\log x + \log y) \qquad (5)$$

4. (4 pts) Consider an arbitrary vector $\vec{A}$; and let the unit vector $\vec{e}$ point along some fixed coordinate direction.

Using words and figures, explain the geometrical significance of the two two terms in the following vector expansion: (You do not have to prove this relation, you just have to explain what it means!)

$$\vec{A} = \vec{e}(\vec{A}\cdot\vec{e}) + \vec{e}\times(\vec{A}\times\vec{e}) \qquad (6)$$

5. *And lastly, a Phys 1110 review question:* (12 pts: 4+4+4)

You're at the park; it's just finished snowing. You're trying to pull your kid sister on a sled (which together have mass m) as fast as possible. The sled is old, so there's some small, constant coefficient of kinetic friction μ_K between the sled bottom and the snow.

(a) Assuming you can pull (at any angle you want!) with some given, fixed arm-force $|\vec{F}_{\text{arm}}|$, at what angle from the horizontal should you pull to accelerate your sister and the sled forward as fast as possible?

(b) After you find the angle - figure out a formula for this maximum possible forward acceleration for your sister and the sled.

Your answers for both angle and acceleration should be in terms of *just* the given constants m, μ_K, and/or $|\vec{F}_{\text{arm}}|$ (and of course g).

(c) Explicitly check both your answers by considering units, and also examining the limit of $\mu_K \to 0$. Here is a much subtler question, which you might find challenging: can you still make sense of your answer in the limit of very *large* μ_K? Explain!

Notes: Remember that we model kinetic friction as being simply proportional to the normal force, $|\vec{F}_{\text{arm}}| = \mu_K|\vec{N}|$. Of course in this problem you need to be careful, the normal force is NOT the weight of the sled system - draw a free body diagram!)

Extra credit (up to 4 bonus points, but won't count off if you don't do it!).

Use your knowledge of the dot product to figure out the angle between the diagonal of one face of a cube and the "body diagonal" of the cube. *Hint: Let your cube have side length one, with one corner at the origin. Can you write down simple expressions for the vectors that represent a face diagonal, and the body diagonal?*