Homework 4

(Due Date: Start of class on Thurs. Feb 9)

1. A car is braking hard. There are two significant resistive forces acting on it, a quadratic (cv^2) air drag, and a constant (μmg) frictional force. If you are interested in finding v(x) (rather than v(t)), there is a commonly used method known as the "v dv/dx rule", which uses the chain rule to rewrite $\dot{v} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$.

(a) Write down the equation of motion for $\dot{v} = f(v)$ and use the "v dv/dx" rule to solve the equation of motion directly for v(x), and show that the distance the car needs for a full stop is:

$$x_{max} = \frac{A^2}{2\mu g} \ln\left(\frac{A^2 + v_0^2}{A^2}\right) \tag{1}$$

here μ is the friction coefficient. What is the constant A in this case (in terms of given parameters in the differential equation?)

(b) My car has mass $m \approx 1600$ kg, and (I estimate) $c \approx 0.6$ kg/m. I'm driving down I-70 towards Moab, traveling 75 mph, when I see a stopped car on the highway 300 m front of me. I slam on the brakes! Assuming a friction coefficient of $\mu = 0.7$, find my stopping distance (do I crash, or am I safe?), and then compare it to what it would have been in the absence of any air drag (i.e. just the road friction alone). Briefly, comment. If I'd been driving a sports car, traveling at twice this speed (but with all other parameters the same) would air drag have made a more significant difference? Again, comment!

(c) Car engines are complicated, they are not simple "constant force" devices, but I estimate that at highway speeds my car might apply a fairly steady forward force against the road of about 1500 N. If the stopped car managed to pull off to the side of the highway when I had slowed to 30 mph and I immediately started re-accelerating with this constant maximum forward force, how much highway distance would I need to get my car back up to 75 mph? (Include the quadratic air resistance, of course!) *Hint: the v dv/dx rule may be useful again*

(d) Make a rough sketch (by hand, not with Mathematica!) of my car's velocity $v(t)$ described by the "story" of parts b and c. *Note: that's v(t), not v(x). Don't calculate it, just sketch what it should look like.* Comment briefly on interesting features of your graph (e.g., signs of slope, signs of concavity, interesting points...)

2. Recall from special relativity that for a particle moving at a relativistic speed, v, the energy $E = \gamma mc^2$, where $\gamma = \dfrac{c}{\sqrt{c^2 - v^2}}$.

(a) In the non-relativistic limit, find the first two nonzero terms in the series expansion of the energy by using the Taylor expansion of $\dfrac{1}{\sqrt{1-x}}$. What is the non-relativistic limit? What is x in this case? About what point are you expanding? What is the second term? When Einstein did what you just did (100 years ago), I think he must have smiled a big smile at that second term - why?

(b) How can an expansion of $\dfrac{1}{\sqrt{x}}$ be made to agree with your answers to part a? Comment.
Hint: you won't be expanding around x = 0 this time! About what point should you expand?

(c) I claim that an expansion of $\dfrac{1}{\sqrt{x}}$ about $x = 0$ makes little sense. Why? Interpret this limit physically.

3. Find the regulation size (and mass, while you're at it) for a soccer ball.

(a) The drag coefficient c_D for a soccer ball is about 0.25. (Note: the c in cv^2 is given by $\frac{1}{2}c_D\rho D^2$) Calculate the numerical value of the quadratic drag constant "c" for a soccer ball traveling in air at STP, and write down the equations of motion required to solve for x(t) and y(t) with (just) quadratic drag.

CONTINUED

(b) On our course calendar, right next to this homework, you can find a sample Mathematica notebook that solves for trajectory with *linear air drag.* Modify that code so that you have *quadratic* drag. For simplicity and concreteness, set the kick angle to 45 degrees, and pick an initial speed of 90 mi/hr (converted to SI metric, of course) Plot the trajectory. What is the ideal (drag-free) range in this case, and what does your code tell you the range is with quadratic drag? (Don't hunt for the optimum range like last week, stick with a 45 degree angle) Also, calculate what fraction of the initial *speed* is lost from when you first hit the ball (at 45 degrees) to when it lands. (Where does the lost energy go?)

(c) A few years ago, there was big controversy when the Soccer Federation banned international soccer competitions in cities above 2500 m elevation (This was out of concern about players' health, and has since been eliminated.) But, what about the impact on the physics of the game? Use your code to estimate how much farther that same kick would take a soccer ball in La Paz, Bolivia (elevation, 3000 m) compared to a game at sea level (which your code was assuming.) Do you think this difference might be measurable/noticeable? *Note: The air density at 3000 m is about 70% what it is at sea level.*

4. Consider a familiar horizontal spring-mass system. Recall that the the solutions for both the velocity and position of the mass are oscillatory.

(a) Write down the second-order differential equation which describes the position of the mass. Then, as we did in the numerical integration tutorial in class, write this differential equation as two first order differential equations (one for dx/dt, one for dv/dt) *(That in-class Tutorial is also available for review/download on this week's course calendar)*

(b) Assume the oscillator starts from rest at $x = +1$ m, with $m = 0.1$ kg and $k = 10$ N/m. Recalling that the period of oscillations is given by $T = 2\pi\sqrt{m/k}$, let's pick the time step for our numerical integration to be 10% of T (so 1 period has 10 time steps). Using the computational tool of your choice, you will be writing a `for` loop to numerically integrate these equations for several periods. (For help writing `for` loops in Mathematica, review the numerical integration tutorial, use the documentation or check out this screencast: `http://youtu.be/UpkcScYeQTc`.)
IMPORTANT:
We'd like you to write TWO codes: (code 1, "Euler-Cromer" method) Compute the force at an instant, update the velocity (using that force), then update the position (using that *updated* velocity!) and then repeat. Alternatively, (code 2, "Simple Euler method") Compute the force at an instant, update the position first, then update the velocity (and repeat.) See the difference? (Compare these two methods by using each of them to integrate the spring mass system and plot the position of the mass as a function of time.) What do you notice?
One anecdote claims the Euler-Cromer method was discovered by a high school physics student. The story is given in this paper.

(c) Try decreasing your integration time step by an order of magnitude. What changes do you have to make to your code? What happens to your solutions? When numerically integrating, what would help you pick the time step? (Why not, say, choose a billionth of a period in this case, to get super accuracy?)

(d) Several good integration algorithms are built in to a lot computational tools (e.g., Runge-Kutta, Dorman-Prince, etc.). Use one of these built-in integration methods to integrate the equations of motion and compare the results to those in part (b) or (c). Mathematica users can use NDSolve, a function which attempts to select the best method given the equation of motion. For help using NDSolve, check out this screencast: `http://youtu.be/zKO6v0w0KdI`. Notice, you don't have to pick a time step!