Homework 12

(Due Date: Start of class on Thurs. Apr 12)

1. Go back to midterm #2, and pick any "lettered" portion (like 7-i or whatever) where you lost more than $\sim 1/3$ of the points. Look it over, redo the problem, and explain what pieces you were missing and why you missed them. Solutions are posted, so we're not so much interested in "the right answer" as your reflections about what you did wrong and why. (For a couple of you, there were no sub-parts where you lost this much credit - just let the grader know.)

2. We often ask you to *check* your answer. "Check" (in this sense) doesn't mean "redo the problem carefully", nor to invent and solve a related problem. It means think about a basic physical result you know must be true *independent* of working out the formula, and then you check that your formulaic answer agrees with that expectation. (E.g, the units are as expected, or the behavior at extreme values of some parameter does what you expect...) To be useful, you must *already know* what that behavior should be (e.g. if friction goes up, terminal velocity should go down. Or, if you get far away, a force should vanish. That sort of thing)

Let's practice! In each case below, I will pose an abbreviated problem and a proposed answer. Your task is NOT to solve the problem(!!) but rather, to simply CHECK the given answer. If you can think of more than one way, all the better. In each case, briefly comment! If the check does NOT work out, does it suggest to you what MIGHT be wrong, where you might look more carefully if you needed to get the answer right?

(a) In a tragic accident, a car (mass m) plunges into an icy lake. The water produces a linear drag force on the car $\vec{f}_d = -b\vec{v}$. Find the velocity of the car as a function of time. The student's response after several lines of careful calculations was:

$$v(t) = v_0 e^{bt/m} \tag{1}$$

(Remember, you are NOT being asked to solve this problem!! You only need to make two **checks** of this answer to see if it is reasonable. If your checks indicate it is not reasonable, do they suggest any possible fix, or something you might look into?)

(b) A formula for potential energy U(x,y,z) was given, and the question asked for the resulting force. The student's response after several lines of careful calculations was:

$$\vec{F}(x,y,z) = cx\ \hat{j} \tag{2}$$

with the constant $c = 4.9/s^2$ (Again, don't solve the problem, just check the solution - and then, can you make any suggestions about what might have gone wrong even without having solved, or in this case even seen, the original problem?)

(c) A space platform is in the shape of an annulus: a flat ring of uniform mass per unit area of σ, with different inner and outer radii given in terms of some distance R. A student on an exam works out the gravitational potential energy of a test mass m at position z above the central axis, and gets

$$U(z) = -2\pi G\sigma(\sqrt{4R^2 + z^2} + \sqrt{R^2 + z^2}) \tag{3}$$

One final time, don't redo the problem to check it - just check this given solution. (I can think of at least two obvious ways, each of which catches a different small but important mistake.)

3. (a) Solve Boas Problem 8.6.12 (That's chapter 8, section 6, problem 12, on page 423). We just want the general solution here.

Please also tell us *explicitly* if the associated (complementary) homogenous problem is (choose one) {over-damped, under-damped, undamped, critically damped} How do you tell?

(b) Now, given that the system starts with $y(0) = 0$, $y'(0) = 0$, solve that problem completely. There should be no undetermined coefficients left!

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(c) Sketch the homogeneous *piece* of the solution in part b) above (by itself). Also sketch the particular solution *piece* (by itself). Finally add them to sketch the full solution - noting any important aspects of the sketch. (Don't worry about precision, this is a sketch, not a plot. We want to know the *features*.) Then, use a computer to plot the actual solution. Print it out and discuss - in particular, discuss if it did not exactly match with your sketch. (The differences may or may not be subtle, depending on the care of your initial sketch)

(d) Let's change the problem slightly. Suppose the middle term (the"4D" term) is *quadratic* rather than *linear* drag. Write down the new ODE (don't use the "D" notation, just write it out as an ODE) This is no longer analytically solvable! But that needn't stop us - keeping all numerical coefficients the same as Boas had, just replacing linear with quadratic drag, use your favorite computational environment to solve and plot the new solution (with the same initial conditions). Did changing linear for quadratic drag change any qualitative aspects? Briefly, comment.

(e) Invent two *physics* problems, for which the ODEs are the ones used in parts a and d respectively. Be explicit - what are the values of all relevant physical parameters, in SI units? What physics would induce you to choose the ODE of part d instead of part a?

4. A series LRC circuit (Taylor Fig 5.10) is connected across the terminals of an AC power supply that produces a voltage $V(t) = V_0 e^{i\omega t}$. The "equation of motion" for the charge $q(t)$ across the capacitor is as follows:

$$L\ddot{q} + R\dot{q} + q\frac{1}{C} = V_0 e^{i\omega t} \tag{4}$$

The above differential equation has a steady-state solution which can be written as:
$q(t) = q_0 e^{i(\omega t+\delta)}$ [Note: The parameters q_0 and δ are actually functions of ω, the frequency of the AC power supply. However, in this problem you will *not* have to solve for or write out these functions in full!]

(a) In terms of q_0, ω, δ and the relevant coefficients from the differential equation, write down the following functions:

i. the potential difference $\Delta V_C(t)$ across the capacitor

ii. the potential difference $\Delta V_R(t)$ across the resistor

iii. the potential difference $\Delta V_L(t)$ across the inductor

iv. Finally, determine the smallest positive values of α, β, and γ (in radians) that satisfy the following Euler relations: $e^{i\alpha} = i$, $e^{i\beta} = -1$, $e^{i\gamma} = -i$

(b) Using your results from part iv, rewrite the functions in part i-iii so that each function can be written as a positive real number times a complex exponential function. Use your rewritten functions to answer the following questions.

i. What is the phase difference between $\Delta V_C(t)$ and $\Delta V_R(t)$?
Do the peaks of $\Delta V_C(t)$ come just *before* ("leads") or just *after* ("lags") the peak of $\Delta V_R(t)$? (Show/explain your reasoning)

ii. What is the phase difference between $\Delta V_R(t)$ and $\Delta V_L(t)$?
Does $\Delta V_R(t)$ lead or lag $\Delta V_L(t)$?

(c) Assuming for simplicity that $\delta = 0$, make a crude sketch on one graph of $\Delta V_R(t)$, and $\Delta V_C(t)$ being very careful to identify which is which. Does your sketch agree with your claim in part b about "leading" or "lagging"? (Briefly, comment)

5. In section 5.6, Taylor states that the amplitude of an oscillator subject to a sinusoidal driving force is

$$A^2 = \frac{f_0^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$

(a) Find an expression for ω such that the amplitude A is maximized. That is, derive our text's Equation 5.73. (*Hint:* Rather than differentiate and extremize the entire expression for $A(\omega)$, make your job easier by judiciously picking which part of that expression to differentiate and extremize.)

Then, use your expression to find the exact maximum amplitude, A_{max}.

Write a second expression for the amplitude A_0 which corresponds to $\omega = \omega_0$.

Briefly, comment on the difference between A_{max} and A_0.

(b) Open up the resonance PhET sim, found at

`http://phet.colorado.edu/en/simulation/resonance` Play around to see what the sim can do.

By investigating the sim in a variety of ways, clearly explain Taylor's Figure 5.19 in your own words, *referring explicitly to "experiments" or setups you made with the sim* to help you connect the figure to the behavior of the objects. (Please discuss briefly what effects "transients" have, and how you dealt with them in the sim)

Extra credit. Play with the sim a little more, and write down one additional (real, interesting) question that you have about something you notice!