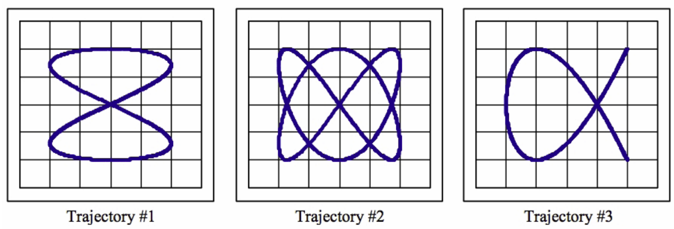
Homework 11

(Due Date: Start of class on Thurs. Apr 5)

IMPORTANT! As part of this homework (i.e. before Apr 5) ONE member of your final project team needs to go to `http://www.colorado.edu/physics/phys2210/preflights/project_v1.html` and fill in the brief form. If you have partners, please coordinate with each other - everyone on the team gets one homework problem credit for this.

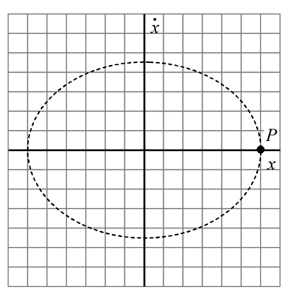
1. Consider the motion of a 2D non-isotropic oscillator, in which the spring constant in the x direction (k_x) is not equal to the spring constant in the y direction (k_y). Each trajectory below depicts the possible motion of a unique non-isotropic oscillator.



(a) determine the (simplest possible) ratio of ω_x /ω_y. Explain your reasoning.

(b) Use your favorite computational environment to generate the three Lissajous plots shown above (ParametricPlot may be handy in Mathematica) We said in class that you can always choose your "x" phase angle to be zero. Can you do that for all 3 figures? Briefly, explain why or why not.

2. The phase space trajectory of an undamped oscillator is shown below. In the diagram, each division along the position axis corresponds to 0.1 m; along the velocity axis, 0.10 m/s.



(a) First, what is the angular frequency ω_0 of the undamped oscillator? Explain how you can tell. Next - a retarding force is now applied to the oscillator for which the damping constant (using Taylor's notation, introduced in Eq. 5.28) is equal to $\beta = 0.0644\omega_0$. By what factor does the amplitude change after a single oscillation? After 10 cycles? Show all work.

(b) On the basis of your results above, carefully sketch the phase space plot for the first cycle of the motion of the damped oscillator, starting at point P. *There is a larger figure on the last page of this homework set that you can print out to do your sketching.*

(c) If this system corresponded to a real pendulum, which effect of damping would be more noticeable - the change of the period or the decrease of the amplitude? Justify your opinion.

3. *Energy considerations in a 1D simple harmonic oscillator:*

(a) Consider a simple harmonic oscillator with period τ. Let $\langle f \rangle$ denote the average value of a function $f(t)$ averaged over one complete cycle:

$$\langle f \rangle = \frac{1}{\tau} \int_0^\tau f(t)dt. \tag{1}$$

Prove that $\langle T \rangle = \langle U \rangle = E/2$, where E is the total energy of the oscillator, T the kinetic energy and U the potential energy.

Hint Start by proving the more general, and extremely useful, result that $\langle \sin^2(\omega t - \delta) \rangle = \langle \cos^2(\omega t - \delta) \rangle = 1/2$. Explain why these two results are almost obvious, then prove them by using trig identities to rewrite $\sin^2\theta$ and $\cos^2\theta$ in terms of $\cos(2\theta)$.

(b) If we add a damping force $F_{damp} = -b\dot{x}$, find the rate of change of the energy $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$ by straightforward differentiation (getting a simple result in terms of x, $\dot{x}$, and/or $\ddot{x}$) and then with the help of Taylor 5.24, show that it is exactly the rate at which work is done by F_{damp}.

