Homework 5

(Due Date: Start of class on Thurs. Feb 16.)

NOTE: Feb 16 is also the day of our first first exam! You may bring one side of one page of paper with your own *hand written* notes, if you like. Please read the LAST homework question first, so you know it's coming. This may require some planning on your part.

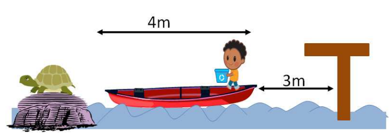
1. *Taylor series are the single most important and common approximation technique used throughout physics. We need to keep practicing with them!*

(a) Suppose $f(x) = 1/(1+x)$. Find an approximate expression for $f(x)$, going out to second order. Use it to estimate f(0.1) and f(5). In both cases, compare the exact result for $f(x)$ with your series approximation, going to first order (terms proportional to x) and also second order (x^2). Does your estimate improve at second order in both cases? Comment on the general criterion about x you would guess tells you when the series approach seems to be a fruitful one here.

(b) Now let $f(x) = (1-x)/(1+x)$. Taylor expand to approximate $f(x)$ out to second order. There are many ways to do this. First, use the usual formal formula for Taylor expansion. Another way is to take your series answer in part a, and multiply it by (1-x). Try it this way too, and use it to check yourself, both methods should agree to all orders! Which method do you prefer?

(c) Now let $f(x) = (a-x)/(a+x)$, where a is a constant with units, like $a = 5m$. Expand to second order. In this case, do NOT start "from scratch" blindly using the math formula. Instead, factor out "a", so that it looks *exactly* like what you had in part b, except the "thing you're expanding in" isn't x. What is it? For this part, what is now the general criterion on x that tells you when this series approach seems to be a fruitful one? Why? (And please do not say "x must be small" or "$x << 1$". Small compared to what? You cannot compare meters to numbers, or apples to oranges!) *(Once you understand this trick you will be spared having to ever painfully Taylor expand much of anything - the front flyleaf of your text will tell you how to expand almost anything you'll ever encounter)*

(d) Taylor's eqn 2.33 gives the velocity of a ball that has been dropped from rest with linear drag. Obtain an approximate expression for v_y(t) in the case of small drag. Include terms to 2nd order (Feel free to use the front flyleaf to save some grief!) What is the physical meaning of the leading nonzero term in your approximate solution? What is your interpretation of the *sign* of the next term? As in the previous question, what exactly do we mean by "small drag" here - what exactly is small, compared to what, here?



2. Danny Jr. (mass m_d) is standing at one edge of a 4 m long canoe (mass M_c) (See Fig. 1). He observes a turtle on top of a rock standing close to the other edge of the canoe. Danny wants to catch the turtle and starts walking towards it. Ignoring water friction:

(a) Qualitatively describe the motion of the system (canoe+ Danny) as he walks forward. If initially the canoe is 3m away from the dock, where is Danny with respect to the dock when he reaches the other end of the canoe? (We want a formula in terms of m_d, M_c, and the given starting dimensions)

(b) If $m_d = 30$ kg and Danny can stretch his arm 1 m away from the canoe's edge, what is the minimum mass the canoe must have to ensure that he can catch the turtle? (Also, what happens in the limit of a very light canoe? A very massive canoe?)

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3. Exam review! (The rest of this homework is required, and you should turn it in. But, for obvious reasons, the grader will not look over what you write in *detail*, each part will be graded simply "credit/no credit") Our exams will likely be 4 or 5 questions (each multiple parts), so we figure ∼15-20 min/question. For the question below: if you have a way to scan or input your work - look for the discussion forum for this problem on our D2L site and upload it! (This isn't required, but would be helpful.)

On our website, in the syllabus, after our "Introduction" paragraph, there is a link to our course goals. Go there, read that page. Then -

(a) Find in the textbook (Taylor, or one of the "recommended" texts available in the Physics library reserve), or invent a plausible exam question covering material we have covered so far this term (You might also explicitly think about what we've covered in Boas, too - can you "hook that in"?) Write it down.

(b) Solve it. (Include your solution with your homework) (If it takes $<\approx 5$ min, how could we have made it a little more interesting/richer/challenging? If it takes $>\approx 20$ min, how could we scaffold/hint/simplify to get at the interesting physics, without making it tedious/grungy?)

(c) Write a brief (just a few words) summary which characterizes what *content* this problem covers (e.g, "position vectors in polar-coordinates", or "Newton's law in tilted Cartesian coordinates", or etc)

(d) Which of the course scale learning goals does your problem address (list them by number) See http://www.colorado.edu/physics/phys2210/phys2210_sp12/course_goals.html

4. Get together with at least one other person from class, and share your made-up exam question. Do theirs, and discuss with them whether you think they got the level and coverage right. (If you can't find someone in person, use our D2L forum which I created for this purpose - but don't wait till the last minute, get this arranged with plenty of time) To turn in to us:

(a) Write down the name(s) of the person you worked with

(b) To show us that you really did this activity (yes, this IS a homework problem, for credit!), write down in your homework the problem of theirs that you did. (If it's really long, paraphrase)