Homework 3

(Due Date: Start of class on Thurs. Feb 2)

NOTE: Show your work and explain what you are doing. (Correct answers, for which we cannot follow the work, are worth no credit). As always, unless otherwise stated, it's 4 points per part

1. Consider a ball that moves vertically under the influences of both gravity and air resistance. For the purposes of this problem, take vertically upward as the positive direction. For each equation of motion below, determine whether that equation applies to (i) a situation in which the ball moves upward, (ii) a situation in which the ball moves downward, (iii) either of these, or (iv) neither of these. Explain your reasoning for each case. (4 pts total on this one)

a) $m(dv_y/dt) = -mg - c_1 v_y$, b) $m(dv_y/dt) = -mg + c_1 v_y$

c) $m(dv_y/dt) = -mg - c_2 v_y^2$, d) $m(dv_y/dt) = -mg + c_2 v_y^2$

2. You're hanging out by a campfire after acing the first Phys 2210 exam. You notice a tiny piece of fly ash that is ejected from the fire vertically upward with initial speed v_0. You remember the motion of objects that are very small can be modeled to a very good approximation by linear drag. Measuring the position of the fly ash, y, upward from the point of release:

(a) Find the time for the fly ash to reach its highest point and its position y_{max} at that time. (Note that velocity $v_y(t)$ and position $y(t)$ have been derived in the lecture notes, and the text. Of course, if you want to rederive them yourself, that's a very good idea, but not required for credit)

(b) Show that as the drag coefficient approaches zero your answer in part b reduces to the well known freshman physics result $y_{max} = \frac{v_o^2}{2g}$. **Hint:** If the drag coefficient is very small the terminal velocity is very big so v_0/v_{ter} is very small. Use the Taylor expansion $\ln(1+\epsilon) \approx \epsilon - \epsilon^2/2 + \ldots$ for $\epsilon \ll 1$.

3. A grain of pollen (mass m) slides down a smooth, flat, tilted solar panel (tilted at θ from horizontal) under the influence of gravity.

(a) If the motion is resisted by a linear drag force $f = kmv$ find an expression for how far down the plane the grain slides in time t, assuming it is released from rest.

Hint: Draw a free body diagram to start!

(b) Show that as the drag coefficient becomes very small, your answer in part a reduces to a simple freshman physics result (which is what?) **Hint:** Here, I claim the useful Taylor expansion you'll want will be $e^x \approx 1 + x + x^2/2 + \ldots$, for $x \ll 1$ And yes, you'll really need to go all the way out to that $x^2/2$ term!

For you to think about. You do not have to write this up for credit, but we'll revisit this idea again this term, it's very important! The power x in e^x must be unitless. "Drag coefficient" has units. So it can not be exactly correct to say the "drag coefficient becomes small", without answering "small compared to what"? What exactly is it that is really the "small thing, x" in this case?

4. In Season 1 Episode 4 of Mythbusters (originally aired in 2003), Jamie and Adam considered the myth that a penny dropped from the Empire State building can kill a person walking on the sidewalk below. You can watch a clip from this episode here: http://youtu.be/PHxvMLoKRWg. Let's do our own investigation!

(a) Look up the size and mass of a penny. Using the drag coefficients given by Taylor pp 44-45, which means we are assuming a spherical penny ◡̈, find the terminal speed of a dropped penny, taking into account both linear and quadratic terms together. Then, write down the differential equation for the motion of a falling penny (Keep both linear and quadratic terms. Clearly articulate your sign conventions)

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(b) Taylor Eq 2.7 compares quadratic and linear drag forces. For what range of speeds will (i) the linear term of air resistance dominate over the quadratic term? (ii) the quadratic term dominate over the linear term? Discuss the relative importance of linear and quadratic drag on this penny as it falls. If you had to pick just ONE term (linear or quadratic) to use, which would you use, and why?

(c) If we keep both linear and quadratic terms, we have a non-linear differential equation which we can only solve numerically. Use your favorite numerical differential equation solver to determine and plot velocity and position of the penny as a function of time, as it falls from the top of the Empire State building (381 m). Also find the time it hits the ground, and the speed with which it hits. Include your code and plots with your homework.
Mathematica users might find this screencast on using `NDSolve` (Mathematica's built-in numerical differential equations solver) helpful: http://youtu.be/zKO6v0w0KdI
Be Elegant: *You could use Mathematica's* `EventLocator` *method to stop the integration when the penny hits the ground. Helpful link for that:* http://goo.gl/Mkz3i

(d) Compare the result for "final velocity" from your numerical results (part c) with what you got in part a, and also what you get by assuming JUST the one dominant drag term you chose in part b. Comment.

(e) Now for the interesting part of all this. Could this penny kill someone? Use some basic freshman physics (e.g., assume your skull applies a constant force to stop the penny during impact) and make some rough estimates to discuss whether you think the myth is busted or not (Human skulls are about 5 mm thick, and I figure if the penny drives any significant fraction of that distance into your skull, you're in trouble. Also, Wikipedia: "hydrostatic shock" says pressure on a skull of $10^7 N/m^2$ can seriously injure you.)

5. Consider a ball thrown at an angle θ above the horizontal ground with an initial speed v_0 in a medium with linear drag. For numerical solutions, computational tools don't deal well with unknown symbols (!), so let's consider a particular case where $v_0 = v_{ter}$ and $v_{ter}^2/g = 1$. We know that in vacuum the maximum range is at $\theta = \pi/4$. Let's try to estimate the maximum range (and angle) when we include air resistance:

(a) Plot Eq.(2.37) in Taylor for different values of θ and try by inspection to figure out the angle at which the range is maximum. (With the assumptions above, most unit-full quantities become "one", but do be careful of the fact that e.g. $v_{0,x}$ is $v_0 \cos(\theta)$, not v_0), etc.

(b) Use a root finder to find the range when $\theta = \pi/4$. Any root finder will require you to make a guess for the root. This is what is called the "neighborhood of the root" or "bracketing the root". A screencast showing how to use Mathematica to find roots is available here: http://youtu.be/673IQ6Z-6Yc

(c) Repeat for different values of θ (homing in on a small range near the angle you estimated in part a). Continue until you know the maximum range and the corresponding angle to two significant figures. Connect to point a) and compare with the ideal vacuum values. Does your answer seem reasonable? Briefly, explain.
Extra credit Check what happens in the limit $v_0 \gg v_{ter}$ (say $v_0 = 100 v_{ter}$) and $v_0 \ll v_{ter}$ (say $v_0 = 0.01 v_{ter}$). Discuss if your results make sense. (+4 points)

Note: Judiciously use "print selection" on a minimalist subset of your MMA notebook so we can see your results, and discussion, without having to wade through pages of preliminary plots and calculations.