Homework 7

(Due Date: Start of class on Thurs. Mar 1)

1. Electrical force is given by $\vec{F} = q\vec{E}$, where $\vec{E}$ is the electric field. Electric force is conservative in "electrostatic" situations (but is *not* conservative in all situations!)

(a) Consider a laboratory setup in which a charge q sits in a field $\vec{E} = (z^2 + 1)\hat{k}$. Prove that the resulting force is conservative, and deduce a formula for the potential energy as a function of position of this charge

(b) You release the charge (it has mass m) at the origin, starting from rest. Describe its motion qualitatively. How fast is it going when it reaches a distance h from the starting point? If the object was in the shape of a bead (*same* mass m and charge q) threaded on a curved, frictionless, rigid wire which started at the origin $(0, 0, 0)$ and ended at some point (x_0, y_0, h) would it also have that same speed you just calculated, or not? (Why?)

2. Electrical potential energy is given by $U = qV$, where V is the voltage. Consider a charge q moving in a region of space where the voltage is approximated by the formula $V(x, y, z) = c(2xz + x^2)$ (Assume the constant c has appropriate units so U is in Joules when x,y, z are in meters)

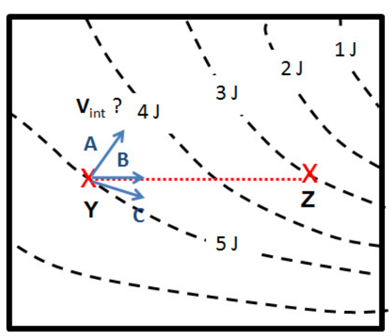
(a) Find the force on the charge q at the origin, and the point (1,1,1)

(b) Does this situation produce a conservative electrical force? Clearly justify your answer.

(c) For an arbitrary scalar function f(x, y, z), evaluate the components of $\vec{\nabla} \times \vec{\nabla} f$ in Cartesian coordinates and show that the result is 0. Does the formal result relate to (b)? (If not, why not? If so, briefly comment)

(d) Explicitly compute the work done by the force in part a on a charge q as you follow a path from the origin (0,0,0) to (1,1,1) along a direct, straight line path. Then, check your answer very simply using the potential function given. Explain the idea of your check.

3. The diagram below shows a region of space. The dashed curves indicate positions of equal potential and are labeled with the value of the potential at that curve. Three vectors originating from point Y are also shown in blue. The vector B points directly from point Y to Z. If you want a particle to be launched from Y and reach Z, which vector represents best the initial velocity? Explain your reasoning. Then, make a (very crude, no calculations required!) sketch/guess of the *path* you expect the particle to follow if launched from point Y with initial **v** given by vector A. (Explain your reasoning, briefly, so we know what you were thinking about)



4. Each diagram in Fig 2 (next page) depicts a force field in a region of space.

(a) For which force fields can you identify a closed path over which $\oint \vec{F} \cdot d\vec{l} \neq 0$? For each such case, clearly indicate an appropriate path on the diagram.

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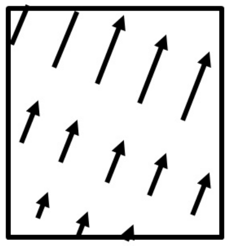
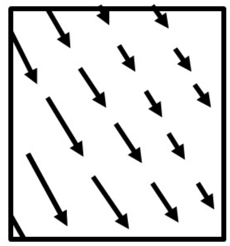
(b) For each case indicate if $\nabla \times \vec{F} = 0$ everywhere in the box? Also, for each case, could the force depicted in the diagram be conservative? Briefly, explain.

(c) For each case, is it possible to draw a self-consistent set of equipotential contours for that situation? If so: Draw them! (Each drawing should clearly show the correct shape of the contour lines, the correct relative spacing of the contours, and label the regions that correspond to highest and lowest potential energy) If not: Explain why drawing such contours is impossible.



Case 1



Case 2

5. A driver traveling *downhill* slams on the brakes and skids 40 m on before hitting a parked car. You have been hired as a physics expert to help the insurance investigators decide if the driver had been traveling faster than the 25 MPH speed limit at the start of this event. The slope of the hill is 5 degrees. Assuming braking friction has the usual form μN, what is the "critical value" of μ for which you would conclude the driver was speeding? Can you convince the investigators this driver was speeding, or do you need more information? (Do you need to know the mass of either vehicle? Road conditions?) While there are multiple ways to solve this problem, *please solve it using work and energy*

6. A simple pendulum consists of a point mass m fixed to the end of a massless rod (length l), whose other end is pivoted from the ceiling to let it swing freely in the vertical plane. The pendulum's position can be identified simply by its angle ϕ from the equilibrium position.

(a) Write the equation of motion for ϕ using Newton's second law. Assuming that the angle ϕ remains small throughout the motion, solve for $\phi(t)$ and show that the motion is periodic. What is the period of oscillation?

(b) Show that the pendulum's potential energy (measured from the equilibrium level) is $U(\phi) = A(1 - \cos\phi)$. Find A in terms of m, g and l. Then, write down a formula for the *total energy* as a function of ϕ and $\dot{\phi}$.

(c) Show that by differentiating the energy with respect to t you can recover the equation of motion you found in (a). (What basic principle of physics are you using, here?)

(d) Picking simple values for parameters (e.g, $m = l = 1, g = 10$), use your favorite computational environment (e.g., NDSolve in Mathematica) to solve for $\phi(t)$ given a fairly small starting angle ϕ_0. Provide output that clearly shows that the period of oscillation is very close to the theoretical prediction from part d. Repeat with $\phi_0 = 2$ rad (which is quite large), and show that the period now deviates from ideal. (Does the period get *larger* or *smaller* as ϕ_0 increases?)

(e) Using your numerical solution to part (d) and your formulas for energy from part (b), generate a single plot which graphs KE, U, and total E as a function of time, for one period, in the case where $\phi_0 = 2$ rad is NOT small. Briefly, comment! (Do we still conserve energy when we cannot use the small angle approximation any more?)