Homework 2

(Due Date: Start of class on Thurs. Jan 26)

NOTE: Show your work and explain what you are doing. (Correct answers, for which we cannot follow the work, are worth no credit). As always, unless otherwise stated, it's 4 points per lettered sub-section.

1. In the great blizzard of '89, a rancher had to drop hay bales from an airplane to feed her cattle. Suppose the plane flew horizontally at a steady 120 km/hr, and dropped the bales from 60 m above the flat range. (Neglect air resistance in the first two parts of this question)

(a) If she wanted the bales of hay to land 20 m behind the cattle (so as to not hit them!), where should she push the bales out of the plane? (Clearly define a coordinate system so we understand your answer)

(b) What is the largest time error she could make while pushing the bales out of the plane, to ensure not hitting the cattle?

(c) If we did NOT neglect air resistance for the dropping hay, describe qualitatively (no detailed calculations, please!) what would change about your answers to both previous questions.

2. A soccer player kicks the ball with a velocity v_0 at an angle 45° (to reach maximum range). She wants to kick it in such a way that it barely passes on top of two opposite team players of height h.

(a) Show that if the separation between the two opponents is $d \leq \frac{v_0}{g}\sqrt{v_0^2 - ?gh}$, the soccer player succeeds. (What is the numerical value of the ? in this formula?)

(b) Use a computational tool of your choice (Mathematica or Python) to plot d as a function of v_0, setting $h = 2.0\ m$. To do this, write the equation as a function rather than "hard-coding" the equation into the plot function.
Additional help on writing functions in Mathematica is available here: http://youtu.be/1A4f91yMVhA

(c) Give a physical explanation of any major features in your plot.

(d) What are some benefits of writing functions to plot over "hard-coding" plots?

3. (8 pts) Trajectory of a particle. A particle moves in a two-dimensional orbit defined by

$$x(t) = \rho_0[1 + \cos(\omega t)] \quad (1)$$

$$y(t) = \rho_0[2 + \sin(\omega t)] \quad (2)$$

(a) Sketch the trajectory. Find the velocity and acceleration (as vectors, and also their magnitudes), and draw the corresponding velocity and acceleration vectors along various points of your trajectory. Discuss the results physically - can you relate your finding to what you know from previous courses? Finally: what would you have to change if you want the motion to go the other way around?

(b) Plot the trajectory using your favorite computational tool. For this plot, set $\rho_0 = 1$ and $\omega = 2\pi$. This is a called a "parametric plot".

(c) Prove (in general, not just for the above situation) that if velocity, $\vec{v}(t)$, of any particle has constant magnitude, then its acceleration is orthogonal to $\vec{v}(t)$. Is this result valid/relevant for the trajectory discussed in part a?
Hint There's a nice trick here - consider the time derivative of $|\vec{v}(t)|^2 = \vec{v} \cdot \vec{v}$.

Note: What you have proven in part c is quite general, and very useful. It explains why, e.g., $d\vec{\hat{r}}/dt$ *must point in the* $\vec{\hat{\phi}}$ *direction in polar coordinates. Do you see why?*

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4. Let's generalize the analysis you did in class (for the motion of a particle in polar coordinates) to *spherical* coordinates. The three unit vectors: $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ which describe spherical coordinates can be written as:

$$\hat{r} = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k}, \tag{3}$$

$$\hat{\theta} = \cos\theta\cos\phi\hat{i} + \cos\theta\sin\phi\hat{j} - \sin\theta\hat{k}, \tag{4}$$

$$\hat{\phi} = -\sin\phi\hat{i} + \cos\phi\hat{j}. \tag{5}$$

(a) First, investigate that the definitions given in Eqs.(**??**-**??**) make sense. Define in your own words what orthonormal vectors are, and then check to see if these vectors $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are indeed orthonormal. Then, convince yourself (and the grader) that e.g. at least the x-component of $\hat{r}$ is correct, using a simple geometric picture. (Taylor's Fig 4.16, or Boas Fig 4.5 should help)

(b) If we constrain the particle to move with $\phi = 0$, state in simple words what this means in terms of the particle motion. Sketch the three spherical unit vectors at some point $\phi = 0$ and $r = R$ for some particular (nonzero) angle θ of your choice.

(c) Show that the velocity of any particle in spherical coordinates is given by:

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\hat{\phi} \tag{6}$$

(d) Let's switch to *cylindrical* polar coordinates. Find expressions for the cylindrical unit vectors ($\hat{\rho}, \hat{\phi}$, and $\hat{z}$) in terms of Cartesian coordinates ($\hat{\imath}, \hat{\jmath}, \hat{k}$) Then, take a derivative with respect to time to find $d\hat{\rho}/dt$ in cylindrical coordinates.

5. Romeo and Juliet are in love, but in our version of this story, the characters are a little different than tradition holds. Let R(t) = Romeo's feelings for Juliet at time t. Large positive values means deep love, large negative values means strong hatred. Similarly, Juliet's feelings for Romeo will be characterized by J(t). Now let's consider a "model for the lovers" governed by the general linear coupled differential equations,

$$\frac{dR(t)}{dt} = aR(t) + bJ(t) \tag{7}$$

$$\frac{dJ(t)}{dt} = cR(t) + dJ(t) \tag{8}$$

If a>0 and b=0, I might describe Romeo as "in love with his love". His emotional state has nothing to do with Juliet's feelings at all; he just falls more in love the more in love he is. . . a very narcissistic individual!

(a) Let's assume none of the parameters are zero. Consider *all four* possible permutations of signs of a and b, and for each, describe the emotional character of Romeo. (For instance, one of the combinations might be called a "emotionally distant jerk", which one? But maybe you have a better or totally different description/interpretation of this same combination!)

(b) Suppose that Juliet has the exact same *emotional character* as Romeo. (I'm not saying J(t)=R(t), but they respond to each other in analogous ways) What would this say about parameters c and d in terms of a and b? In this case are there values for the parameters a and b that would lead to a stable situation? And, is there any stable situation besides "complete mutual indifference", i.e. R=J=0?

You might be wondering why we are asking you to think about such a whimsical question like this in Phys 2210. This question is really just about developing intuitions about ODEs. Physics is full of ODEs! You will need such intuitions in almost every branch of physics. I got the question from Strogatz' "Nonlinear Dynamics and Chaos" text.