(c) *The rest is pure extra credit, worth up to 4 points!*

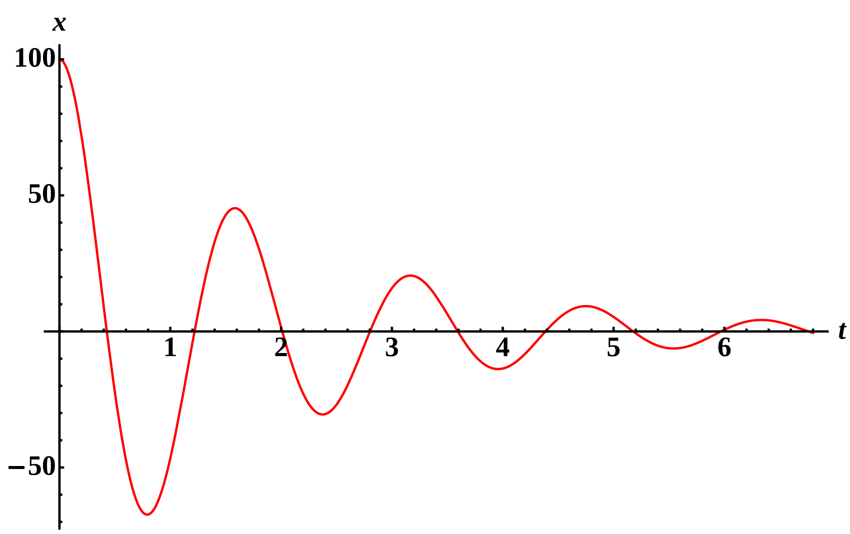
The position of an overdamped oscillator is given by Eq. 5.40 in Taylor's text. Find the constants C_1 and C_2 in terms of the initial position x_0 and velocity v_0. Then, sketch the behavior of $x(t)$ for the two separate cases $x_0 = 0$, and $v_0 = 0$. Finally, show that if you let $\beta = 0$, your solution for x(t) matches the correct solution for undamped motion.

I find this rather remarkable, since the solution you started from is overdamped, it wasn't supposed to work for the undamped case?! As Taylor puts it, the math is sometimes cleverer than we are!

4. In the following pair of problems you will compare our model for a damped harmonic oscillator to real data collected from a video of a spring-mass system immersed in viscous oil. The video from which this position vs time data is collected is here: http://www.youtube.com/watch?v=ZYFVKZPut9w. Data was obtained using free video tracking software (Tracker) and saved in a comma-separated variables (CSV) file.

First, you will describe a few properties of a damped oscillator to assist you in your analysis of the data.

(a) Write down the differential equation for a damped harmonic oscillator with undamped natural frequency, ω_0, and damping parameter, β. Write down the general solution for this differential equation for the underdamped case ($\beta < \omega_0$). Then, consider the following sketch of a solution to this differential equation for a particular choice of β and ω_0 when the oscillator was displaced and let go.



You can obtain an estimate for $\omega_1 = \sqrt{\omega_0^2 - \beta^2}$ using the time between successive zero crossing. Estimate ω_1 in the figure above and explain how you obtained your result.

(b) You can also obtain an estimate for β using successive maxima or minima from the plot. Estimate β and ω_0. Explain how you obtained your result.

(c) If ω_0 were kept constant but β were increased (but still below ω_0), sketch how the plot above would change. What physical change to a spring-mass immersed in oil can be done to increase β? Answer both questions for the case where ω_0 is kept constant but β is decreased. What property of the sketch does β appear to control?

(d) If β were kept constant but ω_0 were decreased (but still above β), sketch how the plot above would change. What physical change to a spring-mass immersed in oil can be done to decrease ω_0? Answer both questions for the case where β is kept constant but ω_0 is increased. What property of the sketch does ω_0 appear to control?

5. You will now analyze a real spring-mass system immersed in viscous oil for which position versus time data was collected for the oscillator.

(a) The position versus time data appears in a comma-separated variables (CSV) file on our course home page and also our D2L page. Importing this data is straight-forward in Mathematica using the `Import` command. You'll need to store the data in a variable, so you can plot it.

For example, `data = Import["/file/to/datafile.csv"]` will import the values in the CSV file into the variable `data`. (You will use Insert → Filepath to generate the right file path on your computer - if you need a little more help, a video describing how to import data into Mathematica is located here: http://youtu.be/MmS3JNk7JE4 .)

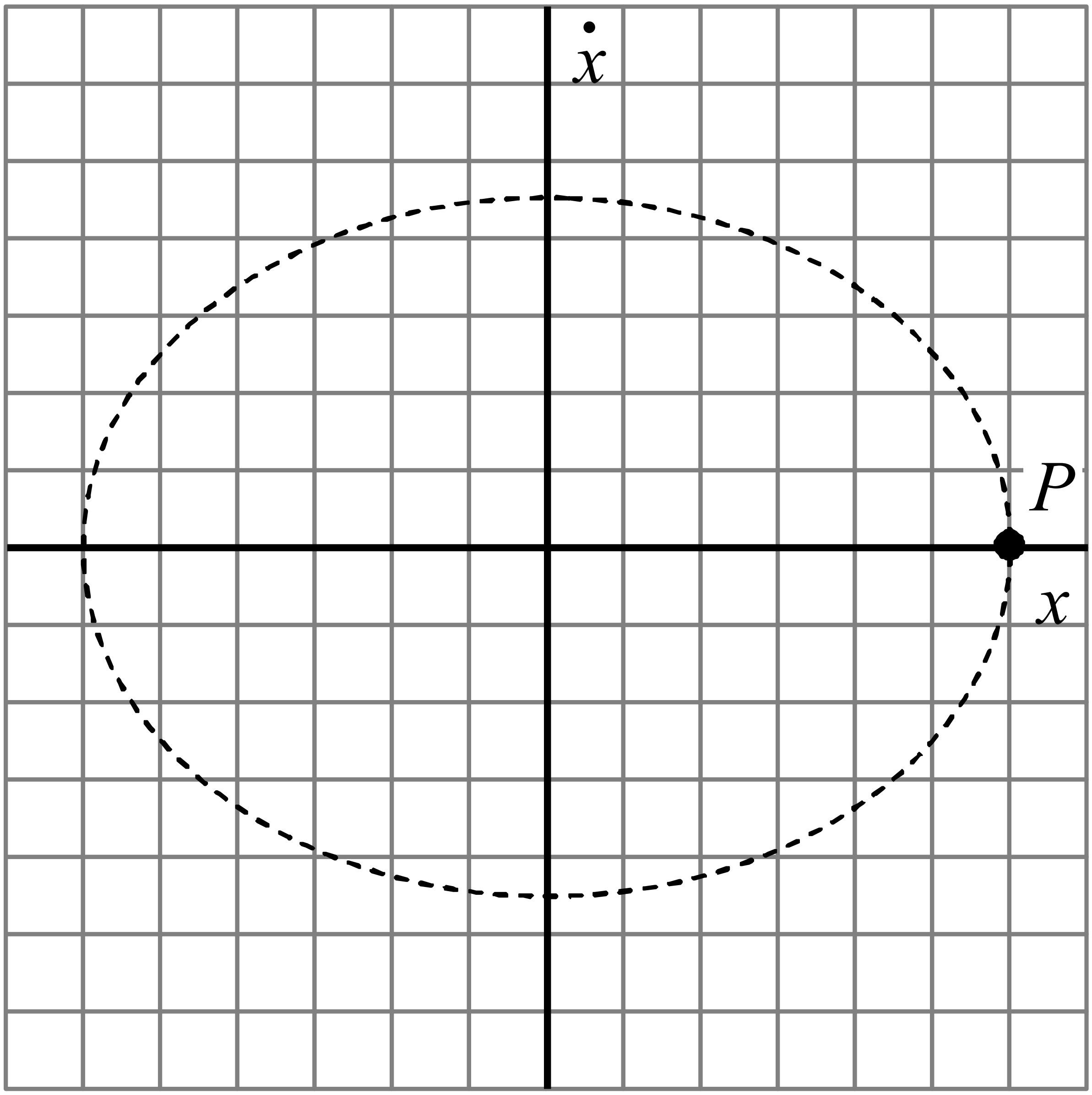
Plot this data using the `ListPlot` command,
`experiment = ListPlot[data[[All, {1, 2}]], PlotRange -> All]`.
You are storing this plot (as `experiment`) for later, so you can display the experimental data and the results from your model on the same plot.

(b) Using the plot of the data, estimate values for ω_1, β, and ω_0. Determine the initial position and initial velocity for the oscillator. This oscillator was displaced and released from rest.

(c) Plot the particular solution to damped harmonic oscillator for your model parameters (ω_1, β, and ω_0) and initial conditions. You should store this plot (e.g., `model = Plot[...]`).

(d) Plot the result of your model and the experimental data on the same axes using the `Show` command. This is why you stored the plots earlier (e.g., `Show[experiment, model]`). How does your model match the experimental data? Can you tweak your model parameters to make the fit better?

(e) Why did we do this? Experimental physicists collect data and often attempt to fit a model of that system to their data. It's likely that you got pretty good but not great agreement with the data that was collected. Can you identify at least 3 aspects of the physical system (spring mass immersed in oil) that might have improved the model? Can you identify at least 2 aspects of the data collection procedure (video tracking software) that might have helped you better estimate your model parameters (ω_1, β, and ω_0)?



$\dot{x}$
P
x